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Robust adaptive distributed optimization for heterogeneous unknown second-order nonlinear multiagent systems

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The distributed optimization problem has drawn significant attention recently since it has a variety of applications in the economic dispatch of smart grids, regression of distributed data, optimal coordination of mobile robot systems (MRSs), and so on. Different from the traditional leaderless consensus and consensus tracking problems in multiagent systems (MASs), the typical distributed optimization problems are not only required to achieve consensus but also to cooperatively minimize a global cost function [1], where the global cost function is the sum of local cost functions and each local cost function is only known to the agent itself.

The scholars initially focused on designing step-size-based discrete-time algorithms to solve the distributed optimization problem [2]. As a counterpart, many practical physical systems usually describe continuous-time dynamics. For example, MRSs, unmanned surface vessel (USV) systems, pendulum systems, and Euler-Lagrange systems are usually described by continuous second-order systems with heterogeneous unknown nonlinear dynamics, disturbances, and unavailable parameters [3]. The asymptotic distributed convex optimization (ADCO) problems of MASs with continuous-time nonlinear dynamics are considered in [4, 5], while the parameters are available. When the upper bound parameters involving nonlinear functions and disturbances are unavailable, the ADCO problem of MASs with continuous-time second-order nonlinear dynamics is difficult and requires further study. We briefly summarize the motivation of this work here, and a more comprehensive literature review is given in Appendix A.

This work addresses the ADCO problem of a second-order MAS via robust adaptive control, where the heterogeneous unknown nonlinear functions and disturbances are considered. A modified command filter approach that obviates the need to compute an analytic derivative of the virtual velocity is applied to design a discontinuous robust adaptive distributed optimization algorithm. To avoid the chattering phenomenon caused by the proposed discontinuous algorithm, a continuous robust adaptive distributed optimization algorithm is further designed. Both the designed discontinuous and continuous algorithms solve the ADCO problem in a fully distributed fashion while guaranteeing strong system robustness.

Problem formulation. Consider that a heterogeneous unknown second-order nonlinear MAS over a graph $\mathcal{G}$ and each agent $i \in \mathbb{N} = \{1, 2, \ldots, N\}$ has a local cost function $f_i(x_i) : \mathbb{R}^n \to \mathbb{R}$ only available to itself. Each agent $i \in \mathbb{N}$ has the following continuous-time dynamics:

$$\dot{x}_i = v_i, \quad \dot{v}_i = \theta_i u_i + h_i(x_i, v_i) + d_i, \tag{1}$$

where $x_i \in \mathbb{R}^n$, $v_i \in \mathbb{R}^n$, and $u_i \in \mathbb{R}^n$ are respectively the position, velocity, and input of agent i, θ_i is a known nonzero constant, $h_i \in \mathbb{R}^n$ and $d_i \in \mathbb{R}^n$ are respectively the heterogeneous unknown nonlinearities and disturbances. This study aims to solve the ADCO problem formulated below.

ADCO problem. Design a fully distributed algorithm such that $\lim_{t\to\infty} \|x_i - x_*\| = 0, \forall i \in \mathbb{N}$, where $x_* \in \mathbb{R}^n$ is the minimizer of the following optimization problem:

$$\min_{x\in\mathbb{R}^n} \quad J = \sum_{i=1}^{N} f_i(x), \tag{2}$$

where x is the common state, J is the global cost function.

Some assumptions are given to solve the ADCO problem.

Assumption 1. The graph $\mathcal{G}$ among N agents is a connected undirected graph.

Assumption 2. There exist three constants ρ_i, ϱ_i, $\varsigma_i \geqslant 0$ such that $\|h_i\| \leqslant \rho_i + \varrho_i\|\varphi_i(x_i, v_i)\|$ and $\|d_i\| \leqslant \varsigma_i$, where the function $\varphi_i \in \mathbb{R}^n$ is available to the agent i only, and φ_i is bounded if x_i and v_i are bounded, $\forall i \in \mathbb{N}$.

