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• Supplementary File •

Observer-based stabilization for discrete nonlinear semi-Markov jump singularly perturbed models with mode-switching delay

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Appendix A Proof of Theorem 1

Proof. Consider

$$\mathbb{V}(\mathcal{S}(\ell), r_\ell, O_\ell = \tau) \triangleq \mathcal{S}^T(\ell)\tilde{P}_{r_\ell}(\ell,\tau)\mathcal{S}(\ell), \tag{12}$$

where $\tilde{P}_{r_\ell}(\ell,\tau) \triangleq \sum_{\gamma=1}^{m_p} F_\ell^\gamma \mathcal{P}_{r_\ell}(\tau)$ with $\tau = \ell - \ell_n + 1$, and F_ℓ^γ is adopted to signify $F^\gamma(S(\ell))$.

For $r_\ell = o$, $\forall \ell \in \mathbb{Z}_{[\ell_n,\ell_{n+1})}$, we can get

$$\xi_1 \parallel \mathcal{S}(\ell) \parallel^2 \leqslant \mathbb{V}(\mathcal{S}(\ell), r_\ell, \tau) \leqslant \xi_2 \parallel \mathcal{S}(\ell) \parallel^2, \tag{13}$$

where $\xi_1 \triangleq \inf_{\forall o\in\mathbb{M},\tau\in\mathbb{Z}_{[1,\mathbb{T}^o_{max}]}}\{\lambda_{min}(\mathcal{P}_{o\tau})\}$ and $\xi_2 \triangleq \sup_{\forall o\in\mathbb{M},\tau\in\mathbb{Z}_{[1,\mathbb{T}^o_{max}]}}\{\lambda_{max}(\mathcal{P}_{o\tau})\}$.

Define $\Phi_1(\|\mathcal{S}(\ell)\|) \triangleq \xi_1 \parallel \mathcal{S}(\ell) \parallel^2$ and $\Phi_2(\|\mathcal{S}(\ell)\|) \triangleq \xi_2 \parallel \mathcal{S}(\ell) \parallel^2$, where Φ_1 and Φ_2 are $\mathcal{K}_\infty$ functions. Then, (7) in Lemma 1 holds.

If $\Re_n = o$, $\Re_{n-1} = j$, $\forall \ell \in \mathbb{Z}_{[\ell_n+1,\ell_{n+1}]}$, we have

$$\begin{aligned}\mathcal{S}(\ell) = \mathcal{S}(\ell_n+\tau) = \Pi_{\gamma=1}^{\tau-1}\bar{\mathfrak{A}}_{o,o}(\ell_n+\gamma)\bar{\mathfrak{A}}_{o,j}(\ell_n+\tau)\mathcal{S}(\ell_n)\\ =\Pi_{\gamma=1}^{\tau-1}(\sum_{\gamma_\gamma=1}^{m_o} F_{\ell_n+\gamma}^{\gamma_\gamma}\mathfrak{A}_{o,o}^{\gamma_\gamma})\sum_{\gamma_\tau=1}^{m_o} F_{\ell_n+\tau}^{\gamma_\tau}\mathfrak{A}_{o,j}^{\gamma_\tau}\mathcal{S}(\ell_n).\end{aligned}$$

Combining $\tilde{P}_o(\ell_n,1) = \sum_{\gamma=1}^{m_o} F_{\ell_n}^\gamma \mathcal{P}_o(1)$ and (10), one has

$$\begin{aligned}&\mathbb{V}(\mathcal{S}(\ell_n+\tau), o, \tau+1) - \delta_o\mathbb{V}(\mathcal{S}(\ell_n), o, 1)\\ &= \mathcal{S}^T(\ell_n)(\sum_{\gamma_1'=1}^{m_o}\cdots\sum_{\gamma_\tau'=1}^{m_o}\sum_{\gamma_1=1}^{m_o}\cdots\sum_{\gamma_\tau=1}^{m_o}\sum_{\gamma_{\tau+1}=1}^{m_o} F_{\ell_n+\tau+1}^{\gamma_{\tau+1}}(\prod_{\gamma=1}^{\tau} F_{\ell_n+\gamma}^{\gamma_\gamma'}F_{\ell_n+\gamma}^{\gamma_\gamma})\\ &[(\mathfrak{A}_{o,j}^{\gamma_\tau'})^T(\prod_{\gamma=1}^{\tau-1}\mathfrak{A}_{o,o}^{\gamma_\gamma'})^T\mathcal{P}_{o\tau}\prod_{\gamma=1}^{\tau-1}\mathfrak{A}_{o,o}^{\gamma_\gamma}\mathfrak{A}_{o,j}^{\gamma_\tau'}])\mathcal{S}(\ell_n) - \delta_o\mathcal{S}^T(\ell_n)\tilde{P}_o(\ell_n,1)\mathcal{S}(\ell_n)\\ &\leqslant \mathcal{S}^T(\ell_n)(\sum_{\gamma_1=1}^{m_o}\cdots\sum_{\gamma_\tau=1}^{m_o}\sum_{\gamma_{\tau+1}=1}^{m_o}(\prod_{\gamma=1}^{\tau+1} F_{o(\ell_n+\gamma)}^{\gamma_\gamma})\\ &[(\mathfrak{A}_{o,j}^{\gamma_\tau})^T(\prod_{\gamma=1}^{\tau-1}\mathfrak{A}_{o,o}^{\gamma_\gamma})^T\mathcal{P}_{o\tau}\prod_{\gamma=1}^{\tau-1}\mathfrak{A}_{o,o}^{\gamma_\gamma}\mathfrak{A}_{o,j}^{\gamma_\tau}] - \delta_o\mathcal{S}^T(\ell_n)\sum_{\gamma=1}^{m_o} F_{\ell_n}^\gamma\mathcal{P}_{o1}\\ &\leqslant \sum_{\gamma_1=1}^{m_o}\cdots\sum_{\gamma_{\tau+1}=1}^{m_o}(\prod_{\gamma=1}^{\tau+1} F_{\ell_n+\gamma}^{\gamma_\gamma})\mathcal{S}^T(\ell_n)[(\mathfrak{A}_{o,j}^{\gamma_\tau})^T\\ &(\prod_{\gamma=1}^{\tau-1}\mathfrak{A}_{o,o}^{\gamma_\gamma})^T\mathcal{P}_{o\tau}\prod_{\gamma=1}^{\tau-1}\mathfrak{A}_{o,o}^{\gamma_\gamma}\mathfrak{A}_{o,j}^{\gamma_\tau} - \delta_o\mathcal{P}_{o1}]\mathcal{S}(\ell_n) < 0,\end{aligned} \tag{14}$$

which further means that (8) holds in Lemma 1.

