SCIENCE CHINA
Information Sciences

• RESEARCH PAPER •

March 2025, Vol. 68, Iss. 3, 132207:1–132207:17
https://doi.org/10.1007/s11432-024-4149-x

Controllability of heterogeneous networked sampled-data systems

Zixuan YANG[1,2], Lin WANG[1,2*], Xiaofan WANG[1,3,4] & Guanrong CHEN[5]

[1]*Department of Automation, Shanghai Jiao Tong University, Shanghai 200240, China*
[2]*Key Laboratory of System Control and Information Processing, Ministry of Education of China, Shanghai 200240, China*
[3]*School of Mechatronic Engineering and Automation, Shanghai University, Shanghai 200444, China*
[4]*School of Electrical and Electronic Engineering, Shanghai Institute of Technology, Shanghai 201418, China*
[5]*Department of Electrical Engineering, City University of Hong Kong, Hong Kong 999077, China*

Received 8 January 2024/Revised 23 March 2024/Accepted 9 September 2024/Published online 8 February 2025

Abstract This study investigates the controllability of a general heterogeneous networked sampled-data system (HNSS) consisting of nonidentical node systems, where the inner coupling between any pair of nodes can be described by a unique matrix. The signals on control and transmission channels are sampled and held by zero-order holders, and the control sampling period of each node can be different. Necessary and sufficient controllability conditions are developed for the general HNSS, using the Smith normal form and matrix equations, respectively. The HNSS in specific topology or dynamic settings is discussed subsequently with easier-to-verify conditions derived. These heterogeneous factors have been determined to independently or jointly affect the controllability of networked sampled-data systems. Notably, heterogeneous sampling periods have the potential to enhance the overall controllability, but not for systems with some special dynamics. When the node dynamics are heterogeneous, the overall system can be controllable even if it is topologically uncontrollable. In addition, in several typical heterogeneous sampled-data multi-agent systems, pathological sampling of single-node systems will necessarily cause overall uncontrollability.

Keywords network controllability, sampled-data system, heterogeneous system, inner coupling, multi-agent system

Citation Yang Z X, Wang L, Wang X F, et al. Controllability of heterogeneous networked sampled-data systems. Sci China Inf Sci, 2025, 68(3): 132207, https://doi.org/10.1007/s11432-024-4149-x

1 Introduction

Controllability is a fundamental issue in control theory, which has been thoroughly investigated since the 1960s [1]. In recent years, this concept has been extended from single systems to networks [2–6]. The complexity of networking is a major challenge to classical controllability criteria, such as the Popov-Belevitch-Hautus (PBH) rank condition and Kalman criterion, for numerous nodes and intricate connections in complex networks that will lead to the curse of dimensionality. Structural controllability theory [7, 8] establishes a direct relationship between the graphical properties of a network and its controllability, which solves the aforementioned problem to some extent. However, the results of structural controllability are not equivalent to those of state controllability when edge weights are fixed or constrained to certain special values.

In networked multiple input-multiple output (MIMO) systems, node states are multidimensional and coupled with each other through multiple transmission channels [9]. Some controllability conditions with less computation have been established. For instance, in [10], a decomposition method is proposed for the eigenspace of state matrices and is applied to the analysis of target controllability [11]. In [12], more convenient conditions are given by dividing eigenvalues into invariant modes and repeated modes. In [13], the controllability of multilayer networks with a deep-coupling mode is investigated. As a special variant of networked MIMO systems, multi-agent systems (MASs) have particular characteristics related to the Laplacian matrix and can be decoupled into two independent parts corresponding to node dynamics and network topology [14]. Li et al. [15] established an equivalence between the controllability of the linearized Turing's model and that of a Laplace dynamic system with first-order agents and determined the minimal number and placement of control nodes in specific graphical structures. In [16, 17], the

* Corresponding author (email: wanglin@sjtu.edu.cn)

controllability of MASs is examined based on graphical structures with limitations analyzed from the perspective of equivalent partition. Strategy equivalence partitioning is further proposed in [18] to ensure the controllability of MASs within a Nash equilibrium framework.

Most of the aforementioned studies assume that node systems are identical, which can be unrealistic in practice. Arreola-Delgado and Barajas-Ramírez [19] demonstrated that the controllability of networked systems can be altered when the node dynamics change from homogeneous to heterogeneous. Xiang et al. [20] derived controllability conditions for networked MIMO systems with heterogeneous node dynamics, and Kong et al. [21] further considered nonidentical inner-coupling matrices. In [22], heterogeneous subsystems are designed to ensure the structural controllability of networked systems. However, the heterogeneity of real-life networked systems often has more complex requirements. For example, in an adaptive cruise control task, unmanned vehicles can be considered nonidentical node systems with multiple states (i.e., acceleration, velocity, and displacement). Therefore, the inner couplings between various pairs of states in the system may be different, and the signal communication patterns are diverse. The constructability of more general heterogeneous networks with completely distinct inner couplings is analyzed in [23] by topology identification. In [24], the observability of heterogeneous MASs consisting of first-order agents and second-order agents is verified. Although the aforementioned studies have investigated other properties similar to controllability, state controllability analysis methods that can be applied to more heterogeneous systems have not yet been developed.

Compared with the continuous-time model, the networked sampled-data system model can more faithfully describe the actual system, where inputs and outputs are converted into digital signals by sample-and-hold circuits, acting on continuous node states through control or transmission channels. Given the differences between controllable and reachable subspaces, the controllability of a discrete-time system may not be equal to the continuous-time system with the same system matrix [25]. Intuitively, sampling leads to information loss, which, in turn, results in compromised system performance. Pathological sampling [26] describes the phenomenon that a continuous-time system loses controllability after periodic sampling. In [27], a sufficient condition on the sampling density that ensures controllability is established using block erasure channels as the main communication platform. In [28], ϵ-controllability is proposed to control a given switched sampled-data system with fewer sampling times in a nonequidistant schedule. In [29], a sparse sampling strategy is presented to preserve observability by taking coarse samples that contain sufficient information. A nonequidistant sampling pattern is proposed in [30] to avoid pathological sampling based on Kronecker's theorem, and a sufficient controllability condition is given in [31] for networks with nonidentical control sampling rates. In [32], the controllability of linear discrete-time impulsive hybrid systems with input delay is analyzed by constructing the null reachable and controllable sets.

Relevant to this work, the structural controllability of MASs with switching topology and sampled data is investigated in [33]. In [34], asynchronous and synchronous protocols are considered for the data-sampling controllability of MASs, where input matrices of agents are only one-dimensional. In [35], the controllability of MASs with the matrix-weight-based signed network is analyzed, which solves the problem of dimensionality increase caused by generalizing weights from scalars to matrices. The controllability of MASs consisting of heterogeneous agents with different time delays due to an absolute protocol is discussed in [36]. Among these existing studies, the data-sampling controllability of MASs consisting of agents with nonidentical dynamics and sampling periods is rarely noticed. Moreover, in [37], we established a more general networked sampled-data system with sample-and-hold circuits not only on control channels but also on transmission channels and determined that pathological sampling of single-node systems can be eliminated by proper design of inner couplings and topology. However, these results are derived based on the assumption of homogeneity and cannot be simply generalized to heterogeneous systems.

Based on the previously presented analysis, we observed that most of the existing studies on the controllability of networked systems and MASs ignore the heterogeneity of inner couplings and rarely examine the nonidentical sampling periods. The main contributions of this paper are threefold.

(1) A more general heterogeneous networked sampled-data system (HNSS) model is formulated, where node dynamics, inner-coupling matrices, and control sampling periods are nonidentical. Correspondingly, the sampled-data MAS model with heterogeneous dynamics and sampling periods is developed.

(2) Controllability conditions are developed for the established model by the Smith normal form and matrix equations, which have lower computational complexity than the classic criteria. A necessary and sufficient criterion is established for general discrete-time linear time-invariant (LTI) systems, which

supplements the PBH rank condition and further improves our results in [37]. For systems with special topology or dynamics, more easy-to-follow conditions are derived correspondingly.

(3) The influence of heterogeneous factors on the overall sampling controllability is depicted, which fills the research gap in the existing literature. The nonpathological sampling condition is extended from a single system to the network setting. We determined that nonidentical control sampling periods can enhance the overall controllability, but not for systems with node dynamics in the one-dimensional or self-loop setting. In addition, the overall system can be controllable even if the network topology is uncontrollable when no node dynamics are involved. The pathological sampling of single-node systems will inevitably lead to the loss of overall controllability in some typical heterogeneous sampled-data MASs.

2 Preliminaries

2.1 Notation and definitions

Notation. Let $\mathbb{R}$, $\mathbb{C}$, $\mathbb{N}$, and $\mathbb{N}^+$ denote the fields of real, complex, natural numbers, and positive integers, respectively. Let $\mathbb{R}^n$ and $\mathbb{C}^n$ denote the n-dimensional real and complex column vectors, respectively, and $\mathbb{R}^{m\times n}$ and $\mathbb{C}^{m\times n}$ denote the sets of $m\times n$ real and complex matrices, respectively. Let $\sigma(A)$ denote the set of all eigenvalues of matrix $A\in\mathbb{C}^{n\times n}$. Let $\mathrm{span}\{v_1,\ldots,v_s\}$ be the space with the basis of row vectors $v_1,\ldots,v_s$. Let $M(\lambda|A)$ denote the eigenspace spanned by left eigenvectors of A with respect to eigenvalue λ. Let $A\otimes B$ denote the Kronecker product of matrices A and B and $\mathrm{diag}\{A_1,\ldots,A_N\}$ denote the block diagonal matrix with $A_1,\ldots,A_N$ on the diagonal. Let A^{T} denote the transpose of matrix A and A^{-1} denote the inverse, with $\mathrm{rank}(A)$ indicating the row rank of A. Let $\mathcal{N}_L(A)$ denote the left null space of matrix $A\in\mathbb{C}^{n\times m}$, which is spanned by all of the row vectors in $\{\xi\in\mathbb{C}^{1\times n}|\xi A=0\}$. Let I_n denote the $n\times n$ identity matrix, and $\mathbf{1}_n$ denote the n-dimensional column vector consisting of entries of 1. The dimension of space V is defined as $\dim(V)$, and the direct sum of spaces V_1 and V_2 is defined as $V_1\oplus V_2$. Always assume that the dimensions of matrices and vectors are compatible in algebraic operations.

Definition 1 ([38])**.** A row vector v_m is an mth-order generalized left eigenvector of $A\in\mathbb{R}^{n\times n}$ corresponding to $\lambda\in\sigma(A)$, if $v_m(A-\lambda I_n)^m=0$ and $v_m(A-\lambda I_n)^{m-1}\neq 0$. $v_1,v_2,\ldots,v_\alpha$ form a left Jordan chain of matrix A, where the maximum value of α is the length of the chain.

Definition 2 ([10])**.** Given $A,B\in\mathbb{C}^{n\times n}$, and $\lambda\in\sigma(A)$, if row vectors $\xi_1,\xi_2,\ldots,\xi_\beta$ satisfy $\xi_1(\lambda I_n-A)=0$, and $\xi_j(\lambda I_n-A)=\xi_{j-1}B$ for $j=2,\ldots,\beta$, then they form a generalized left Jordan chain of A about B corresponding to λ, where the maximum value of β is the length of the chain.

Definition 3 ([25])**.** Given a polynomial matrix $P(s)\in\mathbb{C}[s]^{m\times n}$, the minors of $P(s)$ of order i are the determinants of all $i\times i$ square submatrices of $P(s)$. The determinantal divisor of $P(s)$ is the polynomial $\{D_i(s):0\leqslant i\leqslant r\}$, where $D_0(s)=1$ and $D_i(s)$ is the monic greatest common divisor of all nonzero minors of $P(s)$ of order i. Let $r\triangleq\mathrm{rank}(P(s))$ denote the maximum order of a nonzero minor of $P(s)$.

Definition 4 ([25])**.** Any polynomial matrix $P(s)\in\mathbb{C}[s]^{m\times n}$ can be equivalently transformed into the Smith normal form, which is the diagonal polynomial matrix $S_P(s)=\mathrm{diag}\{d_1(s),\ldots,d_r(s),0,\ldots,0\}$, where $r=\mathrm{rank}(P(s))$ and $d_i(s)\triangleq D_i(s)/D_{i-1}(s)$, $i=1,\ldots,r$, are called the invariant factors of $P(s)$.

Lemma 1 ([25])**.** Given that $P(s)\in\mathbb{C}[s]^{m\times n}$ and the determinantal divisors of $P(s)$ are $\{D_i(s):0\leqslant i\leqslant r\}$, then $D_i|D_{i+1}$, i.e., D_{i+1} is divisible by D_i for $i=0,\ldots,r-1$.

Lemma 2 (The fundamental theorem of algebra [39])**.** If $f(x)$ denotes a nonconstant polynomial in $\mathbb{C}[x]$, then $f(x)$ has at least one complex foot.

