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Gossip-based algorithm for economic dispatch of microgrids integrating isolated and grid-connected modes

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Abstract A gossip-based economic dispatch (ED) algorithm for microgrids is presented in this paper, designed to cope with communication link failures and enable smooth switching of microgrid operation modes. The algorithm is supported by the Push-Pull architecture, which allows its application to direct graphs and relaxes the initial conditions compared to many existing ED algorithms. Under an asynchronous communication network, where only one directed edge is activated at each moment, it has been shown that ED can be achieved with probability 1, provided that the communication graph is strongly connected. Similarly, in a synchronous communication network, where each directed communication link is activated at each moment with a certain probability, ED is also achieved with probability 1 under the condition that the communication graph is strongly connected. This demonstrates that optimal consensus is reached under randomly switched communication networks as long as the expectation of communication graphs is strongly connected, a condition that is less stringent than the B-strongly connected requirement found in many other studies. The algorithm's use of a non-decreasing variable step size enables a transition from a sub-linear convergence rate, associated with a diminishing step size, to a linear convergence rate. This also lays the groundwork for future improvements in convergence rate through online step size optimization based on the communication topology. Finally, the algorithm's effectiveness and its potential application to anti-collusion are demonstrated through simulations.

Keywords distributed optimization, economic dispatch, gossip algorithms, multi-agent systems, consensus algorithm

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1 Introduction

Microgrids, as autonomous and self-healing energy systems, are becoming increasingly critical in the quest for sustainable and resilient power solutions. The economic dispatch (ED) within microgrids is a complex optimization problem that must ensure the efficient generation and distribution of power while minimizing operational costs. This problem is further complicated by the dual-mode operation of microgrids, which can function either in isolation or in conjunction with the main grid, and by the presence of renewable energy sources that introduce variability into the system. Isolated mode and grid-connected mode are two common operation modes of microgrids. The isolated mode allows the microgrid to continue supplying power in the event of a main grid failure or power outage, which improves the reliability of the system, reduces the dependence on the main grid, and also reduces the cost of energy purchases, making more efficient use of local renewable energy sources. The grid-connected mode allows the microgrid to be connected to the main grid, thus enabling support from the main grid to ensure that high-load demands can be met when needed. The switching between these two modes of operation is designed to adapt to different operating conditions and cope with varying demands of power, ensuring that the microgrid operates stably and economically. Therefore, the switching of microgrid operation modes should also be taken into account when studying the microgrid economic dispatch problem (EDP), to achieve a smooth transition.

The centralized approach [1–4] has often been studied and applied to the ED of microgrids. The development of cloud computing has increased the computing capacity, but the centralized algorithms, are

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limited by high latency, unstable network, low bandwidth, privacy and security, large storage, and the presence of a single point of failure. Edge computing has long been a mainstream development trend. End-loaded intelligence has facilitated the development of distributed optimization algorithms, which have been widely studied in the ED of microgrids. A fully distributed consensus-based ED algorithm is given by Yang et al. [5], which does not need a leader to collect the total power mismatch as in [6]. A diminishing step size was used in many articles [7–9] to guarantee a sublinear convergence speed. Benefiting from the improvement of convergence speed, more fixed-step algorithms began to appear, including DIGing [10], ATC-DIGing [11], and distributed PI algorithms [12]. The convergence rate will affect the adaptability of the microgrid to the frequent and rapid fluctuations caused by the variability of renewable energy and the constant change of load. Therefore, algorithms with more stringent convergence speeds such as finite-time [13,14] and fixed-time [15,16] ED algorithms are proposed. To further save communication resources, event-triggered ED algorithms are proposed in [17–20]. Distributed optimization realizes coordination and optimization through communication between agents, so there are problems such as communication delay [21–24], privacy-preserving [25, 26], switching communication topology [7, 8] that need to be addressed.

Most of the algorithms proposed in these articles are synchronous. However, different resources in microgrids may have varying response times and computational capabilities. Thus, asynchronous algorithms are more suitable for handling system heterogeneity and reducing communication delay. A gossip-based asynchronous distributed algorithm for EDP is proposed in [27] to decrease the risk of information congestion. The asynchronous distributed dynamic ED algorithms in [28, 29] enable each aggregator to optimize its subproblem more flexibly and efficiently through limited information exchange with neighbors. An asynchronous distributed primal-dual algorithm to solve the EDP with a linear convergence speed is proposed in [30] which can deal with link failures over stochastic networks.

