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• **Supplementary File** •

Event-Triggered Tracking Control for a Class of Nonlinear Systems Based on High-Order Fully Actuated System Theory

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Appendix A Proof of Theorem 1

To facilitate the proof of Theorem 1, we present the following lemma:

Lemma 1. [1] For any constant $\mu > 0$, there are positive, definite matrix matrices $A_i \in R^{r\times r}$ satisfying the following condition:

$$\mathrm{Re}\lambda_i(\Phi(A^{0\sim n-1})) < -\frac{\mu}{2}.$$

There also exists a positive definite matrix $P(A^{0\sim n-1})$, satisfying

$$\Phi^T(A^{0\sim n-1})P(A^{0\sim n-1}) + P(A^{0\sim n-1})\Phi(A^{0\sim n-1}) \leqslant -\mu P(A^{0\sim n-1}),$$

where $P(A^{0\sim n-1}) = [P_1, P_2, \cdots, P_n]$, $P_i \in R^{nr\times r}$.

For convenience, $e^{(1\sim n)}(t)$, $e^{(0\sim n-1)}(t)$, $z^{(1\sim n)}(t)$, $z^{(0\sim n-1)}(t)$, $H(z^{(0\sim n-1)})$, and $L(z^{(0\sim n-1)})$ are simplified to $e^{(1\sim n)}$, $e^{(0\sim n-1)}$, $z^{(1\sim n)}$, $z^{(0\sim n-1)}$, H, and L, respectively. Based on the above, the proof is derived as follows:

Proof: Choose the following Lyapunov function:

$$V(t) = \frac{1}{2}(e^{(0\sim n-1)})^T Pe^{(0\sim n-1)} + \frac{1}{2}\tilde{\theta}^T(t)\tilde{\theta}(t) + \frac{1}{2}\mathrm{tr}[\tilde{W}^T(t)\tilde{W}(t)]. \tag{A1}$$

Taking derivative leads to:

$$\begin{aligned}\dot{V}(t) =& \frac{1}{2}(e^{(1\sim n)})^T Pe^{(0\sim n-1)} + \frac{1}{2}(e^{(0\sim n-1)})^T Pe^{(1\sim n)} + \dot{\tilde{\theta}}^T(t)\tilde{\theta}(t) + \mathrm{tr}[\tilde{W}^T(t)\dot{\tilde{W}}(t)]\\
=& \frac{1}{2}\Big\{\Lambda(k^{(0\sim n-1)})e^{(0\sim n-1)} - B\{H^T\tilde{\theta}(t) + [\Delta f(z^{(0\sim n-1)}) - \hat{W}^T(t)S(z^{(0\sim n-1)})] - \lambda(t)Lm + Lv_3(t)\}\Big\}^T\\
&\times Pe^{(0\sim n-1)} + \frac{1}{2}(e^{(0\sim n-1)})^T P\Big\{\Lambda(k^{(0\sim n-1)})e^{(0\sim n-1)} - B\{H^T\tilde{\theta}(t) - [\Delta f(z^{(0\sim n-1)})\\
&- \hat{W}^T(t)S(z^{(0\sim n-1)})] - \lambda(t)Lm + Lv_3(t)\}\Big\} + \dot{\tilde{\theta}}^T(t)\tilde{\theta}(t) + \mathrm{tr}[\tilde{W}^T(t)\dot{\tilde{W}}(t)]\\
=& \frac{1}{2}(e^{(0\sim n-1)})^T(\Lambda^T(k^{(0\sim n-1)})P + P\Lambda(k^{(0\sim n-1)}))e^{(0\sim n-1)}\\
&+ (e^{(0\sim n-1)})^T P\Big\{-B\{H^T\tilde{\theta}(t) + [\Delta f(z^{(0\sim n-1)}) - \hat{W}^T(t)S(z^{(0\sim n-1)})] - \lambda(t)Lm + Lv_3(t)\}\Big\}\\
&+ \dot{\tilde{\theta}}^T(t)\tilde{\theta}(t) + \mathrm{tr}[\tilde{W}^T(t)\dot{\tilde{W}}(t)].\end{aligned} \tag{A2}$$