Lemma 3 ([25])**.** The discrete-time LTI system $x(k+1)=Ax(k)+Bu(k)$, where $x(k)\in\mathbb{R}^n, k\in\mathbb{N}$, is state controllable if any one of the following conditions holds:

(1) (PBH rank condition) $\forall\,\lambda\in\sigma(A)$ and $\forall\,\xi\in M(\lambda|A)$ with $\xi\neq 0$, one has $\xi B\neq 0$;

(2) (Kalman criterion) $\mathrm{rank}([B,AB,\ldots,A^{n-1}B])=n$.

Remark 1. In the subsequent text, pair (A,B) is called controllable if and only if matrices A and B satisfy any one of the conditions in Lemma 3. In [25], for the continuous-time LTI system: $\dot{x}(t)=Ax(t)+Bu(t)$, $x(t)\in\mathbb{R}^n$, $t>0$, the PBH rank condition and Kalman criterion are necessary and sufficient. By contrast, for the discrete-time LTI system with a singular matrix A, the PBH rank condition and Kalman criterion are only sufficient but unnecessary. Thus, the controllability of a discrete-time LTI system does not always mean that (A,B) is controllable. Notably, Lemma 3 becomes necessary and sufficient when A is nonsingular.

2.2 Pathological sampling

Pathological sampling refers to the phenomenon that a continuous-time system loses controllability because of data sampling. In [26], a nonpathological condition is derived for the control signal sampling of the LTI system $\dot{x}(t) = Ax(t) + Bu(t)$, $x(t) \in \mathbb{R}^n$, $t > 0$, as stated in Lemma 4. The corresponding sampled-data system can be obtained by periodic sampling using a zero-order holder (ZOH) on the control channel, as $x((k+1)h) = \mathcal{A}x(kh) + \mathcal{B}u(kh)$, where $\mathcal{A} = \mathrm{e}^{Ah}$, $\mathcal{B} = \int_0^h \mathrm{e}^{A\tau}\mathrm{d}\tau B$, with $h > 0$ being the sampling period, $k \in \mathbb{N}$. The analog-to-digital data transform by the ZOH circuit can be understood as a time-triggered mechanism. At each periodic sampling instant, the signal value on the channel is updated; otherwise, it retains the previous value.

Lemma 4 ([26]). The sampling period h is nonpathological about A if $\forall \lambda_p, \lambda_q \in \sigma(A)$,

$$\lambda_p - \lambda_q \neq \frac{2k\pi}{h}\mathrm{i},\ k = \pm 1, \pm 2, \ldots. \tag{1}$$

If the sampling period satisfies the nonpathological condition in Lemma 4, then (A, B) controllable $\Longrightarrow$ $(\mathcal{A}, \mathcal{B})$ controllable. According to the spectrum mapping theorem, if $\sigma(A) = \{\lambda_1, \ldots, \lambda_r\}$, then $\sigma(\mathcal{A}) = \{\mathrm{e}^{\lambda_1 h}, \ldots, \mathrm{e}^{\lambda_r h}\}$. Therefore, an inappropriate selection of the sampling period h will increase the geometric multiplicity of some eigenvalues of the state matrix if $\mathrm{e}^{\lambda_p h} = \mathrm{e}^{\lambda_q h}$ but $\lambda_p \neq \lambda_q$, $p, q \in \{1, \ldots, r\}$, which may reduce the dimension of the controllable subspace.

In this study, in addition to the control signals, the output signals transmitted among node systems are sampled and held by ZOHs. Correspondingly, the notion of pathological sampling is extended from the single control system to the network of systems. Although in [37], we determined that the impacts of pathological sampling of single-node systems can be eliminated by an appropriate design of network topology and inner couplings; however, sampling periods are considered heterogeneous on the control channels, with nonidentical node dynamics and inner couplings, making the system model more complicated. Thus, neither Lemma 4 nor the results presented in [37] are applicable anymore, and new controllability conditions with nonpathological sampling analysis will be developed in the present study.

3 General HNSS

In this section, the general HNSS model is established, with some examples showing the effects of heterogeneity on the overall controllability of the system.

3.1 Model formulation

Consider a continuous-time networked system consisting of N nonidentical nodes, where the dynamics of the ith node is expressed as follows:

$$\begin{cases} \dot{x}_i(t) = A_i x_i(t) + \sum_{j=1, j\neq i}^{N} w_{ij} H_{ij} y_j(t) + \delta_i B_i u_i(t), \\ y_i(t) = C_i x_i(t), \end{cases} \tag{2}$$

where $i = 1, \ldots, N$. In this model, $x_i \in \mathbb{R}^n$, $y_i \in \mathbb{R}^m$, and $u_i \in \mathbb{R}^p$ are the state, output, and external control input, respectively, and $A_i \in \mathbb{R}^{n\times n}$, $B_i \in \mathbb{R}^{n\times p}$, and $C_i \in \mathbb{R}^{m\times n}$ are the state, input, and output matrices, respectively. The inner-coupling matrix $H_{ij} \in \mathbb{R}^{n\times m}$ represents the interactions from node j to node i. As shown in Figure 1, the input signal $u_i(t)$ is sent to node i by the ith actuator through the control channel, whereas the output signal $y_i(t)$ is transmitted from node i to other nodes by the ith sensor through the transmission channels. The network topology is expressed as $\delta_i = 1$ if the control channel to node i exists; otherwise, $\delta_i = 0$; $w_{ij} \neq 0$ if the transmission channel from node j to node i exists; otherwise, $w_{ij} = 0$. Assume that no self-rings exist in the network, i.e., $w_{ii} = 0$.

The corresponding sampled-data system can be obtained by implementing ZOHs to convert the analog input and output signals into digital input and output signals, where control sampling periods are heterogeneous. Specifically, let the sampling period of the ith control channel be h_i and the sampling period of each transmission channel be h, with h being a common multiple of $h_1, \ldots, h_N$, and $h = p_i h_i$,

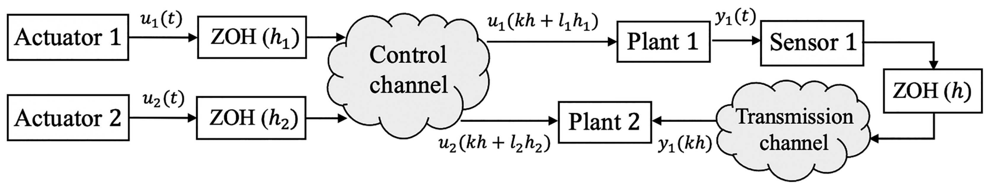

Figure 1 Illustration of the signal transmission process in a general HNSS with two nodes.

$p_i \in \mathbb{N}^+$, $i = 1, \ldots, N$. This system is referred to as the general HNSS, where the dynamics of node i are expressed as follows:

$$\dot{x}_i(t) = A_i x_i(t) + \sum_{j=1, j\neq i}^{N} w_{ij} H_{ij} C_j x_j(kh) + \delta_i B_i u_i(kh + l_i h_i), \tag{3}$$

with $t \in [kh + l_i h_i, kh + (l_i + 1)h_i)$, $k \in \mathbb{N}$, $l_i \in \{0, 1, \ldots, p_i - 1\}$, $i = 1, \ldots, N$. By integrating each interval of length h_i and accumulating the integrals, the system expressed in (3) can be converted into the form of the discrete-time LTI system with intervals of length h, as follows:

$$x_i(kh + h) = \mathrm{e}^{A_i h} x_i(kh) + f(A_i) \sum_{j=1, j\neq i}^{N} w_{ij} H_{ij} C_j x_j(kh) + \sum_{l_i=1}^{p_i} g_{l_i}^{(i)}(A_i) \delta_i B_i u_i(kh + (l_i - 1)h_i), \tag{4}$$

where

$$g_{l_i}^{(i)}(A_i) = \mathrm{e}^{A_i(p_i - l_i)h_i} \int_0^{h_i} \mathrm{e}^{A_i\tau} \mathrm{d}\tau, \quad f(A_i) = \sum_{l_i=1}^{p_i} g_{l_i}^{(i)}(A_i) = \int_0^{h} \mathrm{e}^{A_i\tau} \mathrm{d}\tau. \tag{5}$$

Let $X(kh) = [x_1^{\mathrm{T}}(kh), \ldots, x_N^{\mathrm{T}}(kh)]^{\mathrm{T}}$ be the total state and $\bar{U}(kh) = [\bar{u}_1^{\mathrm{T}}(kh), \ldots, \bar{u}_N^{\mathrm{T}}(kh)]^{\mathrm{T}}$ be the total external control input, where $\bar{u}_i(kh) = [u_i^{\mathrm{T}}(kh), \ldots, u_i^{\mathrm{T}}(kh+(p_i-1)h_i)]^{\mathrm{T}}$, $i = 1, \ldots, N$. Then, the system expressed in (4) can be written in a compact form as follows:

$$X(kh + h) = \Phi X(kh) + \Psi \bar{U}(kh), \tag{6}$$

where

$$\begin{aligned} \Phi &= \begin{bmatrix} \mathrm{e}^{A_1 h} & \cdots & w_{1N} f(A_1) H_{1N} C_N \\ \vdots & \ddots & \vdots \\ w_{N1} f(A_N) H_{N1} C_1 & \cdots & \mathrm{e}^{A_N h} \end{bmatrix}, \\ \Psi &= \mathrm{diag}\{\delta_1 G_1, \delta_2 G_2, \cdots, \delta_N G_N\}, \quad G_i = [g_1^{(i)}(A_i)B_i, g_2^{(i)}(A_i)B_i, \ldots, g_{p_i}^{(i)}(A_i)B_i], \ i = 1, \ldots, N. \end{aligned} \tag{7}$$

Notably, a special case, where $h_1 = h_2 = \cdots = h_N = h$, i.e., the sampling periods are the same for all channels, exists. This case is referred to as heterogeneous networked sampled-data system with identical sampling periods (HNSS-IS) in the subsequent sections, which is still different from the networked sampled-data system investigated in [37] because the node dynamics and the inner couplings are heterogeneous. The HNSS-IS is expressed as follows:

$$X(kh + h) = \Phi X(kh) + \bar{\Psi} U(kh), \tag{8}$$

where $U(kh) = [u_1^{\mathrm{T}}(kh), \ldots, u_N^{\mathrm{T}}(kh)]^{\mathrm{T}}$, and

$$\bar{\Psi} = \mathrm{diag}\{\delta_1 f(A_1) B_1, \ldots, \delta_N f(A_N) B_N\}. \tag{9}$$

3.2 Effects of heterogeneity

Examples 1–3 illustrate the effects of heterogeneous node dynamics, inner couplings, and control sampling periods on the overall controllability of the system, respectively.

Example 1. Consider a chain network consisting of three node systems, where $\delta_1 = 1$, $w_{21} = w_{32} = 1$,

$$A_1 = A_2 = A_3 = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}, B_1 = B_2 = B_3 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}, C_1 = C_2 = C_3 = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}, H_{21} = H_{32} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

Let the sampling periods be $h = \pi$, $h_1 = \pi/2$. The Kalman criterion can verify that this system is controllable with $\text{rank}([\Psi, \Phi\Psi, \Phi^2\Psi, \ldots, \Phi^5\Psi]) = 6$. Then, consider the counterpart in the same system setting, except that the node dynamics is heterogeneous:

$$A_1 = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}, B_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, C_1 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, A_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, B_2 = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix},$$
$$C_2 = \begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix}, A_3 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}, B_3 = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}, C_3 = \begin{bmatrix} 0 & 3 \\ 0 & 0 \end{bmatrix}.$$

The Kalman criterion can verify that this system is uncontrollable, for $0 \notin \sigma(\Phi)$ with $\text{rank}([\Psi, \Phi\Psi, \Phi^2\Psi, \ldots, \Phi^5\Psi]) = 3$.

Example 2. Consider a directed star network consisting of three node systems, where $\delta_1 = \delta_2 = 1$, $w_{21} = w_{31} = 1$, and

$$A_1 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, B_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, C_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, A_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$
$$C_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, A_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, B_3 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, C_3 = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}.$$

Let the sampling periods be $h_1 = \pi/2$, $h_2 = \pi/3$, and $h = \pi$, and the inner-coupling matrices be

$$H_{21} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \ H_{31} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}.$$

The Kalman criterion can verify that this HNSS is controllable with $\text{rank}([\Psi, \Phi\Psi, \Phi^2\Psi, \ldots, \Phi^5\Psi]) = 6$. Then, the sampling rates are changed to be identical, i.e., let $h_1 = h_2 = h = \pi$. The Kalman criterion can verify that $0 \notin \sigma(\Phi)$ and $\text{rank}([\Psi, \Phi\Psi, \Phi^2\Psi, \ldots, \Phi^5\Psi]) = 4$, which means that the controllability of the system is damaged, indicating that the nonidentical control sampling rates can enhance the state controllability of a networked sampled-data system, compared with its variant with identical sampling rates.