Letting $\Re_n = o$, $\Re_{n-1} = j$, $\Re_{n+1} = q$, and $\iota = \ell_{n+1} - \ell_n$ yields

$$\begin{aligned}&E[\mathbb{V}(\mathcal{S}(\ell_{n+1}), q, 1)]|_{\mathcal{S}(\ell_n),\Re_n=o} - \mathbb{V}(\mathcal{S}(\ell_n), o, 1)\\ &= E[\mathcal{S}^T(\ell_{n+1})\tilde{P}_q(\ell_{n+1},1)\mathcal{S}(\ell_{n+1})] - \mathcal{S}^T(\ell_n)\tilde{P}_o(\ell_n,1)\mathcal{S}(\ell_n)\end{aligned}$$

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$$
\begin{aligned}
&= \mathcal{S}^T(\ell_n)\{\sum\nolimits_{\iota=1}^{\mathbb{T}^o_{max}}[\sum\nolimits_{q\in\mathbb{M}_o^A}\frac{\tilde{\pi}_{oq}(\iota)}{\varsigma_o}(\bar{\mathfrak{A}}_{o,j}(\ell_n+\iota))^T(\prod\nolimits_{\gamma=1}^{\iota-1}\bar{\mathfrak{A}}_{o,o}(\ell_n+\gamma))^T\tilde{P}_o(\ell_{n+1},1)\\
&\prod\nolimits_{\gamma=1}^{\iota-1}\bar{\mathfrak{A}}_{o,o}(\ell_n+\gamma)\bar{\mathfrak{A}}_{o,j}(\ell_n+\iota)+\sum\nolimits_{q\in\mathbb{M}_o^U}\frac{\tilde{\pi}_{oq}(\iota)}{\varsigma_o}(\bar{\mathfrak{A}}_{o,j}(\ell_n+\iota))^T\\
&(\prod\nolimits_{\gamma=1}^{\iota-1}\bar{\mathfrak{A}}_{o,o}(\ell_n+\gamma))^T\tilde{P}_o(\ell_{n+1},1)\prod\nolimits_{\gamma=1}^{\iota-1}\bar{\mathfrak{A}}_{o,o}(\ell_n+\gamma)\bar{\mathfrak{A}}_{o,j}(\ell_n+\iota)-\tilde{P}_o(\ell_n,1)\}\mathcal{S}(\ell_n)\\
&\leqslant \mathcal{S}^T(\ell_n)\{\sum\nolimits_{\iota=1}^{\mathbb{T}^o_{max}}[\sum\nolimits_{q\in\mathbb{M}_o^A}\frac{\tilde{\pi}_{oq}(\iota)}{\varsigma_o}(\mathfrak{A}^{\gamma_\iota}_{o,j})^T(\prod\nolimits_{\gamma=1}^{\iota-1}\mathfrak{A}^{\gamma_\gamma}_{o,o})^T\mathcal{P}_{q1}\prod\nolimits_{\gamma=1}^{\iota-1}\mathfrak{A}^{\gamma_\gamma}_{o,o}\mathfrak{A}^{\gamma_\iota}_{o,j}\\
&+\sum\nolimits_{q\in\mathbb{M}_o^U}\frac{\tilde{\pi}_{oq}(\iota)}{\varsigma_o}(\mathfrak{A}^{\gamma_\iota}_{o,j})^T(\prod\nolimits_{\gamma=1}^{\iota-1}\mathfrak{A}^{\gamma_\gamma}_{o,o})^T\mathcal{P}_{q1}\mathfrak{A}^{\gamma_\iota}_{o,j}\prod\nolimits_{\gamma=1}^{\iota-1}\mathfrak{A}^{\gamma_\gamma}_{o}]-\mathcal{P}_{o1}\}\mathcal{S}(\ell_n)\\
&\leqslant \mathcal{S}^T(\ell_n)\{\sum\nolimits_{\iota=1}^{\mathbb{T}^o_{max}}\sum\nolimits_{\gamma_1=1}^{m_o}\dots\sum\nolimits_{\gamma_\iota=1}^{m_o}(\prod\nolimits_{\gamma=1}^{\iota}F^{\gamma_\gamma}_{\ell_n+\gamma})[(\mathfrak{A}^{\gamma_\iota}_{o,j})^T(\prod\nolimits_{\gamma=1}^{\iota-1}\mathfrak{A}^{\gamma_\gamma}_{o,o})^T\frac{\mathfrak{P}_{o\iota}}{\varsigma_o}\prod\nolimits_{\gamma=1}^{\iota-1}\mathfrak{A}^{\gamma_\gamma}_{o,o}\mathfrak{A}^{\gamma_\iota}_{o,j}\\
&+\sum\nolimits_{q\in\mathbb{M}_o^U}\frac{\tilde{\pi}_{oq}(\iota)}{\varsigma_o-\varpi_o}\frac{\varsigma_o-\varpi_o}{\varsigma_o}(\mathfrak{A}^{\gamma_\iota}_{o,j})^T(\prod\nolimits_{\gamma=1}^{\iota-1}\mathfrak{A}^{\gamma_\gamma}_{o,o})^T\mathcal{P}_{q1}\prod\nolimits_{\gamma=1}^{\iota-1}\mathfrak{A}^{\gamma_\gamma}_{o,o}\mathfrak{A}^{\gamma_\iota}_{o,j}]-\mathcal{P}_{o1}\}\mathcal{S}(\ell_n),
\end{aligned}
\tag{15}
$$

where $\varsigma_o\triangleq\sum_{\iota=1}^{\mathbb{T}^o_{max}}\sum_{q\in\mathbb{M}}\tilde{\pi}_{oq}(\iota)$.

