• Supplementary File •

Machine learning empowered UAV-based beamforming design in ISAC systems

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Appendix A ML-based algorithm for beamforming design

To further exploit the characteristics of the input, we map the complex-valued input to real-valued parts as

$$\tilde{\boldsymbol{\Psi}}_{n}^{\tau} = \mathcal{M}([\operatorname{Re}\{\boldsymbol{\Psi}_{n}^{\tau}\}, \operatorname{Im}\{\boldsymbol{\Psi}_{n}^{\tau}\}]), \tag{A1}$$

where $\mathcal{M}(\cdot)$ denotes the mapping function. The term $\Psi_n^{\tau} = [\mathbf{H}_{n-1}, \dots, \mathbf{H}_{n-\tau}]$ is the historical channel information, where $\mathbf{H}_n = [\mathbf{h}_{0,n}, \mathbf{h}_{1,n}, \dots, \mathbf{h}_{K,n}]$ encompasses the channel information of all users and the target. At the output of the network, we map the real-valued parts back to the complex-valued input as

$$\mathbf{W}_n = \mathcal{M}'(g_\omega(\tilde{\mathbf{\Psi}}_n^\tau)),\tag{A2}$$

where $\mathcal{M}'(\cdot)$ represents the mapping function that generates the complex-valued beamforming matrix. $g_{\omega}(\cdot)$ denotes the ED network with trainable parameters ω .

The ML-based method comprises two main phases: offline training and online estimation. In the offline training phase, we transform the unconstrained optimization problem, into a neural network loss function, i.e.,

$$J(\boldsymbol{\omega}) = \frac{1}{N_s} \sum_{i=1}^{N_s} \text{CRLB}\left(d_{k,n}^{(i)}, \mathbf{w}_{k,n}^{(i)}(\boldsymbol{\omega})\right)$$

+ $\lambda_1 \left\{ \max\left(0, R_t - \frac{1}{N_s} \sum_{i=1}^{N_s} \sum_{k=1}^K R_k(\mathbf{h}_{k,n}^{(i)}, \mathbf{w}_{k,n}^{(i)}(\boldsymbol{\omega}))\right) \right\}$
+ $\lambda_2 \left\{ \max\left(0, \frac{1}{N_s} \sum_{i=1}^{N_s} \left\| \mathbf{W}_n^{(i)}(\boldsymbol{\omega}) \right\|_F^2 - P_t \right) \right\},$ (A3)

where $\boldsymbol{\omega}$ denotes a set containing all the parameters of the neural network and $\mathbf{w}_{k,n}^{(i)}$ denotes the *i*-th element of $\mathbf{W}_{k,n}^{(i)}$ generated by the EDnet. The operator $\max(\cdot, \cdot)$ is the maximum operation implemented by a rectified linear (ReLU) function [2], and N_s is the sample size. Accordingly, the EDnet can perform back propagation (BP) to optimize parameters $\boldsymbol{\omega}$ for a well-trained model.

The training set is generated by employing the Monte Carlo method, as given by

$$(\Psi_n^{\tau}, \mathbf{H}_n)_{N_s} = \{(\Psi_n^{\tau(1)}, \mathbf{H}_n^{\tau(1)}), \dots, (\Psi_n^{\tau(N_s)}, \mathbf{H}_n^{\tau(N_s)})\},$$
(A4)

where $(\Psi_n^{(i)}, \mathbf{H}_n^{(i)})$ is the *i*-th training example of the proposed network with $\Psi_n^{(i)}$ being estimated historical channels from time slot $n - \tau$ to n - 1, implicitly including sensing information of interest, i.e., distance. Based on the above, the beamforming design objective can be reformulated as the minimization of the function stipulated in (A3). Consequently, EDnet adopts the stochastic gradient descent algorithm (e.g., Adam [1]) to optimize the parameters $\boldsymbol{\omega}$ for training the model. Based on universal approximation theorems [1], it is possible to optimize the parameters to their optimal values, $\boldsymbol{\omega}_{opt}$, given a sufficiently large training set.