Example 3. Consider a directed network consisting of three node systems, where $\delta_1 = 1$ and $w_{21} = w_{31} = w_{23} = 1$. The dynamics of the three node systems is described by the same matrices, i.e., $A_i, B_i, C_i, i = 1, 2, 3$, as that in Example 2. Let $h_1 = \pi/2$, $h = \pi$, and the inner-coupling matrices be

$$H_{21} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, \ H_{23} = \begin{bmatrix} -1 & 1 \\ 1 & 0 \end{bmatrix}, \ H_{31} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}.$$

The Kalman criterion can verify that this HNSS is controllable with $\text{rank}([\Psi, \Phi\Psi, \Phi^2\Psi, \ldots, \Phi^5\Psi]) = 6$. Recall that, in the heterogeneous system model proposed in [20], although the node dynamics are heterogeneous, all of the inner-coupling matrices are identical. In [21], the inner couplings among nodes are heterogeneous, expressed as H_i, $i = 1, \ldots, N$, which means that all of the signals transmitted to the same node system are through identical inner couplings, even if they are from different nodes. For comparison, consider the model in [21] and let

$$H_{21} = H_{23} \triangleq H_2 = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}.$$

Then, the Kalman criterion can verify that $0 \notin \sigma(\Phi)$ and $\text{rank}([\Psi, \Phi\Psi, \Phi^2\Psi, \ldots, \Phi^5\Psi]) = 5$; thus, this system is uncontrollable. This example shows that the heterogeneous inner couplings modeled as H_{ij}, $i, j = 1, \ldots, N$ can enhance the overall controllability of the networked sampled-data system compared with other models in the literature.

4 Controllability of the general HNSS

In this section, the controllability of the general HNSS is analyzed. First, a necessary and sufficient criterion is derived for discrete-time LTI systems by adding one more condition to the PBH rank condition. Then, necessary and/or sufficient conditions are derived for the general HNSS based on the Smith normal form transformation and matrix equations, respectively.

Lemma 5. The discrete-time LTI system $x((k+1)h) = Ax(kh) + Bu(kh)$, where $x(kh) \in \mathbb{R}^n, u(kh) \in \mathbb{R}^p, k \in \mathbb{N}$, and $h > 0$, is controllable if and only if, for every nonzero $\lambda \in \sigma(A)$ and $\forall\, \xi \in M(\lambda|A)$ with $\xi \neq 0$, one has $\xi B \neq 0$.

Proof. Based on the definition of state controllability, the discrete-time LTI system is controllable if and only if, for any initial state $x(0) \in \mathbb{R}^n$, there exists a finite $k \in \mathbb{N}^+$ and inputs $u(0), u(h), \ldots, u((k-1)h)$, such that

$$0 = x(kh) = A^k x(0) + \sum_{\tau=0}^{k-1} A^{k-\tau-1} Bu(\tau h). \tag{10}$$

The aforementioned condition holds if and only if Eq. (10) has a solution within n steps. Let $\mathcal{C}_{A\perp B} \triangleq [B, AB, \ldots, A^{n-1}B]$ denote the controllability matrix. The necessary and sufficient condition for (10) to have a solution within n steps is $\mathrm{rank}([A^n, \mathcal{C}_{A\perp B}]) = \mathrm{rank}(\mathcal{C}_{A\perp B})$. For any matrix, the dimensionality of its left null space equals its row number minus its rank. Thus, the system is controllable if and only if $\dim(\mathcal{N}_L(\mathcal{C}_{A\perp B})) = \dim(\mathcal{N}_L([\mathcal{C}_{A\perp B}, A^n]))$. Notably, $\mathcal{N}_L([\mathcal{C}_{A\perp B}, A^n]) \subseteq \mathcal{N}_L(\mathcal{C}_{A\perp B})$. Therefore, we only need to prove that $\mathcal{N}_L(\mathcal{C}_{A\perp B}) = \mathcal{N}_L([\mathcal{C}_{A\perp B}, A^n])$ if and only if Lemma 5 holds.

Necessity. If $\exists\, \lambda \in \sigma(A)$, $\lambda \neq 0$, such that its eigenvector ξ satisfies $\xi B = 0$, then $\xi \in \mathcal{N}_L(\mathcal{C}_{A\perp B})$ but $\xi \notin \mathcal{N}_L([\mathcal{C}_{A\perp B}, A^n])$ because $\xi A^n = \lambda^n \xi \neq 0$. Therefore, $\mathcal{N}_L(\mathcal{C}_{A\perp B}) \neq \mathcal{N}_L([\mathcal{C}_{A\perp B}, A^n])$.

Sufficiency. First, Lemma 6 is introduced.

Lemma 6 ([11])**.** The uncontrollable subspace of linear system (A, B), which is the left null space of $\mathcal{C}_{A\perp B}$, can be defined as $\mathcal{N}_L(\mathcal{C}_{A\perp B}) = \mathrm{span}\{\xi_i^j | \xi_i^j B = 0, \ldots, \xi_i^1 B = 0, 1 \leqslant i \leqslant r, 1 \leqslant j \leqslant \alpha_i\}$, where the eigenvalues of A are $\lambda_1, \ldots, \lambda_r$ and the left Jordan chain of λ_i is $\xi_i^1, \ldots, \xi_i^{\alpha_i}$, $i = 1, \ldots, r$.

Then, the sufficient part can be proven as follows: Assume that $\exists\, \xi \in \mathcal{N}_L(\mathcal{C}_{A\perp B})$, such that $\xi A^n \neq 0$. According to Lemma 6, $\mathcal{N}_L(\mathcal{C}_{A\perp B})$ can be spanned by some Jordan chains of A. For $\xi A^n \neq 0$, there exists at least one Jordan chain in $\mathcal{N}_L(\mathcal{C}_{A\perp B})$ related to a nonzero eigenvalue. That is, $\exists\, \lambda_i \in \sigma(A)$, $\lambda_i \neq 0$, $i \in \{1, \ldots, r\}$, such that its eigenvector ξ_i^1 satisfies $\xi_i^1 \mathcal{C}_{A\perp B} = \xi_i^1 [B, AB, \ldots, A^{n-1}B] = 0$. Therefore, $\xi_i^1 B = 0$, which completes the proof.

Remark 2. Notably, the well-known PBH rank condition requires checking the entire eigenspace related to all of the eigenvalues of the state matrix A to verify the state controllability of the LTI system (A, B). Furthermore, the PBH rank condition is only sufficient but unnecessary for the discrete-time LTI system with a singular state matrix [25]. Lemma 5 is a supplement to the PBH rank condition for discrete-time LTI systems, revealing that only nonzero eigenvalues of the state matrix need to be checked, and turns the condition into necessary and sufficient. Based on Lemma 5, more necessary and sufficient controllability conditions can be derived for networked sampled-data systems, which are different from the existing results.

Theorem 1. The general HNSS expressed in (6) and (7) is controllable if and only if the determinantal divisor of $[sI_{Nn} - \Phi, \Psi]$ of order Nn is s^k, i.e., $D_{Nn}(s) = s^k$, $k \in \{0, 1, \ldots, Nn\}$.

Proof. Necessity. Suppose that $D_{Nn}(s)$ contains a divisor $\varphi(s)$, which does not include s^k. According to Lemma 2, there exists $s_0 \in \mathbb{C}$, such that $\varphi(s_0) = 0$. Notably, $s_0 \neq 0$ and $D_{Nn}(s_0) = 0$. Let all of the nonzero minors of $[sI_{Nn} - \Phi, \Psi]$ of order Nn be $\varphi_1(s), \ldots, \varphi_p(s)$. Then, there exists $z_i(s)$, such that $\varphi_i(s) = z_i(s) D_{Nn}(s)$ and $\varphi_i(s_0) = 0$ for $i = 1, \ldots, p$, which leads to $\mathrm{rank}([s_0 I_{Nn} - \Phi, \Psi]) < Nn$. Based on Lemma 5, the system expressed in (6) and (7) is uncontrollable if $D_{Nn}(s) \neq s^k$, $k \in \{0, 1, \ldots, Nn\}$.

Sufficiency. Assume that $D_{Nn}(s) = s^k$, and all of the nonzero minors of $[sI_{Nn} - \Phi, \Psi]$ of Nn order are $s^k \phi_1(s), \ldots, s^k \phi_p(s)$, where $\phi_1(s), \ldots, \phi_p(s)$ are polynomials with no common factors. Therefore, for any nonzero $s_0 \in \mathbb{C}$, there exists $\phi_i(s)$, $i \in \{1, \ldots, p\}$, such that $s_0^k \phi_i(s_0) \neq 0$, which means that $[s_0 I_{Nn} - \Phi, \Psi]$ has full row rank. Thus, the system expressed in (6) and (7) is controllable by Lemma 5, which completes the proof.

Corollary 1. The general HNSS expressed in (6) and (7) is controllable if and only if the Smith normal form of $[sI_{Nn} - \Phi, \Psi]$ is $[\mathrm{diag}\{s^{l_1}, \ldots, s^{l_{Nn}}\}, 0]$, where $l_i \in \mathbb{N}$ for $i = 1, \ldots, Nn$ and $0 \leqslant \sum_{i=1}^{Nn} l_i \leqslant Nn$.

Proof. Necessity. According to Theorem 1, if the system expressed in (6) and (7) is controllable, then $D_{Nn}(s) = s^{k_{Nn}}$, $k_{Nn} \in \{0, 1, \ldots, Nn\}$. According to Lemma 1, $D_{Nn-1}(s)|D_{Nn}(s)$, and thus, $D_{Nn-1}(s) = s^{k_{Nn-1}}$, $k_{Nn-1} \in \{0, 1, \ldots, k_{Nn}\}$. Let $l_{Nn} = k_{Nn} - k_{Nn-1} \in \mathbb{N}$. Then, it follows that $d_{Nn}(s) = D_{Nn}(s)/D_{Nn-1}(s) = s^{l_{Nn}}$. A similar derivation easily obtains $D_i(s) = s^{k_i}$, $d_i(s) = s^{l_i}$, where $l_i = k_i - k_{i-1} \in \mathbb{N}$ for $i = 2, \ldots, Nn$, and $l_1 = k_1 \in \mathbb{N}$. Given that $d_1(s) \cdots d_{Nn}(s) = D_{Nn}(s)$, one has $0 \leqslant \sum_{i=1}^{Nn} l_i = k_{Nn} \leqslant Nn$.

Sufficiency. Assume that $[sI_{Nn} - \Phi, \Psi]$ is equivalent to the Smith normal form $[\mathrm{diag}\{s^{l_1}, \ldots, s^{l_{Nn}}\}, 0]$, where $l_i \in \mathbb{N}$ and $0 \leqslant \sum_{i=1}^{Nn} l_i \leqslant Nn$. Thus, $D_{Nn}(s) = d_1(s) \cdots d_{Nn}(s) = s^{\sum_{i=1}^{Nn} l_i}$ can be verified. Let $k = \sum_{i=1}^{Nn} l_i \in \{0, 1, \ldots, Nn\}$. Then, the system expressed in (6) and (7) is controllable according to Theorem 1, which completes the proof.

Theorem 2. The general HNSS expressed in (6) and (7) is controllable if and only if, for every nonzero $s \in \mathbb{C}$, the equation

$$\begin{cases} \xi_i(sI_n - \mathrm{e}^{A_i h}) = \sum_{j=1, j\neq i}^{N} w_{ji}\xi_j f(A_j) H_{ji} C_i, \\ \delta_i \xi_i G_i = 0 \end{cases} \tag{11}$$

has a unique solution $\xi_i = 0$ for every $i = 1, \ldots, N$, with $\xi_i \in \mathbb{C}^{1\times n}$.

Proof. Based on Lemma 5, the system expressed in (6) and (7) is controllable if and only if, $\forall s \in \mathbb{C}$ and $s \neq 0$, $[sI_{Nn} - \Phi, \Psi]$ has full row rank, which means that $[\xi_1, \ldots, \xi_N](sI_{Nn} - \Phi) = 0$ and $[\xi_1, \ldots, \xi_N]\Psi = 0$ hold if and only if $\xi_i = 0$ for every $i = 1, \ldots, N$, where $\xi_i \in \mathbb{C}^{1\times n}$. Simple matrix operations can be used to check that these equations are equivalent to (11), and the proof is complete.

Corollary 2. Suppose that there exists one node i without incoming edges, $i \in \{1, \ldots, N\}$. The general HNSS expressed in (6) and (7) is controllable only if $\delta_i = 1$ and $(\mathrm{e}^{A_i h}, G_i)$ is controllable.