Assumption 3. Each local cost function $f_i(x_i)$ is twice times continuously differentiable with respect to x_i. There exists $g_i(x_i) = [\nabla^2 f_i(x_i)]^{-1}\nabla f_i(x_i)$ in the form of $g_i(x_i) =$

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$\zeta x_i + \vartheta_i \phi_i(x_i)$, where the Hessian matrix $\nabla^2 f_i(x_i)$ is invertible, ζ and ϑ_i are nonnegative constants, the function $\phi_i \in \mathbb{R}^n$ is available to the agent i only, $\nabla^2 f_i(x_i)$ and ϕ_i are bounded if x_i is bounded, $\forall i \in \mathbb{N}$.

The notations and some remarks about the above assumptions are obtained in Appendix B, as well as other preliminaries on graph theory and useful lemmas, which are obtained in Appendix C.

Command filter design. To solve the ADCO problem, we first introduce a virtual velocity v_i^*. Consider the command filter in the following form:

$$\begin{cases} \dot{z}_{i,1} = \pi_i z_{i,2}, \\ \dot{z}_{i,2} = -2\zeta_i \pi_i z_{i,2} - \pi_i(z_{i,1} - v_i^p), \end{cases} \tag{3}$$

where $v_i^* = z_{i,1}$ and $\dot{v}_i^* = \pi_i z_{i,2}$ are the outputs of each filter with the pseudocontrol signal v_i^p as the input, $\pi_i > 0$ and $\zeta_i \in (0, 1]$ are the filter design parameters, $i \in \mathbb{N}$.

Discontinuous controller design. For each $i \in \mathbb{N}$, let us define the following error variables:

$$\tilde{v}_i = v_i - v_i^*, \quad \bar{v}_i = v_i^* - v_i^p, \quad z_i = x_i - \mu_i, \tag{4}$$

where $\bar{v}_i$ and z_i are respectively the filtered error and compensated tracking error, and μ_i is the compensating signal used to eliminate the influence of the filtered error caused by the command filter. It follows from MAS (1) and the command filter (3) that $\dot{\tilde{v}}_i = \theta_i u_i - \pi_i z_{i,2} + h_i + d_i$. Let $u_i^* = \theta_i u_i - \pi_i z_{i,2}$ be the virtual input. Then the input

$$u_i = \frac{1}{\theta_i}\left(u_i^* + \pi_i z_{i,2}\right), \quad i \in \mathbb{N}. \tag{5}$$

Consider the following discontinuous virtual input for (5):

$$u_i^* = -l_i \tilde{v}_i - \alpha_i \psi_i \mathrm{sign}(\tilde{v}_i), \tag{6}$$

$$\dot{\alpha}_i = \gamma_{i1}\left(\psi_i \|\tilde{v}_i\|_1 - \sigma_{i1}(\alpha_i - \tilde{\alpha}_i)\right), \tag{7}$$

$$\dot{\tilde{\alpha}}_i = \delta_{i1}(\alpha_i - \tilde{\alpha}_i), \quad \alpha_i(0) \geqslant \tilde{\alpha}_i(0) > 0, \tag{8}$$

and the compensating signal μ_i in (4) is formulated as

$$\dot{\mu}_i = -k_i \mu_i - \|\bar{v}_i\| \mathrm{sign}(\mu_i) + \bar{v}_i \tag{9}$$

with the pseudocontrol signal v_i^p being designed as

$$v_i^p = -k_i \mu_i - \gamma_i e_{z_i} - \beta_i \chi_i \mathrm{sign}(e_{z_i}) - g_i(x_i), \tag{10}$$

$$\dot{\beta}_i = \gamma_{i2}\left(\chi_i \|e_{z_i}\|_1 - \sigma_{i2}(\beta_i - \tilde{\beta}_i)\right), \tag{11}$$

$$\dot{\tilde{\beta}}_i = \delta_{i2}(\beta_i - \tilde{\beta}_i), \quad \beta_i(0) \geqslant \tilde{\beta}_i(0) > 0, \tag{12}$$

where $e_{z_i} = \sum_{j \in \mathcal{N}_i} a_{ij}(z_i - z_j)$, $\psi_i = 1 + \|\varphi_i\|$, $\chi_i = 1 + \|\phi_i\|$, and $l_i, k_i, \gamma_i, \gamma_{i1}, \gamma_{i2}, \sigma_{i1}, \sigma_{i2}, \delta_{i1}, \delta_{i2}$ are positive design parameters, $i \in \mathbb{N}$.