It is noted that $\sum_{\iota=1}^{\mathbb{T}^o_{max}}\sum_{q\in\mathbb{M}^U}\tilde{\pi}_{oq}(\iota)=\varsigma_o-\varpi_o$, $\forall o\in\mathbb{M}$, we have $\sum_{\iota=1}^{\mathbb{T}^o_{max}}\sum_{q\in\mathbb{M}^U}\frac{\tilde{\pi}_{oq}(\iota)}{(\varsigma_o-\varpi_o)}=1$ with $0\leqslant\tilde{\pi}_{oq}(\iota)/(\varsigma_o-\varpi_o)\leqslant 1$ and $0\leqslant\varpi_o\leqslant\varsigma_o\leqslant 1$, $\forall o\in\mathbb{M}_o^A$, $\forall q\in\mathbb{M}_o^U$.

Together with (11), it is got that

$$
\begin{aligned}
&E[\mathbb{V}(\mathcal{S}(\ell_{n+1}),q,1)]|_{\mathcal{S}(\ell_n),\Re_n=o}-\mathbb{V}(\mathcal{S}(\ell_n),o,1)\\
&\leqslant \mathcal{S}^T(\ell_n)\sum\nolimits_{\iota=1}^{\mathbb{T}^o_{max}}\sum\nolimits_{q\in\mathbb{M}_o^U}\frac{\tilde{\pi}_{oq}(\iota)}{(\varsigma_o-\varpi_o)}[\sum\nolimits_{\iota=1}^{\mathcal{T}^p_{max}}(\mathfrak{A}^{\gamma_\iota}_{o,j})^T(\prod\nolimits_{\gamma=1}^{\iota-1}\mathfrak{A}^{\gamma_\gamma}_{p,p})^T\frac{\mathfrak{P}_{p\iota}}{\varsigma_o}\prod\nolimits_{\gamma=1}^{\iota-1}\mathfrak{A}^{\gamma_\gamma}_{o,o}\mathfrak{A}^{\gamma_\iota}_{o,j}\\
&+\frac{\varsigma_o-\varpi_o}{\varsigma_o}(\mathfrak{A}^{\gamma_\iota}_{o,j})^T(\prod\nolimits_{\gamma=1}^{\iota-1}\mathfrak{A}^{\gamma_\gamma}_{o,o})^T\mathcal{P}_{q1}\prod\nolimits_{\gamma=1}^{\iota-1}\mathfrak{A}^{\gamma_\gamma}_{o,o}\mathfrak{A}^{\gamma_\iota}_{o,j}-\mathcal{P}_{o1}]\mathcal{S}(\ell_n)\\
&\leqslant \mathcal{S}^T(\ell_n)\sum\nolimits_{\iota=1}^{\mathbb{T}^o_{max}}\sum\nolimits_{q\in\mathbb{M}_o^U}\frac{\tilde{\pi}_{oq}(\iota)}{(\varsigma_o-\varpi_o)}[\sum\nolimits_{\iota=1}^{\mathcal{T}^o_{max}}(\mathfrak{A}^{\gamma_\iota}_{o,j})^T(\prod\nolimits_{\gamma=1}^{\iota-1}\mathfrak{A}^{\gamma_\gamma}_{o,o})^T\frac{\mathfrak{P}_{p\iota}}{\omega_o}\prod\nolimits_{\gamma=1}^{\iota-1}\mathfrak{A}^{\gamma_\gamma}_{o,o}\mathfrak{A}^{\gamma_\iota}_{o,j}\\
&+(1-\varpi_o)(\mathfrak{A}^{\gamma_\iota}_{o,j})^T(\prod\nolimits_{\gamma=1}^{\iota-1}\mathfrak{A}^{\gamma_\gamma}_{o,o})^T\mathcal{P}_{q1}\prod\nolimits_{\gamma=1}^{\iota-1}\mathfrak{A}^{\gamma_\gamma}_{o,o}\mathfrak{A}^{\gamma_\iota}_{o,j}-\mathcal{P}_{o1}]\mathcal{S}(\ell_n)\\
&\leqslant \sum\nolimits_{\iota=1}^{\mathbb{T}^o_{max}}\sum\nolimits_{q\in\mathbb{M}_o^U}\frac{\tilde{\pi}_{oq}(\iota)}{(\varsigma_o-\varpi_o)}\lambda_{min}(-[\sum\nolimits_{\iota=1}^{\mathbb{T}^o_{max}}(\mathfrak{A}^{\gamma_\iota}_{o,j})^T(\prod\nolimits_{\gamma=1}^{\iota-1}\mathfrak{A}^{\gamma_\gamma}_{o,o})^T\frac{\mathfrak{P}_{p\iota}}{\omega_o}\prod\nolimits_{\gamma=1}^{\iota-1}\mathfrak{A}^{\gamma_\gamma}_{o,o}\mathfrak{A}^{\gamma_\iota}_{o,j}\\
&+(\varpi_o-1)(\mathfrak{A}^{\gamma_\iota}_{o,j})^T(\prod\nolimits_{\gamma=1}^{\iota-1}\mathfrak{A}^{\gamma_\gamma}_{o,o})^T\mathcal{P}_{q1}\prod\nolimits_{\gamma=1}^{\iota-1}\mathfrak{A}^{\gamma_\gamma}_{o,o}\mathfrak{A}^{\gamma_\iota}_{o,j}-\mathcal{P}_{o1}])\parallel\mathcal{S}(\ell_n)\parallel^2\\
&\leqslant \sum\nolimits_{\iota=1}^{\mathbb{T}^o_{max}}\sum\nolimits_{q\in\mathbb{M}_o^U}\frac{\tilde{\pi}_{oq}(\iota)}{\varsigma_o-\varpi_o}\xi_3\parallel\mathcal{S}(\ell_n)\parallel^2\\
&=-\xi_3\parallel\mathcal{S}(\ell_n)\parallel^2,
\end{aligned}
$$

