## Appendix A Proof of Theorem 1

In order to prove the equivalence of Problem (15) and Problem (16), we have to demonstrate that they share the identical optimal solution when $f(\eta^*)=0$. It is plain to see that Problem (15) and (16) possess the same the same range of feasible solutions. First, we denote $(\widehat{\mathbf{W}},\widehat{\boldsymbol{\varphi}})\in R_1$ and $(\widehat{\mathbf{W}}^*,\widehat{\boldsymbol{\varphi}}^*)\in R_1$ as feasible and optimal solutions of Problem (15) separately. Then, for any $(\widehat{\mathbf{W}},\widehat{\boldsymbol{\varphi}})$, we have the maximum SSE i.e., $\eta^*$ as follows

$$\begin{aligned}\eta^* &= \frac{\sum_{k\in\mathcal{K}}(\frac{1}{1+e^{-\tau_k\gamma_k(\widehat{\mathbf{W}}_k^*)}}-\frac{1}{1+e^{-\tau_k\widehat{\varphi}_k^*}})}{\sum_{k\in\mathcal{K}}\operatorname{tr}(\widehat{\mathbf{W}}_k^*)}\\ &= \max_{W,\varphi}\frac{\sum_{k\in\mathcal{K}}(\frac{1}{1+e^{-\tau_k\gamma_k(\mathbf{W})}}-\frac{1}{1+e^{-\tau_k\varphi_k}})}{\sum_{k\in\mathcal{K}}\operatorname{tr}(\mathbf{W}_k)}\\ &\geqslant \frac{\sum_{k\in\mathcal{K}}(\frac{1}{1+e^{-\tau_k\gamma_k(\widehat{\mathbf{W}}_k)}}-\frac{1}{1+e^{-\tau_k\widehat{\varphi}_k}})}{\sum_{k\in\mathcal{K}}\operatorname{tr}(\widehat{\mathbf{W}}_k)}.\end{aligned}\tag{A1}$$

Since both $\sum_{k\in\mathcal{K}}\operatorname{tr}(\widehat{\mathbf{W}}_k^*)\geqslant 0$ and $\sum_{k\in\mathcal{K}}\operatorname{tr}(\mathbf{W}_k)\geqslant 0$, we can transform (A1) as

$$\sum_{k\in\mathcal{K}}(\frac{1}{1+e^{-\tau_k\gamma_k(\widehat{\mathbf{W}}_k^*)}}-\frac{1}{1+e^{-\tau_k\widehat{\varphi}_k^*}})-\eta^*\sum_{k\in\mathcal{K}}\operatorname{tr}(\widehat{\mathbf{W}}_k^*)=0,\tag{A2}$$

$$\sum_{k\in\mathcal{K}}(\frac{1}{1+e^{-\tau_k\gamma_k(\widehat{\mathbf{W}}_k)}}-\frac{1}{1+e^{-\tau_k\widehat{\varphi}_k}})-\eta^*\sum_{k\in\mathcal{K}}\operatorname{tr}(\widehat{\mathbf{W}}_k)\leqslant 0.\tag{A3}$$

By combing (A2) and (A3), it is clearly observed that $(\widehat{\mathbf{W}}^*,\widehat{\boldsymbol{\varphi}}^*)$ is the optimal solution for (16). Similarly, we denote $(\breve{\mathbf{W}},\breve{\boldsymbol{\varphi}})\in R_1$ and $(\breve{\mathbf{W}}^*,\breve{\boldsymbol{\varphi}}^*)\in R_1$ be feasible and optimal solutions of Problem (16) respectively. Let $f(\eta^*)=0$ and we have

$$\sum_{k\in\mathcal{K}}(\frac{1}{1+e^{-\tau_k\gamma_k(\breve{\mathbf{W}}_k^*)}}-\frac{1}{1+e^{-\tau_k\breve{\varphi}_k^*}})-\eta^*\sum_{k\in\mathcal{K}}\operatorname{tr}(\breve{\mathbf{W}}_k^*)=0,\tag{A4}$$

$$\sum_{k\in\mathcal{K}}(\frac{1}{1+e^{-\tau_k\gamma_k(\breve{\mathbf{W}}_k)}}-\frac{1}{1+e^{-\tau_k\breve{\varphi}_k}})-\eta^*\sum_{k\in\mathcal{K}}\operatorname{tr}(\breve{\mathbf{W}}_k)\leqslant 0.\tag{A5}$$