Proof. The system expressed in (6) and (7) is controllable only if, $\forall\, s \in \mathbb{C}$, $s \neq 0$, $[sI_{Nn} - \Phi, \Psi]$ has full row rank. Given that node i does not have incoming edges, $w_{ij} = 0$ for $j = 1, \ldots, N$, and the ith block row of Φ is $[0, \ldots, \mathrm{e}^{A_i h}, \ldots, 0] \in \mathbb{R}^{n\times Nn}$. If $\delta_i = 0$, then the ith block row of Ψ is all zero. Consider $s_0 \in \sigma(\mathrm{e}^{A_i h})$, which is obviously nonzero, and $\xi_0 \in M(s_0|\mathrm{e}^{A_i h})$ with $\xi_0 \neq 0$. Then, $[0, \ldots, \xi_0, \ldots, 0][s_0 I_{Nn} - \Phi, \Psi] = [0, \ldots, \xi_0(s_0 I_n - \mathrm{e}^{A_i h}), \ldots, 0] = 0$; thus, the system expressed in (6) and (7) is uncontrollable. Otherwise, if $\delta_i = 1$ and $(\mathrm{e}^{A_i h}, G_i)$ is uncontrollable, then there exist $s_0 \in \sigma(\mathrm{e}^{A_i h})$ and $\xi_0 \in M(s_0|\mathrm{e}^{A_i h})$ with $\xi_0 \neq 0$, such that $\xi_0 G_i = 0$. It follows that $[0, \ldots, \xi_0, \ldots, 0][s_0 I_{Nn} - \Phi, \Psi] = [0, \ldots, \xi_0(s_0 I_n - \mathrm{e}^{A_i h}), \ldots, \xi_0 G_i, \ldots, 0] = 0$. Therefore, the system expressed in (6) and (7) is also uncontrollable, which completes the proof.

Remark 3. The subsystem $(\mathrm{e}^{A_i h}, G_i)$ can be regarded as a variant of the sampled-data control system $(\mathrm{e}^{A_i h}, f(A_i)B_i)$, which has been thoroughly investigated in the literature, such as [26]. Based on the spectrum mapping theorem, the eigenvalues of $\mathrm{e}^{A_i h}$ are $\{\mathrm{e}^{\lambda h}|\lambda \in \sigma(A_i)\}$. Thus, there may exist $\lambda_p, \lambda_q \in \sigma(A_i)$, $\lambda_p \neq \lambda_q$ but $\mathrm{e}^{\lambda_p h} = \mathrm{e}^{\lambda_q h}$. Let the eigenvectors of A_i related to λ_p, λ_q be ξ_p, ξ_q. Given that $\mathrm{e}^{\lambda_p h} = \mathrm{e}^{\lambda_q h}$, any nonzero vector $\xi \in \mathrm{span}\{\xi_p, \xi_q\}$ is an eigenvector of $\mathrm{e}^{A_i h}$ related to $\mathrm{e}^{\lambda_p h}$ or $\mathrm{e}^{\lambda_q h}$. It may happen that $\xi_p B_i \neq 0$ and $\xi_q B_i \neq 0$ (which is equivalent to $\xi_p G_i \neq 0$ and $\xi_q G_i \neq 0$) but $\xi G_i = 0$. That is, when (A_i, B_i) is controllable, $(\mathrm{e}^{A_i h}, G_i)$ can still be uncontrollable when the sampling is pathological. In this manner, the nonpathological sampling condition in Lemma 4 is extended to the system $(\mathrm{e}^{A_i h}, G_i)$, as follows.

Lemma 7 (Nonpathological sampling condition)**.** For $i = 1, \ldots, N$, if $\forall \lambda_p, \lambda_q \in \sigma(A_i)$,

$$\lambda_p - \lambda_q \neq \frac{2k\pi}{h} i,\ k = \pm 1, \pm 2, \ldots,$$

which means that h is nonpathological about A_i, and then (A_i, B_i) controllable $\Longrightarrow (\mathrm{e}^{A_i h}, G_i)$ controllable.

Corollary 3. Suppose that there exists one node i without external inputs. The general HNSS expressed in (6) and (7) is controllable only if $[-w_{i1} f(A_i) H_{i1} C_1, \ldots, sI_n - \mathrm{e}^{A_i h}, \ldots, -w_{iN} f(A_i) H_{iN} C_N]$ has full row rank for every nonzero $s \in \mathbb{C}$.

Proof. If $[-w_{i1} f(A_i) H_{i1} C_1, \ldots, sI_n - \mathrm{e}^{A_i h}, -w_{iN} f(A_i) H_{iN} C_N]$ does not have full row rank for some nonzero $s \in \mathbb{C}$, then there exists the nonzero $\xi_0 \in \mathbb{C}^{1\times n}$, such that $\xi_0[-w_{i1} f(A_i) H_{i1} C_1, \ldots, sI_n - \mathrm{e}^{A_i h}, \ldots, -w_{iN} f(A_i) H_{iN} C_N] = 0$. Given that node i has no external inputs, $\delta_i = 0$ and the ith block row of Ψ is all zero. It follows that $[0, \ldots, \xi_0, \ldots, 0][sI_{Nn} - \Phi, \Psi] = 0$; thus, the system expressed in (6) and (7) is uncontrollable.

Corollary 4. If the HNSS-IS expressed in (8) and (9) is controllable, then the corresponding general HNSS expressed in (6) and (7) must be controllable.

Proof. $\bar{\Psi} = \Psi(\mathrm{diag}\{\mathbf{1}_{p_1}, \ldots, \mathbf{1}_{p_N}\} \otimes I_p)$ can be easily obtained. If the general HNSS expressed in (6) and (7) is uncontrollable, then there exists the nonzero $\theta \in \sigma(\Phi)$ and a related eigenvector η, such that $\eta\Psi = 0$. Thus, $\eta\bar{\Psi} = \eta\Psi(\mathrm{diag}\{\mathbf{1}_{p_1}, \ldots, \mathbf{1}_{p_N}\} \otimes I_p) = 0$, and $[\theta I_{Nn} - \Phi, \bar{\Psi}]$ does not have full row rank, which means that the corresponding HNSS-IS expressed in (8) and (9) is also uncontrollable.

Remark 4. Corollary 4 provides a theoretical analysis that the nonidentical control signal sampling periods can guarantee the controllability of the general HNSS no less than the HNSS-IS and may even enhance the controllability. The only difference between the system expressed in (6) and (7) and the system expressed in (8) and (9) is in the input matrices Ψ and $\bar{\Psi}$. According to the relationship between $f(A)$ and $g^{(i)}_{l_i}(A)$, i.e., $f(A) = \sum_{l_i=1}^{p_i} g^{(i)}_{l_i}(A)$, the controllability of the HNSS-IS expressed in (8) and (9) can be proven sufficient for the corresponding general HNSS expressed in (6) and (7); however, the opposite may not hold.

5 Controllability of the HNSS in specific settings

In this section, the controllability of the HNSS in some specific settings is analyzed based on the results presented in Section 4. First, two special topologies are investigated. Then, some special dynamics is considered.

5.1 Special topological settings

5.1.1 *Chain structure*

Assume that the topology of the HNSS is in a chain structure, where the first node is the root with no incoming edges. From Corollary 2, the HNSS with chain structure is controllable only if $\delta_1 = 1$ and $(\mathrm{e}^{A_1 h}, G_1)$ is controllable. Without loss of generality, assume that the input signal is only exerted on the first node. It follows that

$$\Phi = \begin{bmatrix} \mathrm{e}^{A_1 h} & & & \\ w_{21}\mathcal{H}_{21} & \ddots & & \\ & \ddots & \ddots & \\ & & w_{N,N-1}\mathcal{H}_{N,N-1} & \mathrm{e}^{A_N h} \end{bmatrix}, \tag{12}$$

$$\Psi = \mathrm{diag}\{G_1, 0, \ldots, 0\},\ \mathcal{H}_{i,i-1} = f(A_i)H_{i,i-1}C_{i-1},\ i = 2, \ldots, N.$$

Notably, $\sigma(\Phi) = \sigma(\mathrm{e}^{A_1 h}) \cup \cdots \cup \sigma(\mathrm{e}^{A_N h})$ and $0 \notin \sigma(\Phi)$. For preparation, $\sigma(\Phi)$ is divided into some disjoint subsets, $\sigma^1(\Phi), \ldots, \sigma^N(\Phi)$, as follows.

Definition 5. For the state matrix Φ that satisfies $\sigma(\Phi) = \sigma(\mathrm{e}^{A_1 h}) \cup \cdots \cup \sigma(\mathrm{e}^{A_N h})$, define $\sigma^k(\Phi) = \{\theta | \theta \in \sigma(\mathrm{e}^{A_k h}) \text{ and } \theta \notin \sigma(\mathrm{e}^{A_l h}) \text{ for all } l > k\}$, $k, l = 1, \ldots, N$.

Corollary 5. The HNSS with a chain structure expressed in (6) and (12) is controllable if and only if, for every $\theta \in \{\sigma^k(\Phi) | \, k = 1, \ldots, N\}$ and $\forall \xi_1, \ldots, \xi_k \in \mathbb{C}^{1\times n}$ satisfying $\xi_1(\theta I_n - \mathrm{e}^{A_1 h}) = w_{21}\xi_2\mathcal{H}_{21}, \ldots, \xi_{k-1}(\theta I_n - \mathrm{e}^{A_{k-1} h}) = w_{k,k-1}\xi_k\mathcal{H}_{k,k-1}$, $\xi_k(\theta I_n - \mathrm{e}^{A_k h}) = 0$ with $\xi_k \neq 0$, one has $\xi_1 G_1 \neq 0$.

Proof. Consider the eigenvalue $\theta \in \sigma^k(\Phi), k \in \{1, \ldots, N\}$, with the corresponding eigenvector $\xi = [\xi_1, \ldots, \xi_N], \xi_i \in \mathbb{C}^{1\times n}$, $i = 1, \ldots, N$. Thus, the equivalence of $\xi(\theta I_{Nn} - \Phi) = 0$ to $\xi_1(\theta I_n - \mathrm{e}^{A_1 h}) = w_{21}\xi_2\mathcal{H}_{21}, \xi_2(\theta I_n - \mathrm{e}^{A_2 h}) = w_{32}\xi_3\mathcal{H}_{32}, \ldots, \xi_{N-1}(\theta I_n - \mathrm{e}^{A_{N-1} h}) = w_{N,N-1}\xi_N\mathcal{H}_{N,N-1}$, and $\xi_N(\theta I_n - \mathrm{e}^{A_N h}) = 0$ can be verified. If $\theta \notin \sigma(\mathrm{e}^{A_N h})$, then $\xi_N = 0$. Furthermore, if $\theta \notin \sigma(\mathrm{e}^{A_{N-1} h})$, then $\xi_{N-1} = 0$. By analogy, $\theta \in \sigma^k(\Phi)$ means that $\theta \notin \sigma(\mathrm{e}^{A_l h})$ and $\xi_l = 0$ for all $l > k$. Thus, $\xi(\theta I_{Nn} - \Phi) = 0$ is equivalent to $\xi_1(\theta I_n - \mathrm{e}^{A_1 h}) = \xi_2 w_{21}\mathcal{H}_{21}, \ldots, \xi_{k-1}(\theta I_n - \mathrm{e}^{A_{k-1} h}) = \xi_k w_{k,k-1}\mathcal{H}_{k,k-1}$, and $\xi_k(\theta I_n - \mathrm{e}^{A_k h}) = 0$. Then, the conclusion follows Lemma 5.

5.1.2 *Star structure*

Assume that the topology of the HNSS is in a star structure, where the first node is the center node with no incoming edges. The HNSS with star structure is controllable only if $\delta_1 = 1$ and $(\mathrm{e}^{A_1 h}, G_1)$ is controllable according to Corollary 2. Moreover, assume that the input signal is only exerted on the first node. It follows that

$$\Phi = \begin{bmatrix} \mathrm{e}^{A_1 h} & & & \\ w_{21}\mathcal{H}_{21} & \ddots & & \\ \vdots & & \ddots & \\ w_{N1}\mathcal{H}_{N1} & & & \mathrm{e}^{A_N h} \end{bmatrix}, \tag{13}$$
$$\Psi = \mathrm{diag}\{G_1, 0, \ldots, 0\},\ \mathcal{H}_{i1} = f(A_i)H_{i1}C_1,\ i = 2, \ldots, N.$$

Notably, $\sigma(\Phi) = \sigma(\mathrm{e}^{A_1 h}) \cup \cdots \cup \sigma(\mathrm{e}^{A_N h})$, and $0 \notin \sigma(\Phi)$.