A gossip algorithm is a distributed algorithm for information interaction by randomly selecting neighboring nodes, which can be executed in both asynchronous and synchronous networks. A gossip-based ED algorithm was first proposed by Wang et al. [27,31], where the asynchronous directionless communication network is considered. However, all of the above work has considered only one mode of operation for microgrids. An integrated ED algorithm covering two operating modes is first given by Chen et al. [32], where the use of diminishing step sizes allows the algorithm to achieve only sub-linear convergence rates and the algorithm cannot run for long periods. The fixed-step size ED algorithm realizing the smooth transition between two operation modes of microgrids is given in [33], but it is not guaranteed to be fully distributed. The above two articles only consider undirected communication topologies. But in many real applications, there will be one-way communication due to the different broadcast power of different resources in microgrids. At the same time, with the continuous installation of renewable energy and the abandonment of some old or damaged equipment, the communication topology in the microgrid will change inevitably.

Based on the above analyses, we propose a gossip-based composite ED algorithm, working in both operating modes and allowing smooth switching between them under directed asynchronous and synchronous communication networks that can be time-varying. The main contributions and innovations of this paper are as follows.

(1) The gossip-based autonomous ED algorithms under asynchronous and synchronous communication networks are given, and it is shown that the algorithms can converge to the optimum with probability 1 when the communication topology is strongly connected. The algorithms can cope with communication link failure, and handle communication topology switching. Unlike many algorithms that require the switching topology to be B-strongly connected [7, 8, 23], we only need the expectation of switching topologies to be strongly connected, which allows the algorithm to have a larger application space such as collusion prevention.

(2) In contrast to Chen et al. [32], which uses a diminishing step size for the coupling of the algorithm, we use a fixed step size, which enables the algorithm to leapfrog from sub-linear to linear in convergence speed. Different from Chen et al. [33], which uses a fixed and identical step size, our step size is non-uniform and variable, which makes the algorithm more flexible and lays the foundation for future optimization of the step size based on the communication topology to achieve the fastest convergence speed.

(3) Differently from many papers [31–33] that consider only undirected communication topologies, directed communication networks are considered in this paper. The proposed algorithm applies to unbalanced directed graphs by using the protocol of Push-Pull, where the weight coefficient matrices in the algorithm need to be row stochastic and column stochastic, respectively.

(4) The optimal consensus and smooth transition of operation modes are realized in a fully distributed way. Compared to many papers [5, 9, 23, 31, 34–37] where the initial conditions of the algorithms require the power supply and demand to be balanced and the power mismatch estimate to be 0, we only need the power mismatch estimate to be the same as the true value, which is less demanding on the initial operating state of the microgrid.

Notations. $\mathbf{0}, \mathbf{1}, I$ are vectors of 0 and 1 and the identity matrix with the corresponding dimension. The Kronecker product is represented by $\otimes$. $x^{\mathrm{T}}(M^{\mathrm{T}})$ is the transpose of vector $x \in \mathbb{R}^n$ (matrix $M \in \mathbb{R}^{m\times n}$), where $\mathbb{R}^n(\mathbb{R}^{m\times n})$ is the $n(m \times n)$-dimensional real Euclidean space.

2 Problem formulation and preliminaries

2.1 Graph theory

$G = (V, E, A)$ is used to represent the communication structure between nodes in a network system. $V = [1, 2, \ldots, N]$ denotes the set constituted by N nodes. E designates directed edges between nodes, and $(i, j) \in E$ signifies that node j has access to information from node i. The adjacency matrix A represents the communication relationship between these nodes whose elements are non-negative and $a_{ij} > 0$ means that node i can receive information from node j. The nodes from which node i can receive information and the nodes from which it can send information are indicated by N_i^{in} and N_i^{out}, where the number of elements is called the in-degree d_i^{in} and the out-degree d_i^{out} of node i, respectively. The Laplacian matrix L is obtained by $D - A$, where the diagonal matrix D is derived by summing the elements of each row of A.