By combing (A4) and (A5), it is clearly observed that $(\breve{\mathbf{W}}^*,\breve{\boldsymbol{\varphi}}^*)$ is the optimal solution for (15). Finally, we can verify the equivalence between Problem (15) and (16) only if $f(\eta^*)=0$.

## Appendix B Proof of Theorem 2

We denote $\boldsymbol{V}_k^i=(\mathbf{W}^i,p_t^i,\boldsymbol{\varphi}^i,\boldsymbol{z}^i,\boldsymbol{y}^i)$ and $\boldsymbol{V}_k^{i+1}=(\mathbf{W}^{i+1},p_t^{i+1},\boldsymbol{\varphi}^{i+1},\boldsymbol{z}^{i+1},\boldsymbol{y}^{i+1})$ as the feasible solution sets at the $i$-th and $(i+1)$-th iterations for the problem (28), respectively. So that, we can obtain

$$\begin{aligned}&\sum_{k\in\mathcal{K}}(z_k^{i+1}-T_{e_k}^{i+1})-\eta\sum_{k\in\mathcal{K}}\operatorname{tr}(\mathbf{W}_k^{i+1})\\ &=\max_{\mathbf{W},p_t,\boldsymbol{\varphi},\boldsymbol{z},\boldsymbol{y},\eta}\sum_{k\in\mathcal{K}}(z_k-T_{e_k})-\eta\sum_{k\in\mathcal{K}}\operatorname{tr}(\mathbf{W}_k)\\ &\geqslant\sum_{k\in\mathcal{K}}(z_k^i-\frac{1}{1+e^{-\tau_k\varphi_k^i}})-\eta\sum_{k\in\mathcal{K}}\operatorname{tr}(\mathbf{W}_k^i).\end{aligned}\tag{B1}$$

Moreover, utilizing (17), we can additionally attain

$$\frac{1}{1+e^{-\tau_k\varphi_k^{i+1}}}\leqslant\frac{1}{1+e^{-\tau_k\varphi_k^i}}+\frac{\tau_k e^{-\tau_k\varphi_k^i}}{(1+e^{-\tau_k\varphi_k^i})^2}(\varphi_k^{i+1}-\varphi_k^i).\tag{B2}$$

Subsequently, adhering to the iterative method outlined in (28), we conclude with

$$\begin{aligned}&\sum_{k\in\mathcal{K}}(z_k^{i+1}-\frac{1}{1+e^{-\tau_k\varphi_k^{i+1}}})-\eta\sum_{k\in\mathcal{K}}\operatorname{tr}(\mathbf{W}_k^{i+1})\\ &\geqslant\sum_{k\in\mathcal{K}}(z_k^{i+1}-\frac{1}{1+e^{-\tau_k\varphi_k^i}}-\frac{\tau_k e^{-\tau_k\varphi_k^i}}{(1+e^{-\tau_k\varphi_k^i})^2}(\varphi_k^{i+1}-\varphi_k^i))-\eta\sum_{k\in\mathcal{K}}\operatorname{tr}(\mathbf{W}_k^{i+1})\\ &=\sum_{k\in\mathcal{K}}(z_k^{i+1}-T_{e_k}^{i+1})-\eta\sum_{k\in\mathcal{K}}\operatorname{tr}(\mathbf{W}_k^{i+1})\\ &\geqslant\sum_{k\in\mathcal{K}}(z_k^i-\frac{1}{1+e^{-\tau_k\varphi_k^i}})-\eta\sum_{k\in\mathcal{K}}\operatorname{tr}(\mathbf{W}_k^i).\end{aligned}\tag{B3}$$