Based on the condition $\mathrm{Re}\lambda_i(\Lambda(k^{(0\sim n-1)})) < -\frac{\mu}{2}$ and Lemma 1, there exists the matrix $P > 0$, which makes the following inequality hold:

$$\frac{1}{2}(e^{(0\sim n-1)})^T\big\{\Lambda^T(k^{(0\sim n-1)})P + P\Lambda(k^{(0\sim n-1)})\big\}e^{(0\sim n-1)} \leqslant -\frac{\mu}{2}(e^{(0\sim n-1)})^T Pe^{(0\sim n-1)}.$$

Therefore, $\dot{V}(t)$ can be simplified as the following form:

$$\dot{V}(t) \leqslant -\frac{\mu}{2}(e^{(0\sim n-1)})^T Pe^{(0\sim n-1)} + \dot{V}_1(t) + \dot{V}_2(t) + \dot{V}_3(t), \tag{A3}$$

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where

$$\begin{aligned}
\dot{V}_1(t) =&[-(e^{(0\sim n-1)})^T PBH^T + \dot{\tilde{\theta}}^T(t)]\tilde{\theta}(t),\\
\dot{V}_2(t) =&-(e^{(0\sim n-1)})^T PB[\Delta f(z^{(0\sim n-1)}) - \hat{W}^T(t)S(z^{(0\sim n-1)})] + \mathrm{tr}[\tilde{W}^T(t)\dot{\hat{W}}(t)],\\
\dot{V}_3(t) =&-(e^{(0\sim n-1)})^T PB[\lambda(t)Lm + Lv_3(t)].
\end{aligned}$$

Next, deal with the result of the derivatives. Adding and subtracting $\frac{\mu}{2}\tilde{\theta}^T(t)\tilde{\theta}(t)$ into $\dot{V}_1(t)$, further utilizing $\xi(t) = \tilde{\theta}_{r0} - \tilde{\theta}_{e0} - \tilde{\theta}(t) = 0$ yields:

$$\begin{aligned}
\dot{V}_1(t) =&[-(e^{(0\sim n-1)})^T PBH^T + \dot{\tilde{\theta}}^T(t)]\tilde{\theta}(t) - \frac{\mu}{2}\tilde{\theta}^T(t)\tilde{\theta}(t) + \frac{\mu}{2}\tilde{\theta}^T(t)\tilde{\theta}(t) + (\mu+1)\xi^T(t)\tilde{\theta}(t)\\
=&[-(e^{(0\sim n-1)})^T PBH^T + \dot{\tilde{\theta}}_{r0}^T(t) - \dot{\tilde{\theta}}_{e0}^T(t)]\tilde{\theta}(t) - \frac{\mu}{2}\tilde{\theta}^T(t)\tilde{\theta}(t) + \frac{\mu}{2}\tilde{\theta}^T(t)\tilde{\theta}(t)\\
&+ (\mu+1)(\tilde{\theta}_{r0}^T(t) - \tilde{\theta}_{e0}^T(t) - \tilde{\theta}^T(t))\tilde{\theta}(t)\\
=&[-(e^{(0\sim n-1)})^T PBH^T - \dot{\tilde{\theta}}_{e0}^T(t) - (\mu+1)\tilde{\theta}_{e0}^T(t)]\tilde{\theta}(t) + \dot{\tilde{\theta}}_{r0}^T(t)\tilde{\theta}(t) - \frac{\mu}{2}\tilde{\theta}^T(t)\tilde{\theta}(t) + \frac{\mu}{2}\tilde{\theta}^T(t)\tilde{\theta}(t)\\
&+ (\mu+1)(\tilde{\theta}_{r0}^T(t) - \tilde{\theta}^T(t))\tilde{\theta}(t). \qquad \text{(A4)}
\end{aligned}$$

Therefore, using the adaptive control law (6) and Assumption 2, we have

$$\begin{aligned}
\dot{V}_1(t) =&\dot{\tilde{\theta}}_{r0}^T(t)\tilde{\theta}(t) - \frac{\mu+2}{2}\tilde{\theta}^T(t)\tilde{\theta}(t) + (\mu+1)\tilde{\theta}_{r0}^T(t)\tilde{\theta}(t) - \frac{\mu}{2}\tilde{\theta}^T(t)\tilde{\theta}(t)\\
\leqslant&\frac{1}{2}\|\dot{\tilde{\theta}}_{r0}(t)\|\|\tilde{\theta}(t)\| - \frac{\mu+2}{2}\|\tilde{\theta}(t)\|^2 + \frac{(\mu+1)}{2}\|\tilde{\theta}_{r0}(t)\|\|\tilde{\theta}(t)\| - \frac{\mu}{2}\tilde{\theta}^T(t)\tilde{\theta}(t)\\
\leqslant&\frac{1}{2}[\delta_1^2 + \|\tilde{\theta}(t)\|^2 + (\mu+1)(\delta_0^2 + \|\tilde{\theta}(t))\|^2)] - \frac{\mu+2}{2}\|\tilde{\theta}(t)\|^2 - \frac{\mu}{2}\tilde{\theta}^T(t)\tilde{\theta}(t)\\
\leqslant&\frac{1}{2}[\delta_1^2 + (\mu+1)\delta_0^2] - \frac{\mu}{2}\tilde{\theta}^T(t)\tilde{\theta}(t). \qquad \text{(A5)}
\end{aligned}$$