Corollary 6. The HNSS with a star structure expressed in (6) and (13) is controllable if and only if:

(1) for every $\theta \in \sigma^1(\Phi)$ and $\forall \xi \in M(\theta|\mathrm{e}^{A_1 h})$ with $\xi \neq 0$, one has $\xi G_1 \neq 0$;

(2) otherwise, for every $\theta \in \sigma(\mathrm{e}^{A_{k_1} h}) \cap \cdots \cap \sigma(\mathrm{e}^{A_{k_q} h})$, $k_1, \ldots, k_q \in \{2, \ldots, N\}$, $q \geqslant 1$, and $\forall \xi \in \{\xi_1 \in \mathbb{C}^{1\times n} | \xi_1(\theta I_n - \mathrm{e}^{A_1 h}) = w_{k_1 1}\xi_{k_1}\mathcal{H}_{k_1 1} + \cdots + w_{k_q 1}\xi_{k_q}\mathcal{H}_{k_q 1},\ \xi_{k_j} \in M(\theta|\mathrm{e}^{A_{k_j} h})$ for $j = 1, \ldots, q$ with $[\xi_{k_1}, \ldots, \xi_{k_q}] \neq 0\}$, one has $\xi G_1 \neq 0$.

Proof. Consider the eigenvalue $\theta \in \sigma(\Phi)$, with the corresponding eigenvector $\xi = [\xi_1, \ldots, \xi_N], \xi_i \in \mathbb{C}^{1\times n}$, $i = 1, \ldots, N$. Thus, the equivalence of $\xi(\theta I_{Nn} - \Phi) = 0$ to $\xi_1(\theta I_n - \mathrm{e}^{A_1 h}) = w_{21}\xi_2\mathcal{H}_{21} + \cdots + w_{N1}\xi_N\mathcal{H}_{N1}$ and $\xi_2(\theta I_n - \mathrm{e}^{A_2 h}) = \cdots = \xi_N(\theta I_n - \mathrm{e}^{A_N h}) = 0$ can be verified. For $k = 2, \ldots, N$, if $\theta \notin \sigma(\mathrm{e}^{A_k h})$, then $\xi_k = 0$. Else, if $\theta \in \sigma(\mathrm{e}^{A_1 h})$ and $\theta \notin \sigma(\mathrm{e}^{A_k h})$ for any $k \in \{2, \ldots, N\}$, then $\xi = [\xi_1, 0, \ldots, 0]$ and $\xi(\theta I_{Nn} - \Phi) = 0$ is equivalent to $\xi_1(\theta I_n - \mathrm{e}^{A_1 h}) = 0$. In this manner, the conclusion can be derived from Lemma 5.

5.2 Special dynamic settings

5.2.1 *One-dimensional dynamics*

Assume that the dynamics of each node system in the HNSS is one-dimensional, i.e., $A_i = a_i$, $B_i = b_i$, and $H_{ij}C_j = c_{ij}$ are all scalars, where $i, j = 1, \ldots, N$. In this case, Φ and Ψ in (7) degenerate to the following expression:

$$\Phi = \begin{bmatrix} \mathrm{e}^{a_1 h} & \cdots & w_{1N}f(a_1)c_{1N} \\ \vdots & \ddots & \vdots \\ w_{N1}f(a_N)c_{N1} & \cdots & \mathrm{e}^{a_N h} \end{bmatrix}, \tag{14}$$
$$\Psi = \mathrm{diag}\{\delta_1 G_1, \delta_2 G_2, \ldots, \delta_N G_N\},\quad G_i = [g_1^{(i)}(a_i)b_i, g_2^{(i)}(a_i)b_i, \ldots, g_{p_i}^{(i)}(a_i)b_i],\ i = 1, \ldots, N.$$

Furthermore, $\bar{\Psi}$ in (9) degenerates to the following expression:

$$\bar{\Psi} = \mathrm{diag}\{\delta_1 f(a_1)b_1, \ldots, \delta_N f(a_N)b_N\}. \tag{15}$$

Lemma 8. The functions $f(a)$ and $g_j^{(i)}(a)$ are nonzero $\forall a \in \mathbb{R}$, $i = 1, \ldots, N$, $j = 1, \ldots, p_i$.

Proof. If $a \neq 0$, then $f(a) = a^{-1}(\mathrm{e}^{ah} - 1) \neq 0$, and $g_j^{(i)}(a) = a^{-1}\mathrm{e}^{a(p_i - j)h_i}(\mathrm{e}^{ah_i} - 1) \neq 0$. Otherwise, if $a = 0$, then

$$f(a) = \int_0^h \mathrm{e}^{a\tau}\mathrm{d}\tau = \int_0^h 1\mathrm{d}\tau = h \neq 0,$$
$$g_j^{(i)}(a) = \mathrm{e}^{a(p_i - j)h_i}\int_0^{h_i} \mathrm{e}^{a\tau}\mathrm{d}\tau = \mathrm{e}^{a(p_i - j)h_i}h_i \neq 0.$$

Thus, the proof is complete.

Corollary 7. The HNSS with one-dimensional dynamics expressed in (6) and (14) is controllable if and only if the corresponding HNSS-IS expressed in (8) and (15) is controllable.

Proof. Necessity. If the system expressed in (8) and (15) is uncontrollable, then there exists the nonzero $\xi = [\xi_1, \ldots, \xi_N] \in M(\theta|\Phi)$ with $\theta \neq 0$, such that $\xi\bar{\Psi} = [\xi_1\delta_1 f(a_1)b_1, \ldots, \xi_N\delta_N f(a_N)b_N] = 0$. Given that $f(a_i) \neq 0$, $\xi_i\delta_i b_i = 0$ for $i = 1, \ldots, N$, which leads to $\xi\Psi = [\xi_1\delta_1 G_1, \ldots, \xi_N\delta_N G_N] = 0$, the system expressed in (6) and (14) is uncontrollable.

Sufficiency. If the system expressed in (6) and (14) is uncontrollable, then there exists the nonzero $\xi = [\xi_1, \ldots, \xi_N] \in M(\theta|\Phi)$ with $\theta \neq 0$, such that $\xi\Psi = 0$, which means that $\xi_i\delta_i g_j^{(i)}(a_i)b_i = 0$ for every $i = 1, \ldots, N$, $j = 1, \ldots, p_i$. Given that $g_j^{(i)}(a_i) \neq 0$, one has $\xi_i\delta_i b_i = 0$ for $i = 1, \ldots, N$; thus, it follows that $\xi\bar{\Psi} = [\xi_1\delta_1 f(a_1)b_1, \ldots, \xi_N\delta_N f(a_N)b_N] = 0$. Therefore, the system expressed in (8) and (15) is uncontrollable.

5.2.2 *Self-loop dynamics*

Assume that each node system in the HNSS has self-loop dynamics, i.e., $A_i = a_i I_n$, $i = 1, \ldots, N$. In this case, Φ and Ψ in (7) degenerate to the following expression:

$$\Phi = \begin{bmatrix} \mathrm{e}^{a_1 h} I_n & \cdots & w_{1N} f(a_1) H_{1N} C_N \\ \vdots & \ddots & \vdots \\ w_{N1} f(a_N) H_{N1} C_1 & \cdots & \mathrm{e}^{a_N h} I_n \end{bmatrix},$$
$$\Psi = \mathrm{diag}\{\delta_1 G_1, \delta_2 G_2, \ldots, \delta_N G_N\}, \quad G_i = [g_1^{(i)}(a_i)B_i, g_2^{(i)}(a_i)B_i, \ldots, g_{p_i}^{(i)}(a_i)B_i], \ i = 1, \ldots, N. \tag{16}$$

Correspondingly, $\bar{\Psi}$ in (9) degenerates to the following expression:

$$\bar{\Psi} = \mathrm{diag}\{\delta_1 f(a_1)B_1, \ldots, \delta_N f(a_N)B_N\}. \tag{17}$$

Corollary 8. The HNSS with self-loop dynamics expressed in (6) and (16) is controllable if and only if the corresponding HNSS-IS expressed in (8) and (17) is controllable.

Proof. The proof is similar to that of Corollary 7. According to Lemma 8, $f(a_i)$ and $g_j^{(i)}(a_i)$ are always nonzero for $i = 1, \ldots, N$ and $j = 1, \ldots, p_i$. Thus, the necessary and sufficient controllability condition for the system expressed in (6) and (16) and the system expressed in (8) and (17) is the same, i.e., $\forall\, \xi = [\xi_1, \ldots, \xi_N] \in M(\theta|\Phi)$ with $\theta \neq 0$ and $\xi \neq 0$, one has $\xi_i\delta_i B_i = 0$ for $i = 1, \ldots, N$.

5.2.3 *Identical dynamics*

Assume that the HNSS has identical dynamics, i.e., all nodes have the same system matrices A, B, C, and inner-coupling matrix H. The control sampling periods are still nonidentical in this case. This system is referred to as heterogeneous networked sampled-data system with identical dynamics (HNSS-ID), which can be defined as (6), where

$$\Phi = I_N \otimes \mathrm{e}^{Ah} + W \otimes f(A)HC,$$
$$\Psi = \mathrm{diag}\{\delta_1 G_1, \delta_2 G_2, \ldots, \delta_N G_N\}, \quad G_i = [g_1^{(i)}(A)B, g_2^{(i)}(A)B, \ldots, g_{p_i}^{(i)}(A)B], \ i = 1, \ldots, N, \tag{18}$$

in which $W = [w_{ij}] \in \mathbb{R}^{N\times N}$ and $\Delta = \mathrm{diag}\{\delta_1, \ldots, \delta_N\} \in \mathbb{R}^{N\times N}$ denote the communication and control channels, respectively.

Recall the homogeneous networked sampled-data system (HoNSS) introduced in [37], which can be defined as (8), where

$$\bar{\Psi} = \Delta \otimes f(A)B. \tag{19}$$

Notably, the state matrix Φ of the HNSS-ID expressed in (6) and (18) is the same as that of its homogeneous counterpart, i.e., HoNSS expressed in (8) and (19). The eigenvalues and the corresponding eigenspace of Φ have been obtained in [37], which are presented in Lemmas 9 and 10.

Lemma 9. Suppose that $\sigma(W) = \{\lambda_1, \lambda_2, \ldots, \lambda_r\}$ and $\sigma(E_i) = \{\theta_i^1, \theta_i^2, \ldots, \theta_i^{\rho_i}\}$, where

$$E_i = \mathrm{e}^{Ah} + \lambda_i f(A)HC, \ i = 1, \ldots, r. \tag{20}$$

Then, $\sigma(\Phi) = \{\theta_1^1, \theta_1^2, \ldots, \theta_1^{\rho_1}, \ldots, \theta_r^1, \theta_r^2, \ldots, \theta_r^{\rho_r}\}$.

Let the left Jordan chain of W corresponding to λ_i be $v_i^1, v_i^2, \ldots, v_i^{\alpha_i}$. Let the generalized left Jordan chain of E_i about $f(A)HC$ related to θ_i^j be $\xi_{ij}^1, \xi_{ij}^2, \ldots, \xi_{ij}^{\gamma_{ij}}$, where ξ_{ij}^1 is the top vector and γ_{ij} is the length, where $i = 1, 2, \ldots, r$ and $j = 1, 2, \ldots, \rho_i$.

Lemma 10. The eigenspace of Φ about θ_i^j is $M(\theta_i^j|\Phi) = V(\theta_i^j)$, where $V(\theta_i^j) = \text{span}\{\eta_{ij}^1, \eta_{ij}^2, \ldots, \eta_{ij}^{\beta_{ij}}\}$, $\eta_{ij}^1 = v_i^1 \otimes \xi_{ij}^1$, $\eta_{ij}^2 = v_i^2 \otimes \xi_{ij}^1 + v_i^1 \otimes \xi_{ij}^2, \ldots, \eta_{ij}^{\beta_{ij}} = v_i^{\beta_{ij}} \otimes \xi_{ij}^1 + v_i^{\beta_{ij}-1} \otimes \xi_{ij}^2 + \cdots + v_i^1 \otimes \xi_{ij}^{\beta_{ij}}$, and $\beta_{ij} = \min\{\alpha_i, \gamma_{ij}\}$, $i = 1, 2, \ldots, r$ and $j = 1, 2, \ldots, \rho_i$. Specifically, if $\theta_{i_1}^{j_1} = \cdots = \theta_{i_l}^{j_l} \triangleq \theta_i^j$, $i_k \in \{1, \ldots, r\}$, $j_k \in \{1, \ldots, \rho_{i_k}\}$, $k = 1, \ldots, l$, $l > 1$, then the eigenspace of Φ associated with θ_i^j is the direct sum $M(\theta_i^j|\Phi) = \oplus_{k=1}^l V(\theta_{i_k}^{j_k})$.

Theorem 3. The HNSS-ID expressed in (6) and (18) is controllable if and only if, for every nonzero $\theta_i^j \in \sigma(\Phi)$, $i = 1, \ldots, r$, $j = 1, \ldots, \rho_i$ and for every $\eta \in M(\theta_i^j|\Phi)$ with $\eta \neq 0$, one has $\eta\Psi \neq 0$.