2.2 Economic dispatch

Consider a microgrid containing N bus nodes, where the cost function for each node is denoted by $C_i(P_i), i = 1, 2, \ldots, N$, where P_i denotes the power generated by a distributed generator (DG) of node i. Generally, the generation cost $C_i(P_i)$ is modeled as a quadratic function:

$$C_i(P_i) = \frac{(P_i - \alpha_i)^2}{2\beta_i} + \gamma_i, \tag{1}$$

where $\alpha_i, \beta_i, \gamma_i$ are the parameters of the ith DG.

ED aims to minimize the total operating cost of the entire microgrid system by rationally distributing the generators' power output while satisfying the grid's supply-demand balance and the output constraints of each generator. The mathematical model of an EDP is represented as follows:

$$\begin{aligned} &\min_{P_i, P_{\text{MG}}} \sum_{i=1}^{N} C_i(P_i) + \lambda_0 P_{\text{MG}} \\ &\text{s.t.} \sum_{i=1}^{N} P_i + P_{\text{MG}} = P_D, \\ &\underline{P}_i \leqslant P_i \leqslant \overline{P}_i, \end{aligned} \tag{2}$$

where P_{MG} indicates the power produced by the main grid. $P_D = \sum_{i=1}^{N} P_{Di}$ is the total power demand from all bus nodes, with P_{Di} being the power demand of bus node i. λ_0 implies the price of the main grid determined by the energy router (ER), $\underline{P}_i$, and $\overline{P}_i$ signifies the lower and upper bounds of the output power, respectively. Renewable energy sources like wind turbines and photovoltaics are increasingly used in microgrids. However, since they cost almost nothing to generate power, they operate at maximum power and do not participate in the ED analysis of microgrids. Their output power change is equated to the change in demand from loads.

The Lagrangian function is given by

$$L(P_1, \ldots, P_N, P_{\text{MG}}, \lambda) = \sum_{i=1}^{N} C_i(P_i) + \lambda_0 P_{\text{MG}} + \lambda \left(P_D - \sum_{i=1}^{N} P_i - P_{\text{MG}} \right), \tag{3}$$

where λ is the Lagrange multiplier. Define the incremental cost of ith DG as

$$\lambda_i = \frac{\mathrm{d}C_i(P_i)}{\mathrm{d}P_i} = \frac{P_i - \alpha_i}{\beta_i}. \tag{4}$$

We have the optimal solution based on the Lagrange multiplier method

$$P_i^* = \begin{cases} \beta_i\lambda^* + \alpha_i, & \underline{P}_i \leqslant \beta_i\lambda^* + \alpha_i \leqslant \overline{P}_i, \\ \overline{P}_i, & \beta_i\lambda^* + \alpha_i > \overline{P}_i, \\ \underline{P}_i, & \beta_i\lambda^* + \alpha_i < \underline{P}_i, \end{cases} \tag{5}$$

$$P_{\mathrm{MG}}^* = P_D - \sum_{i=1}^{N} P_i^*, \tag{6}$$

where λ^* is the optimal incremental cost, $\lambda^* = \lambda_0$ when the microgrid is operating in grid-connected mode, and in isolated mode

$$\lambda^* = \frac{P_D - \sum_{i\notin\Omega} P_i - \sum_{i\in\Omega}\alpha_i}{\sum_{i\in\Omega}\beta_i}, \tag{7}$$

where Ω is the subset of DGs that do not exceed the output power limit. P_i^* and P_{MG}^* denote the optimal output power of the ith DG and the optimal power exchanged with the main grid, respectively. It can be seen that in the grid-connected mode, the microgrid achieves ED when the marginal cost $\lambda_i, i = 1, \ldots, N$ is equal to the main grid tariff λ_0, so we can use a leader-following consensus algorithm based on a multi-agent system to achieve ED in a fully distributed manner. In the isolated mode, the same marginal cost of all DGs is a necessary condition for the microgrid to achieve ED. We further give the dual of the original problem in the isolated mode