Here, the following inequality holds:

$$\begin{aligned}
\dot{V}_2(t) =&(e^{(0\sim n-1)})^T PB[-\tilde{W}^T(t)S(z^{(0\sim n-1)}) - \varepsilon^*(z^{(0\sim n-1)})] - \mathrm{tr}[\tilde{W}^T(t)\dot{\hat{W}}(t)]\\
=&-(e^{(0\sim n-1)})^T PB\varepsilon^*(z^{(0\sim n-1)}) + \sigma_1\mathrm{tr}[\tilde{W}^T(t)(W^* - \tilde{W}(t))]\\
\leqslant&\frac{1}{2\sigma_2}(e^{(0\sim n-1)})^T e^{(0\sim n-1)} + \frac{\sigma_2}{2}\|PB\|^2 + \frac{\sigma_2\bar{\varepsilon}}{2} + \sigma_1\mathrm{tr}(\frac{W^{*T}W^*}{2}) - \frac{\sigma_1}{2}\mathrm{tr}(\tilde{W}^T\tilde{W}), \qquad \text{(A6)}
\end{aligned}$$

where σ_2 is an adjusted parameter. Note that in the property of hyperbolic tangent function, the following equality holds:

$$0 \leqslant\mid \varrho \mid -\varrho\tanh(\frac{\varrho}{\varepsilon}) \leqslant 0.2785\varepsilon,$$

where $\varepsilon > 0$ and $\varrho \in R$. We can get

$$\begin{aligned}
\dot{V}_3(t) =&-(e^{(0\sim n-1)})^T PB\{\lambda(t)Lm + L\bar{m}\tanh\frac{L^TB^TPe^{(0\sim n-1)}\bar{m}}{\epsilon}\}\\
\leqslant&\mid (e^{(0\sim n-1)})^T PBL\bar{m} \mid -(e^{(0\sim n-1)})^T PBL\bar{m}\tanh\frac{L^TB^TPe^{(0\sim n-1)}\bar{m}}{\epsilon}\\
\leqslant&0.2785\epsilon. \qquad \text{(A7)}
\end{aligned}$$

Combining the above results (A1)-(A7) yields:

$$\begin{aligned}
\dot{V}(t) \leqslant&-\frac{\mu}{2}(e^{(0\sim n-1)})^T Pe^{(0\sim n-1)} + \frac{1}{2}[\delta_1^2 + (\mu+1)\delta_0^2] - \frac{\mu}{2}\tilde{\theta}^T(t)\tilde{\theta}(t)\\
&+ \frac{1}{2\sigma_2}(e^{(0\sim n-1)})^T e^{(0\sim n-1)} + \frac{\sigma_2}{2}\|PB\|^2 + \frac{\sigma_2\bar{\varepsilon}}{2} + \sigma_1\mathrm{tr}(\frac{W^{*T}W^*}{2}) - \frac{\sigma_1}{2}\mathrm{tr}(\tilde{W}^T\tilde{W}) + 0.2785\epsilon\\
\leqslant&-\beta V(t) + C, \qquad \text{(A8)}
\end{aligned}$$

where $\beta = \min\Big\{\mu - \frac{1}{\sigma_2\lambda_{max}(P)}, \sigma_1\Big\}$, $C = \frac{1}{2}[\delta_1^2 + (\mu+1)\delta_0^2] + \frac{\sigma_2}{2}\|PB\|^2 + \frac{\sigma_2\bar{\varepsilon}}{2} + \sigma_1\mathrm{tr}(\frac{W^{*T}W^*}{2})$. To ensure $\beta > 0$, we chose $\mu - \frac{1}{\sigma_2\lambda_{max}(P)} > 0$ when designing the parameters.