Proof. Theorem 3 can be derived based on Lemma 5, where the eigenspace $M(\theta_i^j|\Phi)$ can be obtained following Lemmas 9 and 10.

Remark 5. The computational complexity of Theorem 2 is no more than $\mathcal{O}(Nn^3) + \mathcal{O}(N^2n^2)$, and Theorem 3 requires only $\mathcal{O}(N^4 + n^4N + N^3n^3)$. Meanwhile, the verification of the Kalman criterion or the PBH rank condition involves $\mathcal{O}(N^4n^4)$.

Then, consider the case that W is diagonalizable, which means that each Jordan block of matrix W is one-dimensional, i.e., $\alpha_i = 1$ for $i = 1, \ldots, N$. In this case, the eigenspace decomposition in Lemma 10 degenerates as follows.

Lemma 11. Assume that W is diagonalizable. Then, $v_i \otimes \xi_i^j$ is the eigenvector of Φ about θ_i^j, where $v_i \in M(\lambda_i|W)$ and $\xi_i^j \in M(\theta_i^j|E_i)$ for $i = 1, \ldots, N$ and $j = 1, \ldots, \rho_i$. Specifically, if $\theta_{i_1}^{j_1} = \cdots = \theta_{i_l}^{j_l} \triangleq \theta_i^j$, $i_k \in \{1, \ldots, N\}$, $j_k \in \{1, \ldots, \rho_{i_k}\}$, $k = 1, \ldots, l$, $l > 1$, then the eigenspace of Φ associated with θ_i^j is $\text{span}\{v_{i_1} \otimes \xi_{i_1}^{j_1}, \ldots, v_{i_l} \otimes \xi_{i_l}^{j_l}\}$.

Theorem 4. Assume that W is diagonalizable. Then, the HNSS-ID expressed in (6) and (18) is controllable if the following conditions hold simultaneously:

(1) (W, Δ) is controllable;

(2) (E_i, G_j) is controllable, $\forall i, j \in \{1, 2, \ldots, N\}$;

(3) If $\theta_{i_1}^{j_1} = \cdots = \theta_{i_l}^{j_l}$, where $i_k \in \{1, \ldots, N\}$, $j_k \in \{1, \ldots, \rho_{i_k}\}$, $k = 1, \ldots, l$, $l > 1$, then $\forall \xi_{i_k}^{j_k} \in M(\theta_{i_k}^{j_k}|E_{i_k})$, $k = 1, \ldots, l$, with $[\xi_{i_1}^{j_1}, \ldots, \xi_{i_l}^{j_l}] \neq 0$, one has $(v_{i_1} \otimes \xi_{i_1}^{j_1} + v_{i_2} \otimes \xi_{i_2}^{j_2} + \cdots + v_{i_l} \otimes \xi_{i_l}^{j_l})\Psi \neq 0$.

Proof. Let $T = [v_1^{\mathrm{T}}, \ldots, v_N^{\mathrm{T}}]^{\mathrm{T}}$ and $v_i = [v_{i1}, \ldots, v_{iN}]$, $i = 1, \ldots, N$. Assume that the HNSS-ID expressed in (6) and (18) is uncontrollable. Then, there exist the nonzero $\theta \in \sigma(\Phi)$ and the corresponding eigenvector η, such that $\eta\Psi = 0$. First, consider the case that $\theta \in \sigma(E_i)$, and the multiplicity is 1. According to Lemma 11, $\eta = v_i \otimes \xi_i^j$, $j \in \{1, \ldots, \rho_i\}$, and $\eta\Psi = (v_i \otimes \xi_i^j)(\text{diag}\{\delta_1 G_1, \ldots, \delta_N G_N\}) = \xi_i^j[\delta_1 v_{i1} G_1, \ldots, \delta_N v_{iN} G_N] = 0$. If $[\delta_1 v_{i1}, \ldots, \delta_N v_{iN}] = v_i\Delta = 0$, then (W, Δ) is uncontrollable, which contradicts the first condition. Otherwise, assume that $v_i\Delta \neq 0$. Then, there exists $k \in \{1, \ldots, N\}$, such that $\xi_i^j G_k = 0$, which contradicts the second condition. Second, assume that the multiplicity of θ is $l > 1$. Let $\theta_{i_1}^{j_1} = \cdots = \theta_{i_l}^{j_l} \triangleq \theta$, where $i_k \in \{1, \ldots, N\}$, $j_k \in \{1, \ldots, \rho_{i_k}\}$, and $k = 1, \ldots, l$. Then, η can be any linear combination of $v_{i_1} \otimes \xi_{i_1}^{j_1}, \ldots, v_{i_l} \otimes \xi_{i_l}^{j_l}$. Let $\eta = \sum_{k=1}^l a_k v_{i_k} \otimes \xi_{i_k}^{j_k}$, where $a_k \in \mathbb{C}$ and $a_1, \ldots, a_l$ are not all zero. Then, $\eta\Psi = (v_{i_1} \otimes a_1\xi_{i_1}^{j_1} + \cdots + v_{i_l} \otimes a_l\xi_{i_l}^{j_l})\Psi = 0$ contradicts the third condition, which completes the proof.

Corollary 9. Assume that W is diagonalizable and $0 \notin \sigma(E_i)$ for $i = 1, \ldots, N$. The HNSS-ID expressed in (6) and (18) is controllable only if (W, Δ) is controllable.

Proof. If (W, Δ) is uncontrollable, then there exists the nonzero $v_i \in M(\lambda_i|W)$, such that $v_i\Delta = 0$. It follows that $[\delta_1 v_{i1}, \delta_2 v_{i2}, \ldots, \delta_N v_{iN}] = 0$. Therefore, $(v_i \otimes \xi_i^j)\Psi = \xi_i^j[\delta_1 v_{i1} G_1, \ldots, \delta_N v_{iN} G_N] = 0$, which indicates that the HNSS-ID expressed in (6) and (18) is uncontrollable, can be verified.

Remark 6. Corollary 9 reveals that the controllability of the network topology is necessary for the HNSS-ID expressed in (6) and (18) to be controllable. In [37], the HoNSS expressed in (8) and (19) is controllable only if (W, Δ) is controllable. However, when the dynamics is heterogeneous, the controllability of the overall networked sampled-data system does not necessarily require the topological pair (W, Δ) to be controllable. The following example illustrates.

Example 4. Consider a star network consisting of three node systems, where $\delta_1 = 1$ and $w_{21} = w_{31} = 1$, and let the sampling period be $h = 0.1$, with

$$A_1 = A_3 = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}, A_2 = \begin{bmatrix} 2 & 1 \\ 4 & 1 \end{bmatrix}, B_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, C_1 = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}, H_{21} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, H_{31} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}.$$

Thus, the topological pair (W, Δ) can be verified to be uncontrollable. However, given that $\mathrm{rank}([\Psi, \Phi\Psi, \ldots, \Phi^5\Psi]) = 6$, the overall networked sampled-data system is still controllable.

6 Controllability of heterogeneous MASs

This section focuses on a special type of HNSS, i.e., the heterogeneous sampled-data MASs, where N agents are classified as N_f followers and $N - N_f$ leaders. Instead of the external inputs in the general HNSS expressed in (6) and (7), in this classical MAS model, the interactions between leaders and followers are regarded as inputs on the followers. The topology of the MAS is weighted and undirected. Notably, heterogeneous here is understood as the difference in system matrices or sampling periods of the agents, in contrast to the definition in [24] referring to different orders of the dynamics of the agents. The controllability of the MAS is defined as follows.

Definition 6 ([40])**.** An MAS is said to be controllable if any initial state of the followers can be steered to any final state in finite time by regulating the motions of the leaders.

6.1 MAS with heterogeneous dynamics

In the MAS with heterogeneous dynamics, each agent can be considered a node system, and the dynamics of agent i is expressed as follows:

$$\begin{aligned} \dot{x}_i(t) &= A_i x_i(t) + B_i u_i(t), \\ u_i(t) &= \sum_{j \in \mathcal{N}_i} w_{ij}(x_j(t) - x_i(t)), \end{aligned} \tag{21}$$

where $\mathcal{N}_i$ denotes the neighbor set of agent i, $i = 1, \ldots, N$. Without loss of generality, let the first N_f agents be the followers and the last $N - N_f$ agents be the leaders. The Laplacian matrix $\mathcal{L}$ can express the neighboring relationship of the MAS as follows:

$$\mathcal{L} = \begin{bmatrix} \mathcal{L}_f & \mathcal{L}_{fl} \\ \mathcal{L}_{lf} & \mathcal{L}_l \end{bmatrix} = [l_{ij}]_{N \times N}, \quad \text{where } l_{ij} = \begin{cases} \sum_{k \in \mathcal{N}_i} w_{ik}, & i = j, \\ -w_{ij}, & j \in \mathcal{N}_i, \\ 0, & j \notin \mathcal{N}_i,\ i \neq j. \end{cases}$$

Let $X^{\mathrm{F}}(t) = [x_1^{\mathrm{T}}(t), \ldots, x_{N_f}^{\mathrm{T}}(t)]^{\mathrm{T}}$ and $X^{\mathrm{L}}(t) = [x_{N_f+1}^{\mathrm{T}}(t), \ldots, x_N^{\mathrm{T}}(t)]^{\mathrm{T}}$ denote the total state of followers and leaders, respectively. The compact form of the continuous-time MAS can be written as follows:

$$\dot{X}^{\mathrm{F}}(t) = \begin{bmatrix} A_1 - l_{11}B_1 & \cdots & -l_{1,N_f}B_1 \\ \vdots & \ddots & \vdots \\ -l_{N_f,1}B_{N_f} & \cdots & A_{N_f} - l_{N_f,N_f}B_{N_f} \end{bmatrix} X^{\mathrm{F}}(t) - \begin{bmatrix} l_{1,N_f+1}B_1 & \cdots & l_{1,N}B_1 \\ \vdots & \ddots & \vdots \\ l_{N_f,N_f+1}B_{N_f} & \cdots & l_{N_f,N}B_{N_f} \end{bmatrix} X^{\mathrm{L}}(t). \tag{22}$$

Assume that the sampling on each channel is synchronously periodic with interval $h > 0$. The corresponding sampled-data MAS can be written as follows:

$$X^{\mathrm{F}}(kh + h) = \Phi X^{\mathrm{F}}(kh) - \Psi X^{\mathrm{L}}(kh), \tag{23}$$

where

$$\begin{aligned} \Phi &= \begin{bmatrix} \mathrm{e}^{A_1 h} - l_{11}f(A_1)B_1 & \cdots & -l_{1,N_f}f(A_1)B_1 \\ \vdots & \ddots & \vdots \\ -l_{N_f,1}f(A_{N_f})B_{N_f} & \cdots & \mathrm{e}^{A_{N_f}h} - l_{N_f,N_f}f(A_{N_f})B_{N_f} \end{bmatrix}, \\ \Psi &= \begin{bmatrix} l_{1,N_f+1}f(A_1)B_1 & \cdots & l_{1,N}f(A_1)B_1 \\ \vdots & \ddots & \vdots \\ l_{N_f,N_f+1}f(A_{N_f})B_{N_f} & \cdots & l_{N_f,N}f(A_{N_f})B_{N_f} \end{bmatrix}. \end{aligned} \tag{24}$$

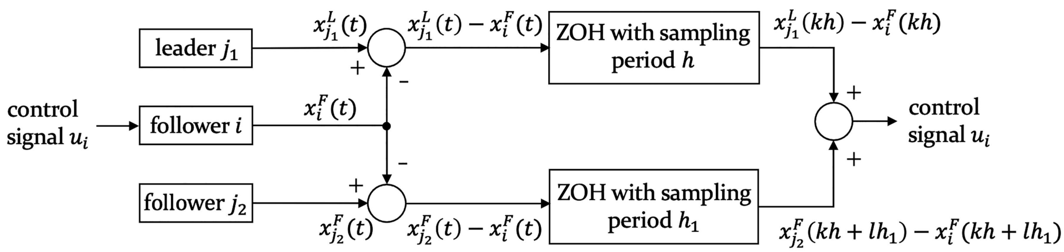

Figure 2 Illustration of the signal transmission process of the MAS with heterogeneous sampling periods.

Theorem 5. The sampled-data MAS with heterogeneous dynamics expressed in (23) and (24) is controllable only if $(\mathrm{e}^{A_ih}, f(A_i)B_i)$ is controllable for every $i = 1, \dots, N_f$.