Based on (B3), it is evident that the iterative process (28) consistently yields solutions that are no worse than the previous iteration, ensuring a monotonically nondecreasing sequence as the number of iterations grows. Furthermore, employing the transmit power constraint $\mathrm{tr}(\mathbf{W}_k) \leqslant p_t$ and utilizing the Cauchy–Schwarz inequality, we can derive an upper bound for the objective function, as illustrated below

$$\sum_{k\in\mathcal{K}} \left(\frac{1}{1+e^{-\tau_k\gamma_k}} - \frac{1}{1+e^{-\tau_k\varphi_k}}\right) - \eta\sum_{k\in\mathcal{K}} \mathrm{tr}(\mathbf{W}_k) \leqslant \sum_{k\in\mathcal{K}} \frac{1}{1+e^{-\tau_k\gamma_k^{\max}}} \tag{B4}$$

where $\gamma_k^{\max} = \frac{p_t \mathrm{tr}(\mathbf{H}_k)}{\sigma^2}$ denotes the maximum value of the signal-to-noise ratio. Upon integrating (B3) and (B4), we can ascertain that the iterative procedure defined in (28) will converge to a $\epsilon$-optimal solution of (16) upon completion of a sufficient number of iterations.

## Appendix C Proof of Theorem 3

We denote the objective function as

$$f(\tau_k) = \frac{1}{\tau_k}\left(\frac{1}{1+e^{-\tau_k\gamma_k}} - \frac{1}{1+e^{-\tau_k\varphi_k}}\right). \tag{C1}$$

We set $\tau_k\varphi_k = u_k$ and then $\tau_k\gamma_k = v_k u_k$, where $v_k = \frac{\gamma_k}{\varphi_k} > 1$. The fomula (C1) can be expressed as

$$f(y_k) = \frac{\varphi_k}{u_k}\left(\frac{1}{1+e^{-v_k u_k}} - \frac{1}{1+e^{-u_k}}\right) = \varphi_k \hat{f}(u_k). \tag{C2}$$

We construct function $g(u_k, v_k)$ and its expression can be written as

$$g(u_k, v_k) = \frac{1}{u_k}\frac{1}{1+e^{-v_k u_k}}, \tag{C3}$$

then

$$\hat{f}(u_k) = g(u_k, v_k) - g(u_k, 1). \tag{C4}$$

The first derivative of $\hat{f}(u_k)$ is

$$\begin{aligned} \frac{d\hat{f}(u_k)}{du_k} &= \dot{g}(u_k, v_k) - \dot{g}(u_k, 1) \\ &= \frac{(v_k u_k - 1)e^{-v_k u_k} - 1}{u_k^2(1+e^{-v_k u_k})^2} - \frac{(u_k - 1)e^{-u_k} - 1}{u_k^2(1+e^{-u_k})^2}, \end{aligned} \tag{C5}$$

where $\dot{g}(u_k, v_k)$ and $\dot{g}(u_k, 1)$ represent the first-order partial derivatives of the function $g(u_k, v_k)$ and $g(u_k, 1)$ with respect to $u_k$ respectively. The derivative of $\dot{g}(u_k, v_k)$ with respect to $v_k$ is

$$\frac{\partial \dot{g}(u_k, v_k)}{\partial v_k} = \frac{g_1 - g_2}{\left(u_k^2(1+e^{-v_k u_k})^2\right)^2}, \tag{C6}$$

where

$$g_1 = u_k^3 e^{-v_k u_k}(2 - v_k u_k)(1+e^{-v_k u_k})^2, \tag{C7}$$

$$g_2 = 2u_k^3 e^{-v_k u_k}(1+e^{-v_k u_k})\left(1 - (v_k u_k - 1)e^{-v_k u_k}\right). \tag{C8}$$

The formula $g_1 - g_2$ can be expanded as

$$g_1 - g_2 = v_k u_k^4 e^{-v_k u_k}(1+e^{-v_k u_k})(e^{-v_k u_k} - 1). \tag{C9}$$

According to (C9), it is evident that $g_1 - g_2$ is less than zero. Then, function $\dot{g}(u_k, v_k)$ monotonically decreases with respect to $v_k$, i.e., $\dot{g}(u_k, v_k) \leqslant \dot{g}(u_k, 1)$, making $\frac{d\hat{f}(u_k)}{du_k} \leqslant 0$. Therefore, $\hat{f}(u_k)$ is a monotonically decreasing function about $u_k$, which is equivalent that $f(\tau_k)$ is a monotonically decreasing function about $\tau_k$.