From inequality (A8) and the Lyapunov stability, it was verified that the tracking error of the closed-loop system is uniformly ultimately bounded, which indicated that the tracking error ultimately converges to nearly zero.

In the following, we will verify that the proposed event-triggered mechanism is Zeno-free. Recalling $\xi(t) = \omega(t) - u(t)$ for $t \in [t_k, t_{k+1})$, we have: $\frac{d|\xi|}{dt} = \frac{d}{dt}(\xi * \xi)^{\frac{1}{2}} = \mathrm{sign}(\xi)\xi \leqslant |\dot{\omega}|$. Noting that $f(\cdot)$, $H(\cdot)$, and $L(\cdot)$ have the first-order continuous partial derivatives, we find that $\dot{\omega}$ is continuous. Additionally, because all signals of the closed-loop system are bounded, there must exist a positive constant, κ, such that $|\dot{\omega}| \leqslant \kappa$. By noting that $\xi(t_k) = 0$ and $\lim\xi(t)_{t\to t_{k+1}} = m$, we obtain the lower boundaries of inter-execution intervals t^*, which must satisfy $t^* \geqslant m/\kappa > 0$. Therefore, Zeno behavior is avoided.

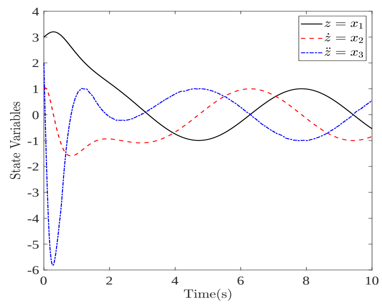

Figure B1 Responses of z, $\dot{z}$ and $\ddot{z}$.

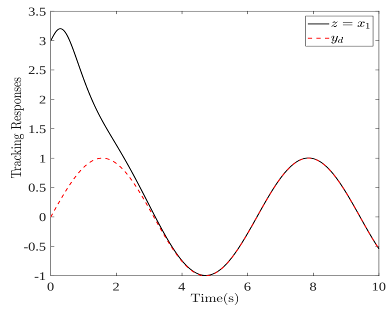

Figure B2 Tracking responses.

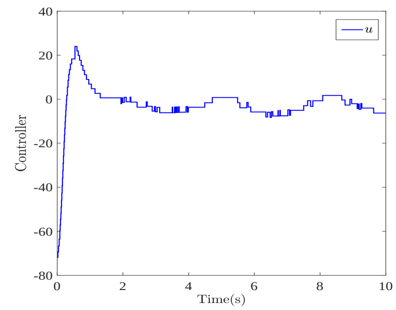

Figure B3 Trajectory of u.

Appendix B Simulation Studies

In this section, two examples are used to verify the effectiveness of the proposed method.
Example 1. In this example, we consider the following system:

$$\begin{cases} \dot{x}_1 = \theta x_1^2 + x_2 \\ \dot{x}_2 = x_3 \\ \dot{x}_3 = u \end{cases}, \tag{B1}$$

which was presented in [2]. Duan [3] converted the parameter θ into a time-varying parameter. Therefore, the following form is presented:

$$z^{(3)} = 2(\dot{z}^2 + z\ddot{z})\theta(t) + \Delta f(z^{(0\sim2)}) + Lu(t), \tag{B2}$$

where $\theta(t) = 2 + 0.2\sin(0.1t)$, $\Delta f(z^{(0\sim2)}) = 0.01(3z^2 + 2z\dot{z} + 98)$, and $L = 1$. The initial values are arranged as $[z, \dot{z}, \ddot{z}, \hat{\theta}(0), \theta_0(t), \dot{\hat{\theta}}(0)] = [3, 1, 2, -1, 1, 2]$. Based on [4], to make the matrix $\Lambda(k^{(0\sim2)})$ have the eigenvalues $-a \pm jb$ and $-c$, we chose $a = 2, b = 1, c = 4$ for $K = [c(a^2 + b^2), a^2 + 2ca + b^2, 2a + c]^T$. After combining this with Lemma 1 and the aforementioned conditions, the parameter vector K and the positive definite symmetric matrix P were chosen as

$$K = \begin{bmatrix} 20 \\ 21 \\ 8 \end{bmatrix}, \qquad P = \begin{bmatrix} 18.3235 & 9.6176 & 0.9412 \\ 9.6176 & 8.5662 & 0.8897 \\ 0.9412 & 0.8897 & 0.1985 \end{bmatrix}.$$