Proof. Assume that $(\mathrm{e}^{A_ih}, f(A_i)B_i)$ is uncontrollable. Then, there exist $\theta \in \sigma(\mathrm{e}^{A_ih})$ and its corresponding eigenvector ξ_i, such that $\xi_i(\theta I_n - \mathrm{e}^{A_ih}) = 0$ and $\xi_i f(A_i)B_i = 0$. Consider $\xi = [0, \dots, \xi_i, \dots, 0] \in \mathbb{C}^{1\times N_f n}$. It follows that $\xi(\theta I_{N_f n} - \Phi) = \xi_i[l_{i1}f(A_i)B_i, \dots, \theta I_n - \mathrm{e}^{A_ih} + l_{ii}f(A_i)B_i, \dots, l_{i,N_f}f(A_i)B_i] = 0$ and $\xi_i\Psi = \xi_i[l_{i,N_f+1}f(A_i)B_i, \dots, l_{iN}f(A_i)B_i] = 0$. Notably, $\theta \neq 0$; thus, the sampled-data MAS expressed in (23) and (24) is uncontrollable, and the proof is complete.

Remark 7. Recall from [37] that the pathological sampling effects of single-node systems sometimes can be eliminated by the network topology and inner couplings. However, this is not possible to accomplish for the system expressed in (23) and (24). From Theorem 5, the pathological sampling of single-node systems (A_i, B_i), where $i = 1, \dots, N_f$, will inevitably cause the overall sampled-data MAS expressed in (23) and (24) to be uncontrollable (a similar conclusion is drawn in Corollary 10). This difference is attributed to the inputs and inner couplings between nodes in the general HNSS being characterized by matrices B_i and H_{ij}, respectively. However, any interaction in the classic MAS is characterized by B_i. Therefore, the controllability matrix of the classic MAS can be decoupled into two independent parts related to node dynamics and network topology, respectively, but not that of the general model.

6.2 MAS with heterogeneous sampling periods

Consider the MAS consisting of identical agents, where the dynamics of the ith agent is expressed as follows:

$$\begin{aligned} \dot{x}_i(t) &= Ax_i(t) + Bu_i(t), \\ u_i(t) &= \sum_{j\in\mathcal{N}_i} w_{ij}(x_j(t) - x_i(t)), \end{aligned} \tag{25}$$

where $i = 1, \dots, N$. The compact form is written as follows:

$$\dot{X}^{\mathrm{F}}(t) = (I_{N_f} \otimes A - \mathcal{L}_f \otimes B)X^{\mathrm{F}}(t) - (\mathcal{L}_{fl} \otimes B)X^{\mathrm{L}}(t). \tag{26}$$

Assume that the sampling periods of this MAS are heterogeneous: the signals from followers and leaders are sampled and held with different periods.

6.2.1 *Followers' faster sampling*

Let the sampling period of signals from the leaders be h and the sampling period of signals from the followers be h_1, with $h = \kappa h_1$ and $h_1 > 0$, $\kappa \in \mathbb{N}^+$. Figure 2 is an illustration of the signal transmission process in this sampled-data MAS. The dynamics of the followers is expressed as follows:

$$\dot{X}^{\mathrm{F}}(t) = (I_{N_f} \otimes A)X^{\mathrm{F}}(t) - (\mathcal{L}_f \otimes B)X^{\mathrm{F}}(kh + lh_1) - (\mathcal{L}_{fl} \otimes B)X^{\mathrm{L}}(kh),$$

where $t \in [kh + lh_1, kh + (l+1)h_1), k \in \mathbb{N}$, $l = 0, \dots, \kappa - 1$. Then, the corresponding discrete-time LTI model can be derived as follows:

$$X^{\mathrm{F}}(kh + h) = \Phi^\kappa X^{\mathrm{F}}(kh) - (\Phi^{\kappa-1}\Psi + \cdots + \Phi\Psi + \Psi)X^{\mathrm{L}}(kh), \tag{27}$$

where

$$\Phi = I_{N_f} \otimes \mathrm{e}^{Ah_1} - \mathcal{L}_f \otimes \int_0^{h_1} \mathrm{e}^{A\tau}\mathrm{d}\tau B, \quad \Psi = \mathcal{L}_{fl} \otimes \int_0^{h_1} \mathrm{e}^{A\tau}\mathrm{d}\tau B. \tag{28}$$

Lemma 12 (Spectral mapping theorem). Let $\theta_1, \ldots, \theta_r$ be the eigenvalues of Φ, and the corresponding eigenvectors be $\xi_1, \ldots, \xi_r$. Then, the eigenvalues of Φ^κ are $\theta_1^\kappa, \ldots, \theta_r^\kappa$, and the eigenvectors are $\xi_1, \ldots, \xi_r$. In particular, if $\theta_{i_1}^\kappa = \cdots = \theta_{i_l}^\kappa \triangleq \theta$, $i_1, \ldots, i_l \in \{1, \ldots, r\}$, $l > 1$, then the eigenspace related to θ is extended to $M(\theta|\Phi) = \mathrm{span}\{\xi_{i_1}, \ldots, \xi_{i_l}\}$.

Theorem 6. The sampled-data MAS with heterogeneous sampling periods expressed in (27) and (28) is controllable if and only if the following conditions hold simultaneously:

(1) The discrete-time system (Φ, Ψ) is controllable;

(2) $\forall \theta \in \sigma(\Phi)$, if $\theta \neq 1$, then $\theta^\kappa \neq 1$;

(3) If there exist the nonzero $\theta_1, \ldots, \theta_q \in \sigma(\Phi)$ and $\theta_1^\kappa = \cdots = \theta_q^\kappa$, $q > 1$, then for any nonzero $\xi \in \mathrm{span}\{\xi_1, \ldots, \xi_q\}$, one has $\xi\Psi \neq 0$, where $\xi_i\Phi = \theta_i\xi_i$ and $\theta_i^\kappa \neq 1$ for $i = 1, \ldots, q$.

Proof. Theorem 6 can be derived based on the spectral mapping theorem and Lemma 5.

Sufficiency. Assume that the system expressed in (27) and (28) is uncontrollable. Then, there exists $\theta^\kappa \in \sigma(\Phi^\kappa)$ with $\theta \neq 0$ and the corresponding eigenvector ξ, such that $\xi\Phi = \theta\xi$, and $\xi(\Phi^{\kappa-1}\Psi + \cdots + \Phi\Psi + \Psi) = (\theta^{\kappa-1} + \cdots + \theta + 1)\xi\Psi = 0$. If $\xi\Psi = 0$, then (Φ, Ψ) is uncontrollable, and the first condition does not hold. Otherwise, if $\theta^{\kappa-1} + \cdots + \theta + 1 = (\theta^\kappa - 1)/(\theta - 1) = 0$, then it contradicts the second condition. Finally, consider the case that there exists the nonzero $\theta_1^\kappa = \cdots = \theta_q^\kappa$, $q > 1$, and $\xi = a_1\xi_1 + \cdots + a_q\xi_q$, $a_1, \ldots, a_q \in \mathbb{C}\backslash\{0\}$, $\xi_i\Phi = \theta_i\Phi$ for $i = 1, \ldots, q$, such that $\xi(\Phi^{\kappa-1}\Psi + \cdots + \Phi\Psi + \Psi) = 0$. According to the second condition, $\theta_i^\kappa \neq 1$, and the aforementioned equation equals to $\sum_{i=1}^q (\theta_i^\kappa - 1)/(\theta_i - 1)a_i\xi_i\Psi = 0$, which contradicts the third condition.

Necessity. If the first condition does not hold, then $\exists\, \theta \in \sigma(\Phi)$ with $\theta \neq 0$ and the corresponding eigenvector ξ, such that $\xi\Psi = 0$. It follows that $\theta^\kappa \in \sigma(\Phi^\kappa)$ and $\xi(\Phi^{\kappa-1}\Psi + \cdots + \Phi\Psi + \Psi) = (\theta^{\kappa-1} + \cdots + \theta + 1)\xi\Psi = 0$. Furthermore, if the second condition does not hold, then $(\theta^{\kappa-1} + \cdots + \theta + 1) = 0$. Thus, the system expressed in (27) and (28) is uncontrollable in both cases. Finally, assume that the third condition is unsatisfied. Then, there exists $\xi = a_1\xi_1 + \cdots + a_q\xi_q$, $a_1, \ldots, a_q \in \mathbb{C}\backslash\{0\}$, where $\xi_i\Phi = \theta_i\xi_i$, $\theta_i \neq 0$ for $i = 1, \ldots, q$, $q > 1$, and $\theta_1^\kappa = \cdots = \theta_q^\kappa \triangleq \theta^\kappa \neq 1$, such that $\xi\Psi = 0$. Let $\bar{\xi} = a_1(\theta_1 - 1)\xi_1 + \cdots + a_q(\theta_q - 1)\xi_q$, such that $\bar{\xi} \in M(\theta^\kappa|\Phi^\kappa)$ with $\bar{\xi} \neq 0$, and $\bar{\xi}(\Phi^{\kappa-1}\Psi + \cdots + \Phi\Psi + \Psi) = (\theta^\kappa - 1)\sum_{i=1}^q a_i\xi_i\Psi = (\theta^\kappa - 1)\xi\Psi = 0$, which means that the system expressed in (27) and (28) is uncontrollable, and the proof is complete.

According to the property of the Laplacian matrix, $\mathcal{L}_f$ is diagonalizable. Assume that $\sigma(\mathcal{L}_f) = \{\lambda_1, \ldots, \lambda_{N_f}\}$. As a specialization of Lemma 11, the eigenspace of Φ can be spanned by $v_1 \otimes \xi_1^1, \ldots, v_1 \otimes \xi_1^{\rho_1}, \ldots, v_{N_f} \otimes \xi_{N_f}^1, \ldots, v_{N_f} \otimes \xi_{N_f}^{\rho_{N_f}}$, where v_i is the eigenvector of $\mathcal{L}_f$ related to λ_i, and $\xi_i^1, \ldots, \xi_i^{\rho_i}$ are eigenvectors of $\mathrm{e}^{Ah_1} - \lambda_i \int_0^{h_1} \mathrm{e}^{A\tau}\mathrm{d}\tau B$, $i = 1, \ldots, N_f$.

Corollary 10. The system expressed in (27) and (28) is controllable only if $(\mathrm{e}^{Ah_1}, \int_0^{h_1} \mathrm{e}^{A\tau}\mathrm{d}\tau B)$ is controllable.

Proof. Assume that $(\mathrm{e}^{Ah_1}, \int_0^{h_1} \mathrm{e}^{A\tau}\mathrm{d}\tau B)$ is uncontrollable, then $\exists\, \theta \in \sigma(\mathrm{e}^{Ah_1})$ and the corresponding eigenvector ξ, such that $\xi \int_0^{h_1} \mathrm{e}^{A\tau}\mathrm{d}\tau B = 0$. Take the nonzero vector $v \in \mathbb{C}^{1\times N_f}$, and it follows that $(v \otimes \xi)\Phi = \theta(v \otimes \xi) - (v\mathcal{L}_f) \otimes (\xi \int_0^{h_1} \mathrm{e}^{A\tau}\mathrm{d}\tau B) = \theta(v \otimes \xi)$. Thus, θ is also an eigenvalue of Φ with the eigenvector $v \otimes \xi$. Then, one has $(v \otimes \xi)\Psi = (v\mathcal{L}_{fl}) \otimes (\xi \int_0^{h_1} \mathrm{e}^{A\tau}\mathrm{d}\tau B) = 0$, which means that (Φ, Ψ) is uncontrollable. According to Theorem 6, the system expressed in (27) and (28) is also uncontrollable.

6.2.2 *Leaders' faster sampling*

Consider the other case that the sampling period of signals from the followers is h and the sampling period of signals from the leaders is h_1, where $h = \kappa h_1$, $h_1 > 0$, and $\kappa \in \mathbb{N}^+$. The dynamics of the followers is expressed as follows:

$$\dot{X}^{\mathrm{F}}(t) = (I_{N_f} \otimes A)X^{\mathrm{F}}(t) - (\mathcal{L}_f \otimes B)X^{\mathrm{F}}(kh) - (\mathcal{L}_{fl} \otimes B)X^{\mathrm{L}}(kh + lh_1),$$

where $t \in [kh + lh_1, kh + (l+1)h_1)$, $k \in \mathbb{N}$, $l = 0, \ldots, \kappa - 1$. Then, the corresponding discrete-time LTI model can be derived as follows:

$$X^{\mathrm{F}}(kh + h) = \Phi X^{\mathrm{F}}(kh) - \Psi \bar{X}^{\mathrm{L}}(kh), \tag{29}$$

where $\bar{X}^{\mathrm{L}}(kh) = [\bar{x}_{N_f+1}^{\mathrm{T}}(kh), \ldots, \bar{x}_N^{\mathrm{T}}(kh)]^{\mathrm{T}}$, $\bar{x}_i(kh) = [x_i^{\mathrm{T}}(kh), x_i^{\mathrm{T}}(kh + h_1), \ldots, x_i^{\mathrm{T}}(kh + (\kappa - 1)h_1)]^{\mathrm{T}}$ for $i = N_f + 1, \ldots, N$, and

$$\Phi = I_{N_f} \otimes \mathrm{e}^{Ah} - \mathcal{L}_f \otimes \int_0^h \mathrm{e}^{A\tau}\mathrm{d}\tau B, \quad \Psi = \mathcal{L}_{fl} \otimes \left[\int_{(\kappa-1)h_1}^h \mathrm{e}^{A\tau}\mathrm{d}\tau B, \ldots, \int_0^{h_1} \mathrm{e}^{A\tau}\mathrm{d}\tau B\right]. \tag{30}$$

Still, assume that $\sigma(\mathcal{L}_f) = \{\lambda_1, \ldots, \lambda_{N_f}\}$. The eigenspace of Φ and can be spanned by $v_1 \otimes \xi_1^1, \ldots, v_1 \otimes \xi_1^{\rho_1}, \ldots, v_{N_f} \otimes \xi_{N_f}^1, \ldots, v_{N_f} \otimes \xi_{N_f}^{\rho_{N_f}}$, where v_i is the eigenvector of $\mathcal{L}_f$ related to λ_i, and $\xi_i^1, \ldots, \xi_i^{\rho_i}$ are eigenvectors of $\mathrm{e}^{Ah} - \lambda_i \int_0^h \mathrm{e}^{A\tau}\mathrm{d}\tau B$, $i = 1, \ldots, N_f$.