where

$$
\begin{aligned}
\xi_3\triangleq&\inf_{o\in\mathbb{M},q\in\mathbb{M}_o^U,\iota\in\mathbb{Z}_{[1,\mathbb{T}^o_{max}]}}\{\lambda_{min}(-[\sum\nolimits_{\iota=1}^{\mathbb{T}^o_{max}}(\mathfrak{A}^{\gamma_\iota}_{o,j})^T(\prod\nolimits_{\gamma=1}^{\iota-1}\mathfrak{A}^{\gamma_\gamma}_{o,o})^T\frac{\mathfrak{P}_{p\iota}}{\omega_o}\prod\nolimits_{\gamma=1}^{\iota-1}\mathfrak{A}^{\gamma_\gamma}_{o,o}\mathfrak{A}^{\gamma_\iota}_{o,j}\\
&+(\varpi_o-1)(\mathfrak{A}^{\gamma_\iota}_{o,j})^T(\prod\nolimits_{\gamma=1}^{\iota-1}\mathfrak{A}^{\gamma_\gamma}_{o,o})^T\mathcal{P}_{q1}\prod\nolimits_{\gamma=1}^{\iota-1}\mathfrak{A}^{\gamma_\gamma}_{o,o}\mathfrak{A}^{\gamma_\iota}_{o,j}-\mathcal{P}_{o1}])\}.
\end{aligned}
$$

Letting $\Phi_3(\parallel\mathcal{S}(\ell_n)\parallel)\triangleq\xi_3\parallel\mathcal{S}(\ell_n)\parallel^2$, (9) can be attributed. Consequently, system (6) is MSS. ■

Appendix B Proof of Theorem 2

Proof. According to (16), one can infer that

$$
\begin{aligned}
&(\mathfrak{A}^{\gamma_\tau}_{o,j})^T\{\sum\nolimits_{\imath=1}^{\tau-1}(\prod\nolimits_{\gamma=\tau-\imath+1}^{\tau-1}\mathfrak{A}^{\gamma_\gamma}_{o,o})^T[(\mathfrak{A}^{\gamma_{\tau-\imath}}_{o,o})^TU_{o(\tau+1)(\imath+2)}\mathfrak{A}^{\gamma_{\tau-\imath}}_{o,o}-U_{o(\tau+1)(\imath+1)}]\prod\nolimits_{\gamma=\tau-\imath+1}^{\tau-1}\mathfrak{A}^{\gamma_\gamma}_{o}\}\mathfrak{A}^{\gamma_\tau}_{o,j}\\
&=(\mathfrak{A}^{\gamma_\tau}_{o,j})^T(\prod\nolimits_{\gamma=1}^{\tau-1}\mathfrak{A}^{\gamma_\gamma}_{o,o})^TU_{o(\tau+1)(\tau+1)}\prod\nolimits_{\gamma=1}^{\tau-1}\mathfrak{A}^{\gamma_\gamma}_{o,o}\mathfrak{A}^{\gamma_\tau}_{o,j}-(\mathfrak{A}^{\gamma_\tau}_{o,j})^TU_{o(\tau+1)2}\mathfrak{A}^{\gamma_\tau}_{o,j}<0.
\end{aligned}
\tag{21}
$$

Combining (17), (21), and $U_{o(\tau+1)(\tau+1)}\triangleq\mathcal{P}_{o(\tau+1)}$, one can get (10).

From (18) and (19), it can be obtained that

$$
\begin{aligned}
&\sum\nolimits_{\jmath=1}^{\mathbb{T}^o_{max}-1}(\mathfrak{A}^{\gamma_\iota}_{o,j})^T(\prod\nolimits_{\gamma=\iota-\jmath+1}^{\iota-1}\mathfrak{A}^{\gamma_\gamma}_{o,o})^T[\sum\nolimits_{\iota=\jmath+1}^{\mathbb{T}^o_{max}}((\mathfrak{A}^{\gamma_{\iota-\jmath}}_{o})^T\mathcal{U}_{o\iota(\jmath+1)}\mathfrak{A}^{\gamma_{\iota-\jmath}}_{o}-\mathcal{U}_{o\iota\jmath})]\prod\nolimits_{\gamma=\iota-\jmath+1}^{\iota-1}\mathfrak{A}^{\gamma_\gamma}_{o}\mathfrak{A}^{\gamma_\iota}_{o,j}\\
&=\sum\nolimits_{\iota=1}^{\mathbb{T}^o_{max}}(\mathfrak{A}^{\gamma_\iota}_{o,j})^T[(\prod\nolimits_{\gamma=1}^{\iota-1}\mathfrak{A}^{\gamma_\gamma}_{o,o})^T\mathcal{U}_{o\iota\iota}\prod\nolimits_{\gamma=1}^{\iota-1}\mathfrak{A}^{\gamma_\gamma}_{o,o}-\mathcal{U}_{o\iota1}]\mathfrak{A}^{\gamma_\iota}_{o,j}<0,
\end{aligned}
\tag{22}
$$

and

$$
\sum\nolimits_{\imath=1}^{\mathbb{T}^o_{max}-1}(\mathfrak{A}^{\gamma_\imath}_{o,j})^T(\prod\nolimits_{\gamma=1}^{\imath-1}\mathfrak{A}^{\gamma_\gamma}_{o,o})^T[(\mathfrak{A}^{\gamma_\imath}_{o})^TU_{q1(\iota-\imath)}\mathfrak{A}^{\gamma_\imath}_{o}-U_{q1(\iota-\imath+1)}]\prod\nolimits_{\gamma=0}^{\imath-1}\mathfrak{A}^{\gamma_\gamma}_{o,o}\mathfrak{A}^{\gamma_\imath}_{o,j}
$$

$$= (\mathfrak{A}_{o,j}^{\gamma_\iota})^T (\prod_{\gamma=1}^{\iota-1} \mathfrak{A}_{o,o}^{\gamma_\gamma})^T U_{q11} \prod_{\gamma=1}^{\iota-1} \mathfrak{A}_{o,o}^{\gamma_\gamma} \mathfrak{A}_{o,j}^{\gamma_\iota} - (\mathfrak{A}_{o,j}^{\gamma_\iota})^T U_{q1\iota} \mathfrak{A}_{o,j}^{\gamma_\iota} < 0. \tag{23}$$