In the online estimation phase, the test dataset $(\dot{\Psi}_n^{\tau}, \dot{\mathbf{H}}_n)$ representing historical vehicle information is fed to the neural network. As a result, the process of online estimation can be expressed as

$$\dot{\mathbf{F}}_n = \mathcal{M}'(g_\omega(\dot{\mathbf{\Psi}}_n^{\tau(i)})). \tag{A5}$$

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Figure B1 The comparison of CRLB performance with various baselines.



Figure B2 The CRLB performance comparison with different antenna numbers.

Appendix B Simulation results

The system parameters are set as follows: $\sigma_u^2 = \sigma_v^2 = -80$ dBm, $N_t = N_r = 32$, and K = 2. For motion parameters, the UAV's position during an ISAC frame is fixed at $\mathbf{v}'_n = [0\,\mathrm{m}, 0\,\mathrm{m}]$, while the target's initial position is given by $\mathbf{v}_n = [x_0 + \Delta x, v_0 + \Delta v]$, where $\Delta x \sim \mathcal{N}(0, 1)$ and $\Delta y \sim \mathcal{N}(0, 1)$ represent random variables. The stationary users¹⁾ are positioned at $[10\,\mathrm{m}, 15\,\mathrm{m}]$ and $[20\,\mathrm{m}, 25\,\mathrm{m}]$, respectively. Within the observation model, the matched-filtering gain is $G_m = 10$, the rate threshold is set as $R_t = 2$ bits/Hz, with both ρ_1 and ρ_2 set to 10^3 . We employ the PyTorch deep learning framework to implement EPDnet. The network is trained using the Adam optimizer with an initial learning rate of 10^{-3} . To enhance training stability, the learning rate is multiplied by 0.95 when fluctuations in the training loss are observed. Additionally, the weight decay in the optimizer is set to 10^{-3} . The training set is generated using the Monte Carlo method with a sample size of 8,000.

To assess the performance of the proposed method for beamforming design, we conduct a comparative analysis with two benchmarks. As shown in Fig. B1, we compare EDnet against both random beamforming and a perfect beamforming design method. It is evident that random beamforming performs poorly due to the misalignment between the target and the UAV. In contrast, EDnet demonstrates satisfactory sensing performance, achieving a CRLB value for distance that closely approaches that of the perfect beamforming method. This enhancement is due to EDnet's capability to effectively map historical channel information to the desired beamforming matrix. Additionally, we observe that sensing performance degrades as the communication rate threshold increases. This outcome is expected, as the objectives of the sensing task and the communication task are not fully aligned within the ISAC system.

To investigate the impact of antenna numbers on sensing performance, we evaluate the CRLB values across different transmit powers and antenna configurations. As illustrated in Fig. B2, the CRLB values consistently decrease as the transmit power increases, eventually saturating in the high-power region. Specifically, when $N_t = N_r = 16$, the CRLB decreases from 9.21×10^{-2} to 3.86×10^{-7} , stabilizing around 3×10^{-7} . This is attributed to the enhanced received SNR at the UAV, which mitigates noise effects and enables more accurate angle estimation. However, when the number of antennas is insufficient (e.g., $N_t = N_r = 4$), the system exhibits degraded performance due to limited beamforming capability and increased interference between beams, which hinders accurate target localization.

References

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- 2 Zhang, X., Liu, C., Yuan, W., Zhang, J.A., Ng, D.W.K.: Sparse prior-guided deep learning for OTFS channel estimation. IEEE Trans. Veh. Techn. (2024)

¹⁾ In mobile user scenarios, tracking user trajectories is essential, necessitating optimization of the network architecture. Note that such optimization does not significantly affect overall system performance, as the system's performance is predominantly determined by the received SINR at the users.