Theorem 7. The sampled-data MAS with heterogeneous sampling periods expressed in (29) and (30) is controllable if and only if the following conditions hold simultaneously:

(1) $(\mathrm{e}^{Ah}, [\int_{(\kappa-1)h_1}^h \mathrm{e}^{A\tau}\mathrm{d}\tau B, \ldots, \int_0^{h_1} \mathrm{e}^{A\tau}\mathrm{d}\tau B])$ is controllable;

(2) $(\mathcal{L}_f, \mathcal{L}_{fl})$ is controllable;

(3) If $v_{i_1} \otimes \xi_{i_1}^{j_1}, \ldots, v_{i_q} \otimes \xi_{i_q}^{j_q}$, $q > 1$ are related to the same eigenvalue, with $i_k \in \{1, \ldots, N_f\}$ and $j_k \in \{1, \ldots, \rho_i\}$ for $k = 1, \ldots, q$, then $\forall\, \eta \in \mathrm{span}\{v_{i_1} \otimes \xi_{i_1}^{j_1}, \ldots, v_{i_q} \otimes \xi_{i_q}^{j_q}\}$, one has $\eta\Psi \neq 0$, where $\eta \neq 0$.

Proof. Sufficiency. Assume that the system expressed in (29) and (30) is uncontrollable, and there exists the eigenvector $v_i \otimes \xi_i^j$, $i \in \{1, \ldots, N_f\}$, $j \in \{1, \ldots, \rho_i\}$, such that $(v_i \otimes \xi_i^j)\Psi = (v_i\mathcal{L}_{fl}) \otimes (\xi_i^j[\int_{(\kappa-1)h_1}^h \mathrm{e}^{A\tau}\mathrm{d}\tau B, \ldots, \int_0^{h_1} \mathrm{e}^{A\tau}\mathrm{d}\tau B]) = 0$. If $v_i\mathcal{L}_{fl} = 0$, then $(\mathcal{L}_f, \mathcal{L}_{fl})$ is uncontrollable, which contradicts the second condition. Otherwise, if $\xi_i^j[\int_{(\kappa-1)h_1}^h \mathrm{e}^{A\tau}\mathrm{d}\tau B, \ldots, \int_0^{h_1} \mathrm{e}^{A\tau}\mathrm{d}\tau B] = 0$, then $\xi_i^j(\sum_{l=0}^{\kappa-1} \int_{lh_1}^{(l+1)h_1} \mathrm{e}^{A\tau}\mathrm{d}\tau)B = \xi_i^j \int_0^h \mathrm{e}^{A\tau}\mathrm{d}\tau B = 0$. Thus, ξ_i^j is also an eigenvector of e^{Ah}, and the first condition does not hold. Else, if $v_{i_1} \otimes \xi_{i_1}^{j_1}, \ldots, v_{i_q} \otimes \xi_{i_q}^{j_q}$, $q > 1$ are related to the same eigenvalue and $\exists\, a_1, \ldots, a_q \in \mathbb{C}\backslash\{0\}$ such that $\sum_{k=1}^q a_k(v_{i_k} \otimes \xi_{i_k}^{j_k})\Psi = 0$. In this case, the third condition does not hold.

Necessity. The proof of the first condition is similar to that of Corollary 10. If $(\mathcal{L}_f, \mathcal{L}_{fl})$ is uncontrollable, then $\exists\, \lambda_i \in \sigma(\mathcal{L}_f)$, such that $v_i\mathcal{L}_{fl} = 0$. Thus, $(v_i \otimes \xi_i^j)\Psi = 0$ and the system expressed in (29) and (30) is uncontrollable. Finally, assume that $v_{i_1} \otimes \xi_{i_1}^{j_1}, \ldots, v_{i_q} \otimes \xi_{i_q}^{j_q}$, $q > 1$ are related to the same eigenvalue. If $\exists\, a_1, \ldots, a_q \in \mathbb{C}\backslash\{0\}$, such that $\eta\Psi = 0$, where $\eta = \sum_{l=1}^q a_l(v_{i_l} \otimes \xi_{i_l}^{j_l})$ is an eigenvector of Φ, then the system expressed in (29) and (30) is also uncontrollable, and the proof is complete.

7 Conclusion

This study analyzes the controllability of the HNSS consisting of nonidentical node systems with distinct inner couplings, where multidimensional inputs and outputs are sampled and held with different periods. Necessary and sufficient conditions are derived for the general HNSS and some variants in special dynamic or topological settings, utilizing the Smith normal form and matrix equations. Notably, heterogeneous factors have various effects on the controllability of networked sampled-data systems. The nonidentical sampling periods can enhance the controllability of the overall system, except for systems with one-dimensional or self-loop dynamics. The controllability of the network topology is unnecessary to guarantee the overall controllability when the node dynamics is heterogeneous. For some heterogeneous sampled-data MASs, the pathological sampling of single-node systems will inevitably lead the entire system to uncontrollability. In the future, more sampling protocols will be investigated for the controllability of networked systems, and optimization strategies will be designed based on the analytical results.

Acknowledgements This work was supported in part by National Natural Science Foundation of China (Grant Nos. 62373245, 62336005), Joint Funds of the National Natural Science Foundation of China (Grant No. U21A20480), National Key R&D Program of China (Grant No. 2023YFB4706800), Project of Science and Technology Commission of Shanghai Municipality, China (Grant No. 22JC1401401), and "Dawn" Program of Shanghai Education Commission, China (Grant No. 21SG14).

References

1 Kalman R E. Canonical structure of linear dynamical systems. Proc Natl Acad Sci USA, 1962, 48: 596–600
2 Xiang L, Chen F, Ren W, et al. Advances in network controllability. IEEE Circuits Syst Mag, 2019, 19: 8–32
3 Liu Y Y, Slotine J J, Barabási A L. Controllability of complex networks. Nature, 2011, 473: 167–173
4 Zhu J, Xiang L, Yu Y, et al. Average controllability of complex networks with Laplacian dynamics. IEEE Trans Circuits Syst I, 2021, 69: 1704–1714
5 Borriello E, Daniels B C. The basis of easy controllability in Boolean networks. Nat Commun, 2021, 12: 5227
6 Montanari A N, Duan C, Aguirre L A, et al. Functional observability and target state estimation in large-scale networks. Proc Natl Acad Sci USA, 2022, 119: e2113750119
7 Lin C T. Structural controllability. IEEE Trans Automat Contr, 1974, 19: 201–208
8 Li J, Chen X, Pequito S, et al. On the structural target controllability of undirected networks. IEEE Trans Automat Contr, 2020, 66: 4836–4843
9 Wang L, Chen G, Wang X, et al. Controllability of networked MIMO systems. Automatica, 2016, 69: 405–409
10 Hao Y, Duan Z, Chen G, et al. New controllability conditions for networked, identical LTI systems. IEEE Trans Automat Contr, 2019, 64: 4223–4228
11 Hao Y, Wang Q, Duan Z, et al. Target controllability of networked LTI systems. IEEE Trans Netw Sci Eng, 2022, 9: 1493–1500
12 Xue M, Roy S. Modal barriers to controllability in networks with linearly coupled homogeneous subsystems. IEEE Trans Automat Contr, 2021, 66: 6187–6193

13 Wu J N, Li X, Chen G. Controllability of deep-coupling dynamical networks. IEEE Trans Circuits Syst I, 2020, 67: 5211–5222
14 Si Y, Wang J R. Relative controllability of multi-agent systems with input delay and switching topologies. Syst Control Lett, 2023, 171: 105432
15 Li T, Zhang R, Liu Z, et al. Controllability of networked multiagent systems based on linearized Turing's model. Automatica, 2024, 162: 111507
16 Guo J H, Ji Z J, Liu Y G. Sufficient conditions and limitations of equivalent partition in multiagent controllability. Sci China Inf Sci, 2022, 65: 132204
17 Qu J, Ji Z, Shi Y. The graphical conditions for controllability of multiagent systems under equitable partition. IEEE Trans Cybern, 2020, 51: 4661–4672
18 Guo J H, Ji Z J, Liu Y G. Controllability of game-based multi-agent system. Sci China Inf Sci, 2023, 66: 222206
19 Arreola-Delgado A, Barajas-Ramírez J G. On the controllability of networks with nonidentical linear nodes. IMA J Math Control Inf, 2021, 38: 39–53
20 Xiang L, Wang P, Chen F, et al. Controllability of directed networked MIMO systems with heterogeneous dynamics. IEEE Trans Control Netw Syst, 2019, 7: 807–817
21 Kong Z, Cao L, Wang L, et al. Controllability of heterogeneous networked systems with nonidentical inner-coupling matrices. IEEE Trans Control Netw Syst, 2021, 9: 867–878
22 Mu J, Wu J, Li N, et al. Structural controllability of networked systems with general heterogeneous subsystems. Asian J Control, 2022, 24: 1321–1332
23 van Waarde H J, Tesi P, Camlibel M K. Topology identification of heterogeneous networks: identifiability and reconstruction. Automatica, 2021, 123: 109331
24 Liu B, Shen X, Wu L, et al. Observability of heterogeneous multi-agent systems. IEEE Trans Netw Sci Eng, 2021, 8: 1828–1841
25 Hespanha J P. Linear Systems Theory. Princeton: Princeton University Press, 2018
26 Chen T, Francis B A. Optimal Sampled-Data Control Systems. London: Springer Science & Business Media, 2012
27 Zhao P, Wang L Y, Yin G. Controllability and adaptation of linear time-invariant systems under irregular and Markovian sampling. Automatica, 2016, 63: 92–100
28 Hamidoğlu A, Göksu G. ϵ-controllability/ϵ-observability of linear switched sampled-data systems with the non-equidistant time schedule. ISA Trans, 2023, 139: 167–178
29 Amini A, Mousavi H K, Sun Q, et al. Space-time sampling for network observability. IEEE Trans Control Netw Syst, 2022, 10: 1159–1171
30 Hamidoğlu A, Mahmudov E N. On construction of sampling patterns for preserving observability/controllability of linear sampled-data systems. Int J Control, 2022, 95: 206–217
31 Pasand M M S, Montazeri M. Controllability and stabilizability of multi-rate sampled data systems. Syst Control Lett, 2018, 113: 27–30
32 Yan J, Hu B, Guan Z H. Controllability criteria on discrete-time impulsive hybrid systems with input delay. IEEE Trans Syst Man Cybern Syst, 2022, 53: 2304–2316
33 Lu Z, Ji Z, Zhang Z. Sampled-data based structural controllability of multi-agent systems with switching topology. J Franklin Inst, 2020, 357: 10886–10899
34 Zhao B, Guan Y. Data-sampling controllability of multi-agent systems. IMA J Math Control Inf, 2020, 37: 794–813
35 Liu B, Su H, Wu L, et al. Controllability for multi-agent systems with matrix-weight-based signed network. Appl Math Computation, 2021, 411: 126520
36 Liu B, Ping Y, Wu L, et al. Controllability of discrete-time multi-agent systems based on absolute protocol with time-delays. Neurocomputing, 2020, 409: 316–328
37 Yang Z, Wang X, Wang L. Controllability of networked sampled-data systems. IEEE Trans Automat Contr, 2024, 69: 5081–5093
38 Roman S, Axler S, Gehring F. Advanced Linear Algebra. New York: Springer, 2005
39 Fine B, Rosenberger G. The Fundamental Theorem of Algebra. Berlin: Springer Science & Business Media, 1997
40 Ji Z, Chen T, Yu H. Controllability of sampled-data multi-agent systems. In: Proceedings of the 33rd Chinese Control Conference, Nanjing, 2014