Combining (20), (22), and (23), one has

$$\begin{aligned}
&\sum_{\iota=1}^{\mathbb{T}^o_{max}} (\mathfrak{A}_{o,j}^{\gamma_\iota})^T (\prod_{\gamma=1}^{\iota-1} \mathfrak{A}_{o,o}^{\gamma_\gamma})^T \frac{\mathcal{U}_{o\iota\iota}}{\omega_o} \prod_{\gamma=1}^{\iota-1} \mathfrak{A}_{o,o}^{\gamma_\gamma} \mathfrak{A}_{o,j}^{\gamma_\iota} - U_{o11} \\
&+ (1-\varpi_o)(\mathfrak{A}_{o,j}^{\gamma_\iota})^T (\prod_{\gamma=1}^{\iota-1} \mathfrak{A}_{o,o}^{\gamma_\gamma})^T U_{q11} \prod_{\gamma=1}^{\iota-1} \mathfrak{A}_{o,o}^{\gamma_\gamma} \mathfrak{A}_{o,j}^{\gamma_\iota} \\
&= \frac{1}{\omega_o} [\sum_{\iota=1}^{\mathbb{T}^o_{max}} (\mathfrak{A}_{o,j}^{\gamma_\iota})^T (\prod_{\gamma=1}^{\iota-1} \mathfrak{A}_{o,o}^{\gamma_\gamma})^T \mathcal{U}_{o\iota\iota} \prod_{\gamma=1}^{\iota-1} \mathfrak{A}_{o,o}^{\gamma_\gamma} \mathfrak{A}_{o,j}^{\gamma_\iota} - \sum_{\iota=1}^{\mathbb{T}^o_{max}} (\mathfrak{A}_{o,j}^{\gamma_\iota})^T \mathcal{U}_{o\iota1} \mathfrak{A}_{o,j}^{\gamma_\iota}] \\
&+ (1-\varpi_o)[(\mathfrak{A}_{o,j}^{\gamma_\gamma})^T (\prod_{\gamma=1}^{\iota-1} \mathfrak{A}_{o,o}^{\gamma_\gamma})^T U_{q11} \prod_{\gamma=1}^{\iota-1} \mathfrak{A}_{o,o}^{\gamma_\gamma} \mathfrak{A}_{o,j}^{\gamma_\gamma} - (\mathfrak{A}_{o,j}^{\gamma_\gamma})^T U_{q1\iota} \mathfrak{A}_{o,j}^{\gamma_\gamma}] \\
&+ \frac{\sum_{\iota=1}^{\mathbb{T}^o_{max}} (\mathfrak{A}_{o,j}^{\gamma_\iota})^T \mathcal{U}_{o\iota1} \mathfrak{A}_{o,j}^{\gamma_\iota}}{\omega_o} + (1-\varpi_o)(\mathfrak{A}_{o,j}^{\gamma_\iota})^T U_{q1\iota} \mathfrak{A}_{o,j}^{\gamma_\iota} - U_{o11} < 0.
\end{aligned} \tag{24}$$

Replacing U_{o11}, U_{q11} and $\mathcal{U}_{o\iota\iota}$ with $\mathcal{P}_{o1}$, $\mathcal{P}_{q1}$ and $\mathfrak{P}_{o\iota}$, (11) can be ensured by (24). ■

Appendix C Proof of Theorem 3

Proof. From (25) and Lemma 1, we can get that

$$\begin{bmatrix} \Delta_{1*} & \daleth_o^T (\mathfrak{A}^{\gamma_{\tau-\imath}})^T \\ * & -O_{o(\tau+1)(\imath+2)} \end{bmatrix} < 0, \tag{30}$$

where $\Delta_{1*} = F_o \mathbb{E}_\epsilon O_{o(\tau+1)(\imath+1)} \mathbb{E}_\epsilon F_o^T - sym\{\daleth_o F_o\}$.

Because

$$(F_o \mathbb{E}_\epsilon O_{o(\tau+1)(\imath+1)} \mathbb{E}_\epsilon - \daleth_o^T)(\mathbb{E}_\epsilon O_{o(\tau+1)(\imath+1)} \mathbb{E}_\epsilon)^{-1} (F_o \mathbb{E}_\epsilon O_{o(\tau+1)(\imath+1)} \mathbb{E}_\epsilon - \daleth_o^T)^T \geqslant 0,$$

one has

$$-\daleth_o^T (\mathbb{E}_\epsilon O_{o(\tau+1)(\imath+1)} \mathbb{E}_\epsilon)^{-1} \daleth_o F_o \mathbb{E}_\epsilon O_{o(\tau+1)(\imath+1)} \mathbb{E}_\epsilon F_o^T - sym\{\daleth_o F_o\}.$$

From (30), we can get

$$\begin{bmatrix} -\daleth_o^T (\mathbb{E}_\epsilon O_{o(\tau+1)(\imath+1)} \mathbb{E}_\epsilon)^{-1} \daleth_o & \daleth_o^T (\mathfrak{A}_o^{\gamma_{\tau-\imath}})^T \\ * & -O_{o(\tau+1)(\imath+2)} \end{bmatrix} < 0. \tag{31}$$

Performing congruence transformation to (31) by $diag\{\daleth_o^{-T}, I\}$ and using Schur complement, one has

$$(\mathfrak{A}^{\gamma_{\tau-\imath}})^T O_{o(\tau+1)(\imath+2)}^{-1} \mathfrak{A}^{\gamma_{\tau-\imath}} - (\mathbb{E}_\epsilon O_{o(\tau+1)(\imath+1)} \mathbb{E}_\epsilon)^{-1} < 0,$$

which means that (16) holds.

