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Observer-based stabilization for discrete nonlinear semi-Markov jump singularly perturbed models with mode-switching delay

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Semi-Markov jump systems (S-MJMs) not only characterize hybrid systems but also address the limitations of Markov jump systems (MJMs) [1–3]. Due to their ability to exhibit multi-time-scale features, singularly perturbed models (SPMs) effectively model practical systems influenced by multiple time-scale phenomena [4]. In this study, the observer-based output feedback controller is asynchronous with the original system due to the time delay in the controller mode switching. A nonlinear plant with singularly perturbed parameters (SPPs) is represented using an interval type-2 (IT2) fuzzy model [5].

Currently, most studies on nonlinear S-MJSPMs adopt the traditional T-S fuzzy modeling approach, which neglects the impact of uncertainty on modeling accuracy. Existing research on IT2 fuzzy S-MJMs typically assumes an ideal synchronous mechanism between the controller mode and the system mode. However, no previous research has addressed observer-based control law design for discrete IT2 fuzzy S-MJSPMs while considering the mode-switching delay. Therefore, this study aims to design a suitable observer-based control strategy to enhance the dynamic performance of IT2 fuzzy S-MJSPMs with incomplete semi-Markov kernel (SMK). The main contributions are summarized below. (i) Unlike previous IT2 fuzzy S-MJMs, which assume synchronous switching between the controller and the system, this study accounts for the asynchronous phenomenon caused by the mode-switching delay in the controller. (ii) Based on fuzzy rules, SPPs, and SMK method, sufficient conditions for the existence of a fuzzy controller are established to ensure mean-square stability.

Preliminaries. Consider the IT2 fuzzy S-MJSPMs as follows:

$$\begin{aligned} s(\ell+1) &= \sum\nolimits_{\gamma=1}^{m_{r_\ell}} F^\gamma(S(\ell))[A^\gamma_{r_\ell} E_\epsilon s(\ell) + B^\gamma_{r_\ell}\mu(\ell)], \\ y(\ell) &= \sum\nolimits_{\gamma=1}^{m_{r_\ell}} F^\gamma(S(\ell)) C^\gamma_{r_\ell} E_\epsilon s(\ell), \end{aligned} \tag{1}$$

where $s(\ell) = [s_s^{\mathrm{T}}(\ell)\ s_f^{\mathrm{T}}(\ell)]^{\mathrm{T}} \in \mathbb{R}^{n_s}$, $s_s(\ell) \in \mathbb{R}^{n_{ss}}$, $s_f(\ell) \in \mathbb{R}^{n_{sf}}$ represent the slow and fast states, respectively. The term $E_\epsilon = \mathrm{diag}\{I^{n_{ss}}, \epsilon I^{n_{sf}}\}$ and $\epsilon > 0$ defines the SPP. The variables $\mu(\ell) \in \mathbb{R}^{n_\mu}$ and $y(\ell) \in \mathbb{R}^{n_y}$ represent the control input and system output, respectively. The matrices $A^\gamma_{r_\ell}$, $B^\gamma_{r_\ell}$ and $C^\gamma_{r_\ell}$ are known and appropriately dimensional matrices, where the semi-Markov chain $\{r_\ell\}_{\ell\in\mathbb{Z}_+}$ takes values in the set $\mathbb{M} \triangleq \{1, 2, \ldots, M\}$. The evolution follows the SMK $\tilde{\Pi}(\iota) = [\tilde{\pi}_{oq}(\iota)]_{o,q\in\mathbb{M}}$. The index $\gamma \in \mathbb{I}_p \triangleq \{1, 2, \ldots, m_o\}$ denotes the fuzzy rule number, where m_o signifies the number of IF-THEN rules for oth system mode when $r_\ell = o \in \mathbb{M}$. The firing strength of the γth fuzzy rule is defined as $F^\gamma_{r_\ell}(S(\ell))$. For all $o, q \in \mathbb{M}$ and $\iota \in \mathbb{Z}_{\geqslant 1}$, define $\mathbb{M}^A_p \triangleq \{p \in \mathbb{M} | \text{if } \tilde{\pi}_{pq}(\iota) \text{ is available}\}$, $\mathbb{M}^U_p \triangleq \{p \in \mathbb{M} | \text{if } \tilde{\pi}_{pq}(\iota) \text{ is unavailable}\}$, where $\mathbb{M}^A_i \triangleq \{\boldsymbol{M}^A_i(1), \boldsymbol{M}^A_i(2), \ldots, \boldsymbol{M}^A_i(M^A_i)\}$ with $1 \leqslant \boldsymbol{M}^A_i(1) < \boldsymbol{M}^A_i(2) < \cdots < \boldsymbol{M}^A_i(M^A_i) \leqslant M$, M^A_i represents the total number of elements in $\mathbb{M}^A_i$, and $\boldsymbol{M}^A_i(n)$ denotes the $\boldsymbol{M}^A_i(n)$th number of $\mathbb{M}$ and nth available element in $\mathbb{M}^A_i$.