$$\min_{\lambda\in\mathbb{R}} -\sum_{i=1}^{N}\Psi_i(\lambda) - \lambda\sum_{i=1}^{N} P_{Di}, \tag{8}$$

where

$$\Psi_i(\lambda) = \min_{P_i} C_i(P_i) - \lambda P_i. \tag{9}$$

It is easy to obtain the gradient of the dual problem concerning λ as $\sum_{i=1}^{N} P_i - \sum_{i=1}^{N} P_{Di}$, which is the mismatch between the power production and requirement. Since the objective function is strongly convex and the constraints are affine, there exists an optimal solution to the problem and the dual gap between the original problem and the pairwise problem is 0. After decentralizing the gradient, we can obtain the optimal solution by solving the dual problem through distributed optimization.

3 Gossip-based ED algorithm under asynchronous network and synchronous network

In this section, the ED algorithm that can handle microgrids in both modes of operation simultaneously is presented. The algorithms under asynchronous communication networks are first given and then similar conclusions can be easily obtained under synchronous communication networks.

3.1 Gossip-based ED algorithm of microgrids integrating isolated and grid-connected modes with asynchronous network

In the framework of energy internet, each bus node is regarded as an agent, and each agent contains an intelligent control unit (ICU), load, and DG as shown in Figure 1. As a bridge of information exchange and energy transfer between the microgrid and the outside main grid, the ER can obtain the electricity price λ_0 of the main grid and adjust the operation mode g_0 of the microgrid. Let $g_0 = 1$ indicate that the microgrid is operating in grid-connected mode and $g_0 = 0$ indicate that the microgrid is operating in isolated mode. Considering an energy internet with N agents, ER is noted as agent 0 and $a_{i0} = 1$ means that ER can send information to node i. Similarly, $a_{0i} = 1$ means that node i can send information to ER. Let λ_i, P_i, y_i, P_{Mi}, and $\Delta\hat{P}_{ai}, i = 1, 2, \ldots, N$ be the state variables of each node, where y_i is the local

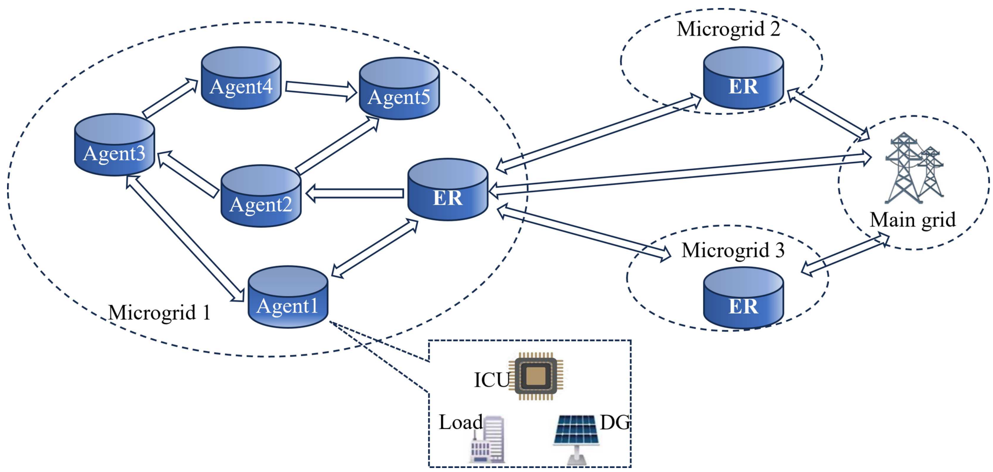

Figure 1 (Color online) Energy internet structure with the ERs based on multi-agent systems.

average estimate of the power mismatch value of node i and $\Delta\hat{P}_{ai}$ is the average estimate of the power mismatch $\Delta P_i = P_{Di} - P_i - P_{Mi}$ after considering the replenishment of the power that the neighbors of the ER can get directly from the ER. P_{Mi} represents the power supplement obtained by node i from the main grid through ER. The update weight $\sigma_i \in (0,1)$, and the optimization step η_i is non-negative and small enough. The initial states are set as $\Delta\hat{P}_{ai}(0) = \Delta P_i(0) = P_{Di} - P_i - P_{Mi}, i = 1,\ldots,N$. For such a system with N agents, suppose that at each time instant k, only one communication link $(j,i), i,j = 1,2,\ldots,N$ is activated with probability p_{ij}. Along this edge, the state variables λ_j and y_j are sent from node j to node i. The gossip-based algorithm for the EDP is designed as follows.