From (26) and Lemma 1, it is got that

$$\begin{bmatrix} \Delta_{2*} & \daleth_o^T (\mathfrak{A}^{\gamma_\tau})^T \\ * & -O_{o(\tau+1)2} \end{bmatrix} < 0, \tag{32}$$

where $\Delta_{2*} = F_o \mathbb{E}_\epsilon \delta_o O_{o11} \mathbb{E}_\epsilon F_o^T - sym\{\daleth_o F_o\}$. Furthermore,

$$\begin{bmatrix} -\daleth_o^T (\mathbb{E}_\epsilon \delta_o O_{o11} \mathbb{E}_\epsilon)^{-1} \daleth_o & \daleth_o^T (\mathfrak{A}_o^{\gamma_\tau})^T \\ * & -O_{o(\tau+1)2} \end{bmatrix} < 0. \tag{33}$$

Performing congruence transformation to (33) by $diag\{\daleth_o^{-T}, I\}$ and using Schur complement, one has

$$(\mathfrak{A}^{\gamma_\tau})^T O_{o(\tau+1)2}^{-1} \mathfrak{A}^{\gamma_\tau} - (\mathbb{E}_\epsilon \delta_o O_{o11} \mathbb{E}_\epsilon)^{-1} < 0.$$

Letting $O_o(\tau, \imath) = (\mathbb{E}_\epsilon U_o(\tau, \imath) \mathbb{E}_\epsilon)^{-1}$, it is got that (26) guarantees (17).

According to (27), one has

$$(\mathfrak{A}^{\gamma_{\iota-\jmath}})^T \mathbb{E}_\epsilon \mathcal{U}_{o(\jmath+1)(\jmath+1)} \mathbb{E}_\epsilon \mathfrak{A}^{\gamma_{\iota-\jmath}} + \sum_{\iota=\jmath+2}^{\mathbb{T}^o_{max}} (\mathfrak{A}^{\gamma_{\iota-\jmath}})^T \mathbb{E}_\epsilon \mathcal{U}_{o(\iota)(\jmath+1)} \mathbb{E}_\epsilon \mathfrak{A}^{\gamma_{\iota-\jmath}} - \sum_{\iota=\jmath+1}^{\mathbb{T}^o_{max}} \mathcal{U}_{o\iota\jmath} < 0, \tag{34}$$

where $\mathcal{U}_{o(\jmath+1)(\jmath+1)} = \sum_{q \in \mathbb{M}_o^U} \tilde{\pi}_{oq}(\jmath+1) U_{o(\jmath+1)(\jmath+1)}$.

Then, (18) is ensured. Similarly, (19) and (20) can be guaranteed from (28) and (29). ■

Appendix D Simulation

In this part, we consider a tunnel diode circuit model from [1] as shown in Figure D1. Firstly, according to the correlation characteristics of the circuit model, the tunnel diode circuit model is modeled by IT2 fuzzy semi-Markov model. Secondly, owing to unmeasurable system state, the feedback controller based on fuzzy observer is designed. Finally, the relevant control gain is solved from MATLAB/LMI toolbox. Therefore, the feedback controller based on fuzzy observer is applied to the tunnel diode circuit, and the final stability of the relevant parameters of the tunnel diode circuit is obtained through the simulation image, and the effectiveness of the proposed method is verified. The block diagram of control system is shown in Figure D2.

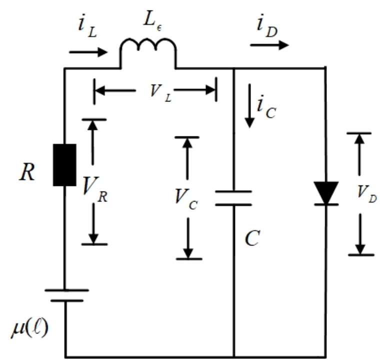

Figure D1 Tunnel diode circuit

The dynamic model is characterized by

$$\begin{aligned} &C\dot{s_s}(\ell) = -\varepsilon s_s(\ell) - \nu_o s_s^3(\ell) + s_f(\ell), \\ &L_\epsilon \dot{s_f}(\ell) = -s_s(\ell) - R s_f(\ell) + \mu(\ell), \\ &y(\ell) = \epsilon s_f(\ell), \end{aligned} \tag{35}$$

where $s_s(\ell) = V_C$ is the voltage, $s_f(\ell) = i_L$ is the current, $\mu(\ell)$ is the input voltage, and $\epsilon = 0.1$ is the SPP. Choose $\mathcal{C}, L_\epsilon$, and R are given as 20 mF, $100mH$, and 10Ω. Defining $\varepsilon = 0.002 + \Delta\varepsilon s_s^2(\ell)$, and $\Delta\varepsilon \in [0.01,\ 0.03]$, system (35) is rewritten as

$$\begin{aligned} &\dot{s_s}(\ell) = -\frac{\varepsilon}{C} s_s(\ell) - \frac{\nu_o}{C} s_s^3(\ell) + \frac{1}{C} s_f(\ell), \\ &\epsilon \dot{s_f}(\ell) = -s_s(\ell) - R s_f(\ell) + \mu(\ell), \\ &y(\ell) = \epsilon s_f(\ell). \end{aligned} \tag{36}$$

Consider the parameter ν_o subject to $r_\ell \in \mathbb{M} \triangleq \{1, 2\}$, and $\nu_1 = 0.01$, $\nu_2 = 0.02$. Assuming $|s_s(k)| \leqslant 3$, we can get $\varepsilon_{min} = 0.002$ and $\varepsilon_{max} = 0.272$. The system model and membership functions are given as

$$\begin{aligned} \bar{E}_\epsilon \dot{s}(\ell) &= \sum\nolimits_{\gamma=1}^{2} F^\gamma(S(\ell))[A_o^\gamma s(\ell) + B_o^\gamma \mu(\ell)], \\ y(\ell) &= \epsilon C_o^\gamma s(\ell), \end{aligned} \tag{37}$$

where

$$A_1^1 = \begin{bmatrix} -\frac{\epsilon_{min}}{\mathcal{C}} & 50 \\ -1 & -10 \end{bmatrix}, A_1^2 = \begin{bmatrix} -\frac{\epsilon_{max}}{\mathcal{C}} & 50 \\ -1 & -10 \end{bmatrix}, A_2^1 = \begin{bmatrix} -4.6 & 50 \\ -1 & -10 \end{bmatrix}, A_2^2 = \begin{bmatrix} -22.6 & 50 \\ -1 & -10 \end{bmatrix},$$

$$B_1^1 = B_1^2 = B_2^1 = B_2^2 \begin{bmatrix} 0 & 1 \end{bmatrix}^T, C_o^\gamma = \begin{bmatrix} 0 & 1 \end{bmatrix}, \overline{f}_o^1(s_s(\ell)) = 1 - e^{-\frac{0.1 s_s^2(t)}{0.5}}, \underline{f}_o^1(s_s(\ell)) = 1 - e^{-\frac{0.1 s_s^2(t)}{0.9}},$$

$$\overline{f}_o^2(s_s(\ell)) = e^{-\frac{0.1 s_s^2(t)}{0.9}}, \underline{f}_o^2(s_s(\ell)) = e^{-\frac{0.1 s_s^2(t)}{0.5}}, \underline{h}_o^\gamma = sin^2(s_s(\ell)), \overline{h}_o^\gamma = 1 - sin^2(s_s(\ell)), \bar{E}_\epsilon = diag\{1, \epsilon_M\}.$$