Next, the IT2 fuzzy observer-based feedback controller is designed as follows:

$$\begin{aligned} \hat{s}(\ell+1) &= \sum\nolimits_{\gamma=1}^{m_{r_\ell}} F^\gamma(S(\ell))[A^\gamma_{\tilde{r}_\ell} E_\epsilon \hat{s}(\ell) + B^\gamma_{\tilde{r}_\ell}\mu(\ell) \\ &\quad + L^\gamma_{\tilde{r}_\ell}(y(\ell) - \hat{y}(\ell))], \\ \hat{y}(\ell) &= \sum\nolimits_{\gamma=1}^{m_{r_\ell}} F^\gamma(S(\ell)) C^\gamma_{\tilde{r}_\ell} E_\epsilon \hat{s}(\ell), \\ \mu(\ell) &= \sum\nolimits_{\gamma=1}^{m_{r_\ell}} F^\gamma(S(\ell)) K^\gamma_{\tilde{r}_\ell} E_\epsilon \hat{s}(\ell), \end{aligned} \tag{2}$$

where $\hat{s}(\ell)$ and $\hat{y}(\ell)$ denote the observer state and output, respectively. The terms $K^\gamma_{r_\ell}$ and $L^\gamma_{r_\ell}$ represent the controller and observer gains, respectively. The notation $\tilde{r}_\ell$ indicates the observer and controller modes with a one-step

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mode delay relative to the system, where $\tilde{r}_\ell = r_{\ell-1}$. Defining $e(\ell) = s(\ell) - \hat{s}(\ell)$ and $\mathcal{S}(\ell) = [s^{\mathrm{T}}(\ell)\ e^{\mathrm{T}}(\ell)]^{\mathrm{T}}$ yields:

$$\mathcal{S}(\ell+1) = \sum\nolimits_{\epsilon=1}^{m_o} F'(S(\ell))\mathfrak{A}'_{r_\ell,\tilde{r}_\ell}\mathbb{E}_\epsilon \mathcal{S}(\ell), \tag{3}$$

where

$$\mathfrak{A}'_{r_\ell,\tilde{r}_\ell} = \begin{bmatrix} A'_{r_\ell} + B'_{r_\ell}K'_{\tilde{r}_\ell} & -B'_{r_\ell}K'_{\tilde{r}_\ell} \\ A'_{r_\ell} + B'_{r_\ell}K'_{\tilde{r}_\ell} - L'_{\tilde{r}_\ell}C'_{r_\ell} - D'_{\tilde{r}_\ell} & D'_{\tilde{r}_\ell} - B'_{r_\ell}K'_{\tilde{r}_\ell} \end{bmatrix},$$

$$\mathbb{E}_\epsilon = \begin{bmatrix} E_\epsilon & 0 \\ 0 & E_\epsilon \end{bmatrix},\ D'_{\tilde{r}_\ell} = A'_{\tilde{r}_\ell} + B'_{\tilde{r}_\ell}K'_{r_\ell} - L'_{\tilde{r}_\ell}C'_{\tilde{r}_\ell}.$$

Stability analysis.

Theorem 1. For given $\delta_o > 0$, $\mathbb{T}^o_{\max} \in \mathbb{Z}_{\geqslant 1}$, $o \in \mathbb{M}$, $q \in \mathbb{M}^A_o$, system (3) is mean-square stable if there exist matrices $\mathcal{P}_{o\tau} > 0$, $o \in \mathbb{M}$, $\tau \in \mathbb{Z}_{[1,\mathbb{T}^o_{\max}]}$, such that $\forall \gamma_0, \gamma_1, \ldots, \gamma_\tau \in \mathbb{I}_o$, $\forall \tau \in \mathbb{Z}_{[1,\mathbb{T}^o_{\max}-1]}$ and $\iota \in \mathbb{Z}_{[1,\mathbb{T}^o_{\max}]}$,

$$\begin{aligned}&(\mathfrak{A}^{\gamma_\tau}_{o,j})^{\mathrm{T}} \left(\prod\nolimits_{\gamma=1}^{\tau-1} \mathfrak{A}^{\gamma_\gamma}_{o,o}\right)^{\mathrm{T}} \mathcal{P}_{o(\tau+1)} \prod\nolimits_{\gamma=1}^{\tau-1} \mathfrak{A}^{\gamma_\gamma}_{o,o}\mathfrak{A}^{\gamma_\tau}_{o,j} \\ &- \delta_o \mathcal{P}_{o1} < 0,\end{aligned} \tag{4}$$