Theorem 1. Under Assumptions 1–3, the discontinuous robust adaptive distributed optimization controller (5) with (3) and (6)–(12) solves the ADCO problem (2) of the second-order MAS (1). Each control signal u_i given by (5) is bounded; moreover, $\lim_{t\to\infty} \alpha_i(t) = \lim_{t\to\infty} \tilde{\alpha}_i(t) = \alpha_i^*$ and $\lim_{t\to\infty} \beta_i = \lim_{t\to\infty} \tilde{\beta}_i = \beta_i^*$, where α_i^* and β_i^* are two positive constants, for each $i \in \mathbb{N}$.

Proof. See Appendix D.

Continuous controller design. Consider the input u_i in (5) with the following continuous virtual input (each $i \in \mathbb{N}$):

$$u_i^* = -l_i \tilde{v}_i - \frac{\alpha_i \psi_i^2 \tilde{v}_i}{\psi_i \|\tilde{v}_i\| + p_i}, \tag{13}$$

$$\dot{\alpha}_i = \gamma_{i1}\left(\frac{\psi_i^2 \|\tilde{v}_i\|^2}{\psi_i \|\tilde{v}_i\| + p_i} - \sigma_{i1} p_i\right), \tag{14}$$

$$\dot{p}_i = -\delta_{i1}(\sigma_{i1} + 1) p_i, \quad \alpha_i(0) \geqslant \frac{\gamma_{i1}\sigma_{i1} p_i(0)}{\delta_{i1}(\sigma_{i1} + 1)} > 0, \tag{15}$$

and the compensating signal μ_i in (4) is formulated as

$$\dot{\mu}_i = -k_i \mu_i - \frac{\|\bar{v}_i\|^2 \mu_i}{\|\bar{v}_i\| \|\mu_i\| + o_i} + \bar{v}_i, \tag{16}$$

$$\dot{o}_i = -\delta_{i3} o_i, \quad o_i(0) > 0 \tag{17}$$

with the pseudocontrol signal v_i^p being designed as

$$v_i^p = -k_i \mu_i - \frac{\beta_i \chi_i^2 e_{z_i}}{\chi_i \|e_{z_i}\| + q_i} - g_i(x_i), \tag{18}$$

$$\dot{\beta}_i = \gamma_{i2}\left(\frac{\chi_i^2 \|e_{z_i}\|^2}{\chi_i \|e_{z_i}\| + q_i} - \sigma_{i2} q_i\right), \tag{19}$$

$$\dot{q}_i = -\delta_{i2}(\sigma_{i2} + 1) q_i, \quad \beta_i(0) \geqslant \frac{\gamma_{i2}\sigma_{i2} q_i(0)}{\delta_{i2}(\sigma_{i2} + 1)} > 0, \tag{20}$$

where $\delta_{i3} > 0$ and other parameters are the same as those in (6)–(12), $i \in \mathbb{N}$.

Similar to the derived main result in Theorem 1, we next derive another main result of this work.

Theorem 2. Under Assumptions 1–3, the continuous robust adaptive distributed optimization controller (5) with (3) and (13)–(20) solves the ADCO problem (2) of the second-order MAS (1). Each control signal u_i given by (5) is bounded and continuous everywhere; moreover, both α_i and β_i converge to some finite steady-state values asymptotically, for each $i \in \mathbb{N}$.

Proof. See Appendix E.

Simulation. We consider a USV system with six vessels as an example to illustrate the validity of the proposed algorithms. The detailed results of this simulation example can be observed in Appendix F.

Conclusion. The ADCO issue of a second-order MAS with nonidentical nonlinear functions and disturbances has been examined in this work. A modified command filter approach has been applied to the design of both discontinuous and continuous singularity-free asymptotic distributed optimization algorithms. The proposed distributed optimization algorithms not only ensure all agents' states asymptotically converge to a global minimizer in a fully distributed fashion but also have strong robustness. The proposed continuous distributed optimization algorithm is free of chattering. The extensions to the non-convex optimization problem and directed topology are interesting future research topics.

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Supporting information Appendixes A–F. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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