With the rapid development of science and technology, the combination of nonlinear system and actual computational control system becomes more and more important. Although the actual computer control system is a hybrid system composed of the pure discrete computer system and the pure continuous controlled object, it is usually treated as a discrete system to facilitate the analysis and design. Under the sampling time $T = 0.1$, one has

$$\begin{aligned} s(\ell+1) &= \sum\nolimits_{\gamma=1}^{2} F^\gamma(S(\ell))[A_o^\gamma \bar{E}_\epsilon s(\ell) + B_o^\gamma \mu(\ell)], \\ y(\ell) &= C\bar{E}_\epsilon s(\ell), \end{aligned} \tag{38}$$

where

$$A_1^1=\begin{bmatrix}0.8144 & 2.8909\\ -0.0578 & 0.2420\end{bmatrix}, A_1^2=\begin{bmatrix}0.1871 & 1.4194\\ -0.0284 & 0.2893\end{bmatrix}, A_2^1=\begin{bmatrix}0.5037 & 2.2417\\ -0.0448 & 0.2616\end{bmatrix}, A_2^2=\begin{bmatrix}0.0646 & 0.9629\\ -0.0193 & 0.3072\end{bmatrix},$$
$$B_1^1=\begin{bmatrix}0.1763 & 0.0582\end{bmatrix}^T, B_1^2=\begin{bmatrix}0.1147 & 0.0596\end{bmatrix}^T, B_2^1=\begin{bmatrix}0.1511 & 0.0587\end{bmatrix}^T, B_2^2=\begin{bmatrix}0.0906 & 0.0602\end{bmatrix}^T,$$
$$C=\begin{bmatrix}0 & 1\end{bmatrix}.$$

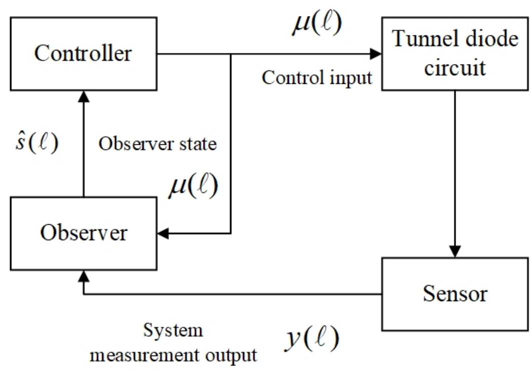

Figure D2 System structure diagram

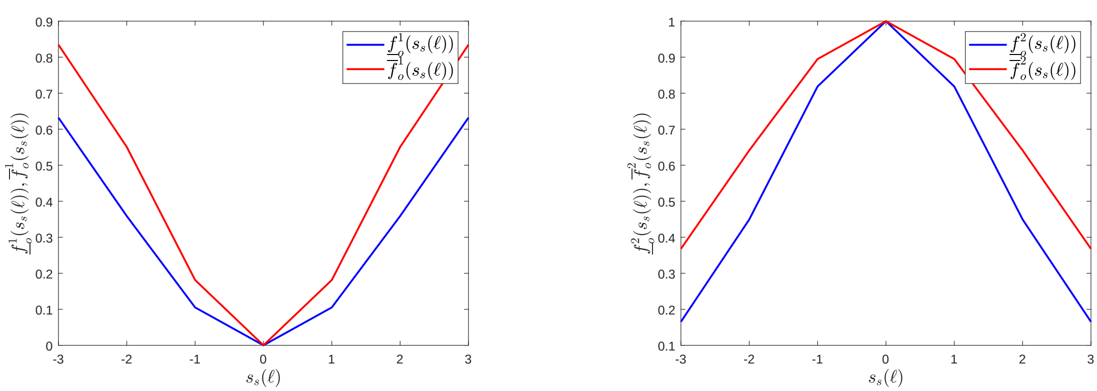

Figure D3 Membership functions

By solving Theorem 3, one has

$$K_1^1=\begin{bmatrix}-1.6551 & -28.6731\end{bmatrix}, K_1^2=\begin{bmatrix}-1.1505 & -48.2906\end{bmatrix},$$
$$K_2^1=\begin{bmatrix}-0.1669 & -13.5600\end{bmatrix}, K_2^2=\begin{bmatrix}0.0195 & -32.9168\end{bmatrix},$$
$$L_1^1=\begin{bmatrix}0.6443\\ 0.1777\end{bmatrix}, L_1^2=\begin{bmatrix}1.2860\\ 0.2685\end{bmatrix}, L_2^1=\begin{bmatrix}0.1178\\ 0.1549\end{bmatrix}, L_2^2=\begin{bmatrix}0.4766\\ 0.3512\end{bmatrix}.$$

In the problem of choosing the value of $\mathcal{T}^o_{max}$, too large value of $\mathcal{T}^o_{max}$ will increase the amount of calculation, and too small value of $\mathcal{T}^o_{max}$ will affect the conservatism. Finally, by weighing the relationship between calculated weights and conservatism, we choose the moderate $\mathcal{T}^o_{max}$. Consider $\mathbb{T}^1_{max}=\mathbb{T}^2_{max}=5$, $\delta_1=\delta_2=1$, $[\psi_{oq}]=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$. The available ST-PDF is $\varphi_{12}(\iota)=0.8^{\iota-1}\cdot 0.2^{8-\iota}\cdot 7!/[(8-\iota)!(\iota-1)!]$, while $\varphi_{21}(\iota)$ is not a fully available ST-PDF with available $\varphi_{21}(2)=0.5$, $\varphi_{21}(4)=0.2$, $\varphi_{21}(5)=0.1$, and unavailable $\varphi_{21}(1)$ and $\varphi_{21}(3)$.