$$\begin{aligned}&\sum\nolimits_{\iota=1}^{\mathbb{T}^o_{\max}} (\mathfrak{A}^{\gamma_\iota}_{o,j})^{\mathrm{T}} \left(\prod\nolimits_{\gamma=1}^{\iota-1} \mathfrak{A}^{\gamma_\gamma}_{o,o}\right)^{\mathrm{T}} \frac{\mathfrak{P}_{o\iota}}{\omega_o} \prod\nolimits_{\gamma=1}^{\iota-1} \mathfrak{A}^{\gamma_\gamma}_{o,o}\mathfrak{A}^{\gamma_\iota}_{o,j} \\ &+ (1-\varpi_o)(\mathfrak{A}^{\gamma_\iota}_{o,j})^{\mathrm{T}} \left(\prod\nolimits_{\gamma=1}^{\iota-1} \mathfrak{A}^{\gamma_\gamma}_{o,o}\right)^{\mathrm{T}} \mathcal{P}_{q1} \prod\nolimits_{\gamma=1}^{\iota-1} \mathfrak{A}^{\gamma_\gamma}_{o,o}\mathfrak{A}^{\gamma_\iota}_{o,j} \\ &- \mathcal{P}_{o1} < 0,\end{aligned} \tag{5}$$

where $\mathfrak{P}_{o\iota} \triangleq \sum_{q \in \mathbb{M}^A_o} \tilde{\pi}_{oq}(\iota)\mathcal{P}_q(0)$, $\omega_o \triangleq \varpi_o + (1-\tilde{\psi}_o)\rho$ with $\varpi_o = \sum_{\iota=1}^{\mathbb{T}^o_{\max}} \sum_{c \in \mathbb{M}^A_o} \tilde{\pi}_{oc}(\iota)$ and $\tilde{\psi}_o = \sum_{q \in \mathbb{M}^A_o} \psi_{oq}$.

Proof. See Appendix A.

By Theorem 1, we establish the mean-square stability of system (3), which involves a power of the matrix $\mathfrak{A}^{\gamma_\gamma}_{o,o}$. This stability result facilitates the controller design. Theorem 2 introduces additional matrix variables to address this problem.

Theorem 2. For given $\delta_o > 0$, $\mathbb{T}^o_{\max} \in \mathbb{Z}_{\geqslant 1}$, $\forall o \in \mathbb{M}$, $q \in \mathbb{M}^U_o$, system (3) is mean-square stable if there exist matrices $U_{o\tau\imath} > 0$ and $\mathcal{U}_{o\iota\jmath} > 0$, $\forall o \in \mathbb{M}$, $q \in \mathbb{M}^U_o$, $\tau \in \mathbb{Z}_{[1,\mathbb{T}^o_{\max}-1]}$, $\forall \imath \in \mathbb{Z}_{[0,\tau-1]}$, $\forall \iota \in \mathbb{Z}_{[1,\mathbb{T}^o_{\max}]}$, $\forall \jmath \in \mathbb{Z}_{[1,\iota-1]}$, such that $\forall o \in \mathbb{M}$, $\forall \iota \in \mathbb{Z}_{[1,\mathbb{T}^o_{\max}]}$,

$$(\mathfrak{A}^{\gamma_{\tau-\imath}}_{o,o})^{\mathrm{T}} U_{o(\tau+1)(\imath+2)} \mathfrak{A}^{\gamma_{\tau-\imath}}_{o,o} - U_{o(\tau+1)(\imath+1)} < 0, \tag{6}$$

$$(\mathfrak{A}^{\gamma_\tau}_{o,j})^{\mathrm{T}} U_{o(\tau+1)2} \mathfrak{A}^{\gamma_\tau}_{o,j} - \delta_o U_{o11} < 0, \tag{7}$$

$$\sum\nolimits_{\iota=\jmath+1}^{\mathbb{T}^o_{\max}} ((\mathfrak{A}^{\gamma_{\iota-\jmath}}_{o,o})^{\mathrm{T}} \mathcal{U}_{o\iota(\jmath+1)} \mathfrak{A}^{\gamma_{\iota-\jmath}}_{o,o} - \mathcal{U}_{o\iota\jmath}) < 0, \tag{8}$$

$$(\mathfrak{A}^{\gamma_\imath}_{o,o})^{\mathrm{T}} U_{q1(\iota-\imath)} \mathfrak{A}^{\gamma_\imath}_{o,o} - U_{q1(\iota-\imath+1)} < 0, \tag{9}$$

$$\begin{aligned}&\frac{\sum_{\iota=1}^{\mathbb{T}^o_{\max}} (\mathfrak{A}^{\gamma_\iota}_{o,j})^{\mathrm{T}} \mathcal{U}_{o\iota 1} \mathfrak{A}^{\gamma_\iota}_{o,j}}{\omega_o} + (1-\varpi_o)(\mathfrak{A}^{\gamma_\iota}_{o,j})^{\mathrm{T}} U_{q1\iota} \mathfrak{A}^{\gamma_\iota}_{o,j} \\ &- U_{o11} < 0,\end{aligned} \tag{10}$$

where ω_o and ϖ_o are defined in Theorem 1.