In addition, other parameters were selected as $\mu = 2$, $\sigma = 0.01$, $\epsilon = 0.01$, $m = 2$, $\bar{m} = 2.5$, and $y_d(t) = \sin(t)$. The simulation results are shown in Figures B1, B2, and B3, which illustrate the effectiveness of the controller scheme proposed in this article. Figure B2 shows the trajectory of tracking responses, which demonstrates that the proposed method has good tracking performance. Figure B3 displays the trajectory of the controller, in which event-triggered strategy is significant in practical production, especially in energy conservation.

Example 2. In this example, the RLC circuit system was used to verify the effectiveness of the proposed method. Based on the Kirchhoff's Law of Voltage and Current, the RLC circuit can be modeled as

$$LC\ddot{x} + \Delta(x, \dot{x}) + RC\dot{x} = u,$$

where $x = u_c$V, $\dot{x} = \frac{du_c}{dt}$V/t, $\ddot{x} = \frac{d^2u_c}{dt^2}$V/t^2, $u = u_r$V. Other parameters were chosen as $L = 0.5$H, $C = 1$F, $R = 0.5\Omega$, $\Delta(x, \dot{x}) = -(0.25\sin(x^2) + 0.5\dot{x}^2)$. The above expression can be transferred into the following HOFA form:

$$\ddot{z} = f(z^{(0\sim1)}(t)) + \Delta f(z^{(0\sim1)}(t)) + H^T(z^{(0\sim1)}(t))\theta(t) + L(z^{(0\sim1)}(t))u(t),$$

where $f(z^{(0\sim1)}(t)) = -2\dot{z}$, $\Delta f(z^{(0\sim1)}(t)) = 0.5\sin(z^2) + \dot{z}^2$, $H(z^{(0\sim1)}(t)) = 2z$, $\theta(t) = 1$, $L(z^{(0\sim1)}(t)) = 2$. The objective of this example is for all signals of the closed-loop system to approach zero. Therefore, we set the desired output as $y_d = 0$. The initial values were arranged as $[z, \dot{z}, \hat{\theta}(0), \theta_0(t), \dot{\hat{\theta}}(0)] = [20, -3, -1, 1, 0]$. Figure B4 shows the triggering times, from which it is apparent that the controller is triggered at different times. The state responses are shown in Figure B5, which shows that all signals of the closed-loop system approach zero. Figure B6 shows that the parameter estimation error $\tilde{\theta}$ converges to zero, which reflects the good performance of the proposed method.

This simulation was run on a version of MATLAB R2018b with Simulink 9.2. Based on the program tic-operation-toc, the time required for the code's execution was obtained as 1.1 s and 11.8 s based on an asymmetric tan-type barrier Lyapunov function in [5], respectively. In contrast, the method based on the HOFA significantly reduces the computational burden.

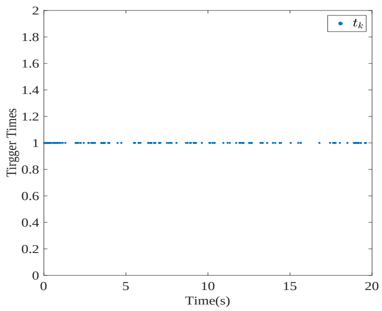

Figure B4 Trigger times.

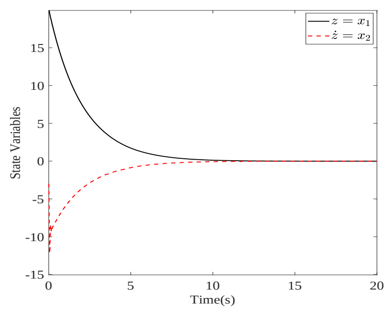

Figure B5 Responses of z, $\dot{z}$.

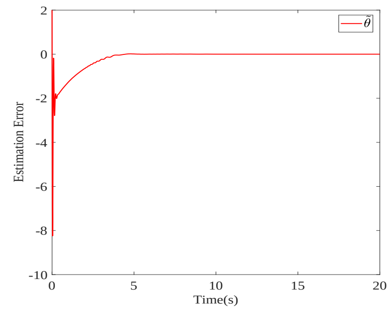

Figure B6 Trajectory of $\tilde{\theta}$.

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