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Finite-time H_∞ control and energy cost optimization for nonlinear delayed systems through switching analysis and interval matrix method

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Stability, as one of the most important research directions, has caught the attention of many scholars [1–3]. Simultaneously, in domains such as vessel control and vehicle systems, external disturbances are unavoidable. The research challenge thus emerges: "How can one attain the desired system performance in a finite time while controlling the impact of external disturbances on output variables?". Finite-time H_∞ control (H_∞ FTC) is a salient solution to this problem.

As opposed to time-invariant systems, the stability of nonlinear time-varying systems is more meaningful and practical since it is no longer simply determined by the spectral nature of the nominal system matrix. The stability of a special class of time-varying systems, such as upper triangular time-varying systems, has been extensively studied [3]. Ref. [4] proposed simplifying switching systems to time-varying systems, incorporating an optimization algorithm to solve the optimal switching problem by targeting cost minimization. Inspired by the above, this study ponders nonlinear delayed systems (NDSs) as a class of special switching systems in which the system coefficients exhibit continuity at switching points and the subsystems do not recur. Interval matrix method (IMM) has been applied in memristive neural networks to address connecting weights [2, 5], but it is now rarely adopted in time-varying systems. The essence of IMM can be briefly formulated as an uncertainty analysis of systems with convex analysis and matrix theory. The salient contributions of this study are encapsulated below: (1) Switching analysis and IMM are exploited to handle the time-varying parameters of systems, thereby time-varying systems are regarded as a special class of uncertain switching subsystems with time-varying parameters which are contained in the closures. (2) Building upon the above switching analysis and IMM, a novel type of piecewise state feedback controllers (PSFCs) is proposed for imposing more precise control on the subsystems. (3) Two switching strategies and a grid search algorithm (GSA)-based optimization algorithm were introduced to pinpoint these switching points with the goal of minimizing energy cost globally.

Problem formulation. Consider the following NDSs:

$$\begin{cases} \dot{\mathsf{x}}(t) = \mathcal{H}(\mathsf{x}(t), \mathsf{x}(t-\iota(t)), t) + \mathcal{B}v(t) + \mathcal{G}u(t), \\ \mathsf{z}(t) = E(t)\mathsf{x}(t) + \mathcal{W}v(t), \end{cases} \tag{1}$$

where $\mathsf{x}(t)$ indicates the state vector over $\mathbb{R}^n$; $\iota(t) \in \mathbb{R}^\iota$ is a time-varying delay which satisfies $\dot{\iota}(t) \leqslant h < 1$ and $0 \leqslant \iota(t) \leqslant \iota$; $v(t) \in \mathbb{R}^v$ stands for an exogenous disturbance; $u(t) \in \mathbb{R}^u$ as well as $\mathsf{z}(t) \in \mathbb{R}^z$ indicates the control input and controlled output, respectively. Furthermore, $\mathcal{H}(\mathsf{x}(t), \mathsf{x}(t-\iota(t)), t) := \mathcal{A}(t)\mathsf{x}(t) + \mathcal{A}_d(t)\mathsf{x}(t-\iota(t)) + f(\mathsf{x}(t), t) + g(\mathsf{x}(t-\iota(t)), t)$, where $E(t)$, $\mathcal{A}(t)$, and $\mathcal{A}_d(t)$ are the matrix-valued functions related to t; $\mathcal{B}$, $\mathcal{G}$, $\mathcal{W}$ denote constant matrices with compatible dimensions; $f(\cdot,\cdot) : \mathbb{R}^n \times \mathbb{R}_+ \to \mathbb{R}^n$, $g(\cdot,\cdot) : \mathbb{R}^n \times \mathbb{R}_+ \to \mathbb{R}^n$ are nonlinear functions. Without loss of generality, suppose $\mathsf{x}(0) = \varsigma = \{\varsigma(s) : -\iota \leqslant s \leqslant 0\}$, where $\varsigma(s)$ denotes the real-valued function. Moreover, the time-varying control gain $K(t)$ is introduced, such that the PSFCs are described as $u(t) = K(t)\mathsf{x}(t)$. The energy cost required for achieving control of the NDSs is defined as $\mathcal{J} = \int_{t_0}^{T_f} |u(s)|^2 \mathrm{d}s$. We next proceed with switching analysis on NDSs, transforming the systems into a special class of nonlinear delayed switching systems (NDSSs) as follows:

$$\begin{cases} \mathrm{d}\mathsf{x}(t) = [\mathcal{A}_{\sigma(t)}(t)\mathsf{x}(t) + \mathcal{A}_{d\sigma(t)}(t)\mathsf{x}(t-\iota(t)) + f_{\sigma(t)}(\mathsf{x}(t), t) \\ \qquad\qquad + g_{\sigma(t)}(\mathsf{x}(t-\iota(t)), t) + \mathcal{B}v(t) + \mathcal{G}u_{\sigma(t)}(t)]\mathrm{d}t, \\ \mathsf{z}(t) = E_{\sigma(t)}(t)\mathsf{x}(t) + \mathcal{W}v(t), \end{cases} \tag{2}$$

where the switching signal is defined as $\sigma(t) : [t_0, T_f] \to \mathbb{M} \equiv \{1, 2, \ldots, m\}$, $m \in \mathbb{Z}_+$. Following [4], the switching

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sequence is recognized as $\mathcal{C} = ((t_0, i_0), (t_1, i_1), \ldots, (t_q, i_q))$, with $t_0 \leqslant t_1 \leqslant \cdots \leqslant t_q \leqslant T_f$, where t_0 and T_f refer to the times of initiation and finalization, respectively. q indicates the number of switching, which is a positive integer. One step further, (t_p, i_p) signifies the switch from the subsystem i_{p-1} to i_p at time t_p, where $p \in \{1, 2, \ldots, q\}$. Considering the switching function, $i(t)$ is provided by $i(t) = i_p$ when $t_p \leqslant t \leqslant t_{p+1}$, and t_{q+1} is specified as T_f.

Remark 1. The core concept of switching analysis delineated in this study is considering the time-varying parameters of NDSs through segmentation, transforming the systems (1) into a series of special NDSSs. Consequently, precise control can be attained to better cope with the challenges posed by abrupt variations in coefficients.

Besides, when considering subsystems active, the energy cost of NDSSs can be defined as

$$\mathcal{J}(t_1, t_2, \ldots, t_{m-1}) = \int_{t_0}^{T_f} |u(s)|^2 \mathrm{d}s = \int_{t_0}^{t_1} |u_1(s)|^2 \mathrm{d}s + \cdots + \int_{t_{m-1}}^{T_f} |u_m(s)|^2 \mathrm{d}s = \sum_{i=1}^{m} \int_{t_{i-1}}^{t_i} |u_i(s)|^2 \mathrm{d}s. \tag{3}$$

Remark 2. In general, if control cost is not taken into account, it always allows the controllable system to remain stabilized. On the other hand, although a system may possess controllability in principle, its practical controllability could be rendered impossible if it necessitates inexhaustible energy cost. In [1], they refer to the switching controllers in accordance with the state values of the systems. Instead, we contemplate designing a novel type of PSFCs that relies on time-switching to attain the reduction in energy cost.

When focusing on the ith subsystem, IMM analysis is employed to re-express the NDSSs below:

$$\begin{cases} \mathrm{d}\mathsf{x}(t) \in \{[\mathcal{A}_i + H_{ai}L_{ai}(t)M_{ai} + \mathcal{G}K_i]\mathsf{x}(t) + [\mathcal{A}_{di} + H_{di}L_{di}(t)M_{di}]\mathsf{x}(t - \iota(t)) + f_i(\mathsf{x}(t), t) + g_i(\mathsf{x}(t - \iota(t)), t) + \mathcal{B}v(t)\}\mathrm{d}t, \\ \mathsf{z}(t) \in [E_i + H_{ei}L_{ei}(t)M_{ei}]\mathsf{x}(t) + \mathcal{W}v(t). \end{cases} \tag{4}$$

The details of IMM analysis and preparations are presented in Appendix A.