Proof. See Appendix B.

Controller design.

Theorem 3. For given $\delta_o > 0$, $\mathbb{T}^o_{\max} \in \mathbb{Z}_{\geqslant 1}$, $\forall o \in \mathbb{M}$, and $\mathcal{F}_o$, if we find matrices $\daleth_o \in \mathbb{R}^{2n_S}$, $O_{o\tau\imath} \in \mathbb{R}^{2n_S}$ with $\mathfrak{I}_3 O_{o\tau\imath} \mathfrak{I}_3^{\mathrm{T}} \geqslant 0$, $\mathcal{O}_{o\iota\jmath} \in \mathbb{R}^{2n_S}$ with $\mathfrak{I}_3 \mathcal{O}_{o\iota\jmath} \mathfrak{I}_3^{\mathrm{T}} \geqslant 0$, $\forall o \in \mathbb{M}$, $q \in \mathbb{M}^U_o$, $\tau \in \mathbb{Z}_{[0,\mathcal{T}^o_{\max}-1]}$, $\forall \imath \in \mathbb{Z}_{[0,\tau]}$, $\forall \iota \in \mathbb{Z}_{[1,\mathcal{T}^o_{\max}]}$, $\forall \jmath \in \mathbb{Z}_{[0,\iota]}$ and sets of matrices U^γ_o, N^γ_o, $\mathcal{N}^\gamma_j$, such that $\forall o, j \in \mathbb{M}$, $q \in \mathbb{M}^U_o$,

$$\begin{bmatrix} \Delta_1 & (\Lambda^{\gamma_{\tau-\imath}}_{o,o})^{\mathrm{T}} \\ * & -O_{o(\tau+1)(\imath+2)} \end{bmatrix} < 0, \tag{11}$$

$$\begin{bmatrix} \Delta_2 & (\Lambda^{\gamma_\tau}_{o,j})^{\mathrm{T}} \\ * & -O_{o(\tau+1)2} \end{bmatrix} < 0, \tag{12}$$

$$\begin{bmatrix} \Theta_1 & L^{\mathrm{T}}_{o(\jmath+1)}[(\Lambda^{\gamma_{\iota-\jmath}}_{o,o})^{\mathrm{T}} \otimes I_{\boldsymbol{M}^A_o(M^A_o)+1}] \\ * & -\mathrm{diag}\{\tilde{O}_o, \tilde{\mathcal{O}}_{o(\jmath+1)}\} \end{bmatrix} < 0, \tag{13}$$

$$\begin{bmatrix} \Upsilon_1 & (\Lambda^{\gamma_\imath}_{o,o})^{\mathrm{T}} \\ & -O_{q1(\iota-\imath)} \end{bmatrix} < 0, \tag{14}$$

$$\begin{bmatrix} \Xi_1 & (\Lambda^{\gamma_\iota}_{o,j})^{\mathrm{T}} & L^{\mathrm{T}}_{o1}[(\Lambda^{\gamma_\iota}_{o,j})^{\mathrm{T}} \otimes I_{\boldsymbol{M}^A_o(M^A_o)+1}] \\ * & -U^*_{q1\iota} & 0 \\ & * & \frac{-\mathrm{diag}\{\tilde{O}_o, \tilde{\mathcal{O}}_{o(\jmath+1)}\}}{\omega_o} \end{bmatrix} < 0, \tag{15}$$

then system (3) is mean-square stable with the controller gain and the observer gains given by $K'_p = U'_p \daleth_p^{-1}$ and $L'_p = N'_p \daleth_p^{-1}(C'_p)^{-1}$.

Proof. See Appendix C.

Conclusion. The design of an observer-based output feedback controller has been proposed for IT2 fuzzy S-MJSPMs under an incomplete SMK, incorporating a mode-switching delay. The main contribution of this work is the formulation of an observer-based feedback control scheme with modal delays to improve dynamic performance. Nonlinear S-MJMs have been derived using the IT2 fuzzy approach, and an observer-based controller has been designed to establish the stability criterion. Compared with the case in which the SMK is fully available, the underlying mean-square stability has been developed. Simulation results, presented in Appendix D, demonstrate the effectiveness of the proposed control strategy. In future work, these results will be extended to IT2 fuzzy S-MJSPMs with time-varying delays.

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Supporting information Appendixes A–D. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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