For $s(0)=\begin{bmatrix}-0.5 & -1\end{bmatrix}^T$, $\hat{s}(0)=\begin{bmatrix}0 & -0.4\end{bmatrix}^T$, the inductor L_ϵ serves as the upper bound on SPP with $\epsilon \leqslant \epsilon_M$. The membership functions are shown in Figure D3. Over 100 realizations, under the given parameters and the designed observer-based controller, the system state of the closed-loop system is shown in Figure D4. It can be seen that over a period of

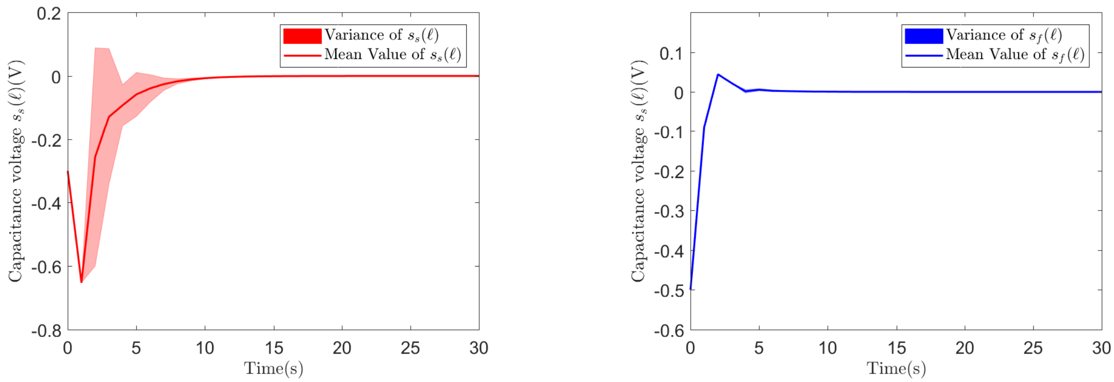

Figure D4 System state

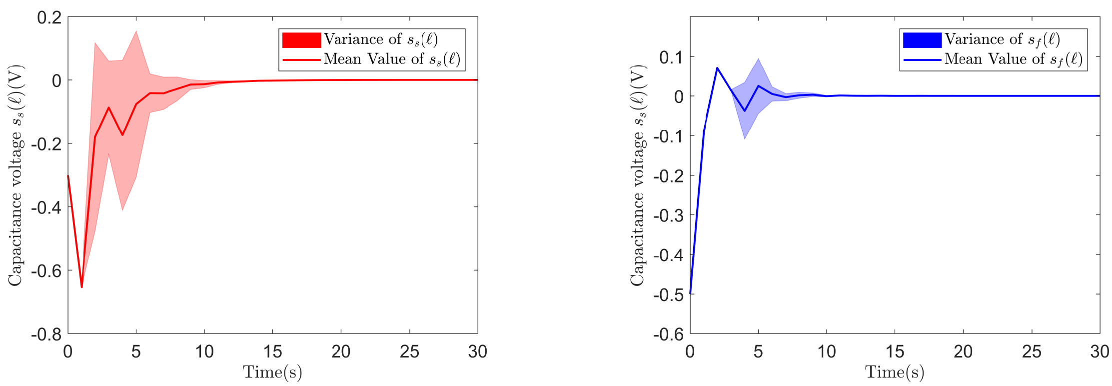

Figure D5 System state without mode-switching delay [2]

time, the capacitor voltage and the inductive current eventually tend to zero, ensuring the mean-square stability. Figure D5 shows the state of the system without modal delay [2]. Obviously, the stability of the system increases after considering the existence of modal delay, which further demonstrates the superiority of the proposed design method. The estimation error is described in Figure D6 and Figure D7 shows the control input. After a period of time, the control input and error trajectory converge to zero, showing the effectiveness of the proposed control strategy.

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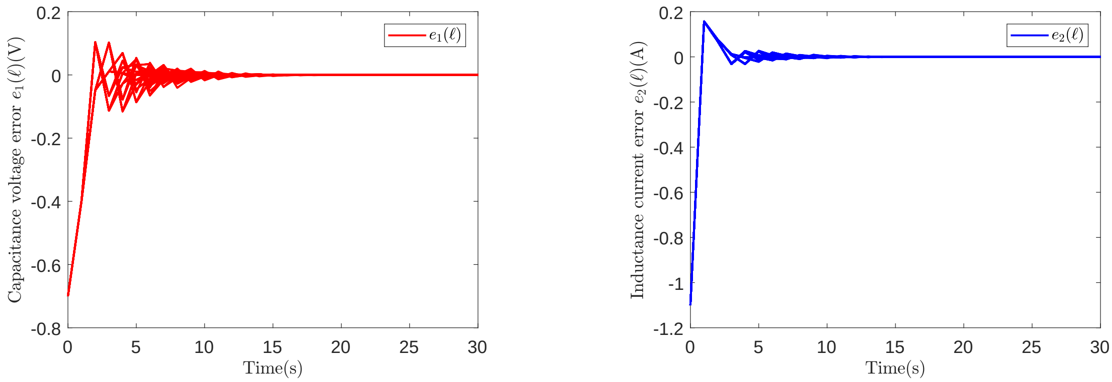

Figure D6 Estimation error

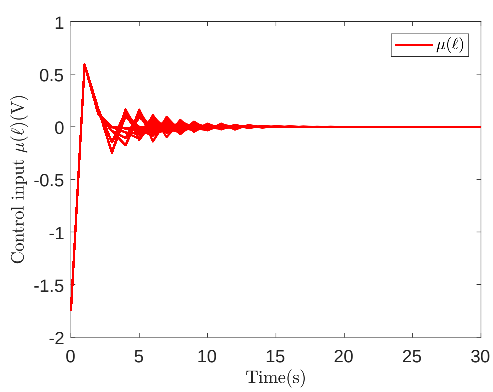

Figure D7 Control input