## Appendix D Proof of Lemma 1

Firstly, the SINR expression in (37a) can be rewritten as

$$\gamma_{k,n} = \frac{\left|\mathbf{h}_{k,n}^H \mathbf{w}_k\right|^2}{\sum_{i\in\mathcal{K},i\neq k}\left|\mathbf{h}_{k,n}^H \mathbf{w}_i\right|^2 + \sigma_n^2} \leqslant \gamma_e, \tag{D1a}$$

$$\Leftrightarrow \mathbf{h}_{k,n}^H(\mathbf{W}_k - \gamma_e \sum_{i\in\mathcal{K},i\neq k}\mathbf{W}_i)\mathbf{h}_{k,n} \leqslant \gamma_e\sigma_n^2, \tag{D1b}$$

$$\Leftrightarrow \mathbf{h}_{k,n}^H\mathbf{Q}_k\mathbf{h}_{k,n} + \gamma_e\sigma_n^2 \geqslant 0. \tag{D1c}$$

where we denote $\mathbf{Q}_k \triangleq \gamma_e\sum_{i\in\mathcal{K},i\neq k}\mathbf{W}_i - \mathbf{W}_k$. By substituting $\mathbf{h}_{k,n} = \hat{\mathbf{h}}_{k,n} + \triangle\mathbf{h}_{k,n}$ into (D1c), we get the equivalent expression as follows

$$(D1c) \Leftrightarrow (\hat{\mathbf{h}}_{k,n} + \triangle\mathbf{h}_{k,n})^H\mathbf{Q}_k(\hat{\mathbf{h}}_{k,n} + \triangle\mathbf{h}_{k,n}) + \gamma_e\sigma_n^2 \geqslant 0, \tag{D2a}$$

$$\Leftrightarrow \underbrace{\triangle\mathbf{h}_{k,n}^H\mathbf{Q}_k\triangle\mathbf{h}_{k,n}}_{f_1} + 2\mathbf{Re}\{\underbrace{\hat{\mathbf{h}}_{k,n}^H\mathbf{Q}_k\triangle\mathbf{h}_{k,n}}_{f_2}\} + \underbrace{\hat{\mathbf{h}}_{k,n}^H\mathbf{Q}_k\mathbf{h}_{k,n} + \gamma_e\sigma_n^2}_{c_{k,n}} \geqslant 0. \tag{D2b}$$

We represent the CSI error $\triangle\mathbf{h}_{k,n}$ as $\triangle\mathbf{h}_{k,n} = \mathbf{\Sigma}_{k,n}^{1/2}\mathbf{v}_{k,n}$ , where $\mathbf{v}_{k,n} \sim \mathcal{CN}(\mathbf{0},\mathbf{I})$, $\mathbf{\Sigma}_{k,n} = \mathbf{\Sigma}_{k,n}^{1/2}\mathbf{\Sigma}_{k,n}^{1/2}$ and $(\mathbf{\Sigma}_{k,n}^{1/2})^H = \mathbf{\Sigma}_{k,n}^{1/2}$, so that the expression $f_1$ in (D2b) can be reformulated as

$$f_1 = (\mathbf{\Sigma}_{k,n}^{1/2}\mathbf{v}_{k,n})^H\mathbf{Q}_k(\mathbf{\Sigma}_{k,n}^{1/2}\mathbf{v}_{k,n}), \tag{D3a}$$