Theorem 1. For given several positive constants d_1, β, T_f, m, α, l_{1i}, l_{2i}, and a matrix $\mathsf{R} > 0$, the NDSSs (2) achieve finite-time boundedness (FTB) regarding (d_1, d_2, β, T_f, R, σ), if there exist positive scalars $\mu_i > 1$, d_2, ϵ_{fi}, ϵ_{gi}, and positive definite matrices P_i, S_i, satisfying

$$\begin{aligned} &\begin{bmatrix} \Gamma & \Xi \\ * & \Theta \end{bmatrix} < 0, \\ &\Omega < 0, \\ &P_i < \mu_i P_j, S_i < \mu_i S_j, j = i-1, \forall i, j \in \mathbb{M}. \end{aligned} \tag{5}$$

Theorem 2. For given some positive constants d_1, β, T_f, m, α, l_{1i}, l_{2i}, γ, the NDSSs (2) achieve H_∞ FTB with respect to (d_1, d_2, β, T_f, R, σ) by setting H_∞ performance index $\gamma^\star = (\mathrm{e}^{\alpha T_f} \prod_{i=2}^m \mu_i)^{\frac{1}{2}}\gamma$, if there exist positive scalars $\mu_i > 1$, d_2, ϵ_{fi}, ϵ_{gi}, and positive definite matrices P_i, S_i, such that

$$\begin{aligned} &\begin{bmatrix} \Pi & \acute{\Xi} \\ * & \Theta \end{bmatrix} < 0, \\ &\tilde{\Omega} < 0, \\ &P_i < \mu_i P_j, S_i < \mu_i S_j, j = i-1, \forall i, j \in \mathbb{M}. \end{aligned} \tag{6}$$

Theorem 3. Combining PSFCs, for given some positive constants d_1, β, T_f, m, α, l_{1i}, l_{2i}, γ, if there exist positive scalars $\mu_i > 1$, ϵ_{fi}, ϵ_{gi}, ϵ_{ai}, ϵ_{di}, ϵ_{ei}, d_2, and positive definite matrices P_i, S_i, satisfying the following inequalities and the condition $\tilde{\Omega} < 0$, the closed-loop systems (4) will be H_∞ FTC regarding (d_1, d_2, β, T_f, R, σ), with the same H_∞ performance index:

$$\begin{aligned} &\begin{bmatrix} \Psi & \Sigma \\ * & \tilde{\Theta} \end{bmatrix} < 0, \\ &X_j < \mu_i X_i, U_j < \mu_i U_i, j = i-1, \forall i, j \in \mathbb{M}. \end{aligned} \tag{7}$$

The details and proofs of Theorems 1–3 are provided in Appendixes B–D. The resolution of the above Theorem 3 intricately intertwines with two switching strategies. Strategy 1 contemplates the idea of average segmentation while Strategy 2 pinpoints optimal switching points through an optimized GSA-based algorithm. Two strategies are specifically described and the optimized GSA-based algorithm is proposed in Appendix E.

Remark 3. The introduction of switching Strategies 1 and 2 induces the enforcement of PSFCs, where Strategy 2 is even more oriented to minimize energy cost. An integration of switching analysis and Strategy 2 is undertaken to impose constraints on the linear matrix inequalities (LMIs) for minimizing the redundancy in energy cost. Synthesizing the above discussions, we describe this optimization problem as follows:

$$\min \mathcal{J}(t_1, \ldots, t_{m-1}) \ \text{ s.t.} \begin{cases} \text{LMIs: (7)}, \\ \tilde{\Omega} \text{ is satisfied.} \end{cases} \tag{8}$$

Simulations are discussed in Appendix F.

Conclusion. This work delved into the H_∞ FTC problem for a class of NDSs. Switching analysis and IMM on NDSs were implemented for transforming the time-varying system into a series of special uncertain subsystems. Two switching strategies and PSFCs were devised to derive matrix conditions for H_∞ FTC and identified optimal switching points of examples.

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Supporting information Appendixes A–F. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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