$$= \mathbf{v}_{k,n}^H\mathbf{\Sigma}_{k,n}^{1/2}\mathbf{Q}_k\mathbf{\Sigma}_{k,n}^{1/2}\mathbf{v}_{k,n}, \tag{D3b}$$

$$\triangleq \mathbf{v}_{k,n}^H\mathbf{A}_{k,n}\mathbf{v}_{k,n}, \tag{D3c}$$

where $\mathbf{A}_{k,n} \triangleq \mathbf{\Sigma}_{k,n}^{1/2}\mathbf{Q}_k\mathbf{\Sigma}_{k,n}^{1/2}$.
Similarily, the expression $f_2$ in (D2b) can be reformulated as

$$f_2 = \hat{\mathbf{h}}_{k,n}^H\mathbf{Q}_k\mathbf{\Sigma}_{k,n}^{1/2}\mathbf{v}_{k,n}, \tag{D4a}$$

$$\triangleq \mathbf{u}_{k,n}^H\mathbf{v}_{k,n}, \tag{D4b}$$

where $\mathbf{u}_{k,n}^H \triangleq \hat{\mathbf{h}}_{k,n}^H\mathbf{Q}_k\mathbf{\Sigma}_{k,n}^{1/2}$. And the expression $c_{k,n}$ in (D2b) can be restructured as

$$c_{k,n} = \mathrm{tr}(\hat{\mathbf{h}}_{k,n}^H\mathbf{Q}_k\mathbf{h}_{k,n}) + \gamma_e\sigma_n^2. \tag{D5}$$

By substituting (D3c), (D4b) and (D5) into (D2b), we have

$$\mathbf{v}_{k,n}^H\mathbf{A}_{k,n}\mathbf{v}_{k,n} + \mathbf{u}_{k,n}^H\mathbf{v}_{k,n} + c_{k,n} \geqslant 0. \tag{D6}$$

Therefore, the per-eavesdropper secrecy outage probability constraint can be given by

$$Pr_{h_{k,n}}\{\mathbf{v}_{k,n}^H\mathbf{A}_{k,n}\mathbf{v}_{k,n} + 2\mathbf{Re}\{\mathbf{u}_{k,n}^H\mathbf{v}_{k,n}\} + c_{k,n} \geqslant 0\} \geqslant 1 - \rho_k. \tag{D7}$$

The equivalence stated in (D7) indicates that the outage probability defined in (37a) can be described through a quadratic inequality pertaining to the Gaussian random vector $\mathbf{v}_{k,n}$.
By invoking the BTI, we can convert the chance constraint (D7) into a deterministic constraint, which furnishes (D7) with a reliable approximation and is given by

$$(D7) \Leftrightarrow \mathrm{tr}(\mathbf{A}_{k,n}) - \sqrt{2\ln(1/\rho_{k,n})}\sqrt{\left\|\mathbf{A}_{k,n}\right\|_F^2 + 2\left\|\mathbf{u}_{k,n}\right\|^2} + \ln(\rho_{k,n})\lambda^+(-\mathbf{A}_{k,n}) + c_{k,n} \geqslant 0, \tag{D8a}$$

$$\Leftrightarrow \begin{cases} \mathrm{tr}(\mathbf{A}_{k,n}) - \sqrt{-2\ln(\rho_{k,n})}p_{k,n} + \ln(\rho_{k,n})q_{k,n} + c_{k,n} \geqslant 0, \\ \left\|\begin{bmatrix}\mathrm{vec}(\mathbf{A}_{k,n}) \\ \sqrt{2}\mathbf{u}_{k,n}\end{bmatrix}\right\|_2 \leqslant p_{k,n}, \\ q_{k,n}\mathbf{I} + \mathbf{A}_{k,n} \succeq 0, q_{k,n} \geqslant 0. \end{cases} \tag{D8b}$$

where $\lambda^+(\mathbf{A}_{k,n}) = \max(\lambda_{\max}(\mathbf{A}_{k,n}), 0)$. $p_{k,n}$ and $q_{k,n}$ are slack variables.