(1) For node i:

$$\lambda_i(k+1) = \lambda_i(k) + \sigma_i[\lambda_j(k) - \lambda_i(k) + g_0 a_{i0}(\lambda_0 - \lambda_i(k))] + (1 - g_0 a_{i0})\eta_i(k)\Delta\hat{P}_{ai}(k), \tag{10}$$

$$P_i(k+1) = \begin{cases} \beta_i\lambda_i(k+1) + \alpha_i, & \underline{P_i} \leqslant \beta_i\lambda_i(k+1) + \alpha_i \leqslant \overline{P_i}, \\ \overline{P_i}, & \beta_i\lambda_i(k) + \alpha_i > \overline{P_i}, \\ \underline{P_i}, & \beta_i\lambda_i(k) + \alpha_i < \underline{P_i}, \end{cases} \tag{11}$$

$$y_i(k+1) = y_i(k) + y_j(k) + \Delta P_i(k+1) - \Delta P_i(k), \tag{12}$$

$$P_{Mi}(k+1) = g_0 a_{i0}[P_{Mi}(k) + a_{0i} y_i(k+1)], \tag{13}$$

$$\Delta\hat{P}_{ai}(k+1) = y_i(k+1) + a_{i0}[P_{Mi}(k) - P_{Mi}(k+1)]. \tag{14}$$

(2) For node j: $\lambda_j(k+1) = \lambda_j(k)$ and $y_j(k+1) = 0$. The other state variables are updated the same as node i.

(3) For other nodes $l \in \{V - \{i,j\}\}$: $\lambda_l(k+1) = \lambda_l(k)$ and $y_l(k+1) = y_l(k)$. The other state variables are updated in the same way as shown in (11), (13), and (14).

Finally, the ER needs to calculate the sum of the power supplements that need to be obtained from the main grid based on its neighbor's information:

$$P_{\mathrm{MG}}(k+1) = \sum_{i=1}^{N} P_{Mi}(k+1). \tag{15}$$

Remark 1. The fact that only one communication link is activated at each moment means that our algorithm is asynchronous. The proposed asynchronous algorithm allows each DG to communicate and compute based on its own conditions without the need for global synchronization and therefore handles heterogeneous systems well and improves overall efficiency. Since nodes communicate and update their states whenever they have the opportunity, without needing to synchronize, the given asynchronous algorithm also reduces the impact of communication on system performance, making dispatch more real-time and flexible. In addition, some nodes in a microgrid may temporarily fail or experience communication interruptions. The above asynchronous algorithm can continue to operate under these conditions, improving the robustness of the system.

Remark 2. Both islanded and grid-connected operation modes of microgrids are considered, thus the proposed algorithm can adapt to multiple modes and realize the seamless transition of operation modes. The algorithm can now respond to main grid disconnection and restoration, which lays the foundation for predicting possible off-grid scenarios in the future and making appropriate energy scheduling and resource allocation in time.

Remark 3. Different from the algorithm in [32] which uses a decreasing step size to decouple the algorithm in grid-connected and isolated modes, we use a non-attenuating step size, thus improving the convergence speed of the algorithm. Compared to [26, 31], our step size can be not only fixed and consistent but also time-varying and non-consistent, thus providing more options for the algorithm's step size selection. In the future, the fastest convergence of the algorithm can be achieved by distributedly optimizing steps according to the communication topology so that the second largest eigenvalue of the iteration coefficient matrix is minimized.

Remark 4. When a microgrid is in grid-connected operation, the ER is important in managing the power supply. The ER distributes required power supplements to its reachable neighbors. This process ensures that the power mismatch of the nodes that receive power supplements from the ER is always 0. However, when the microgrid switches to isolated mode, the power supplements from the main grid instantly stop. Reflected in (13), the neighboring nodes of the ER instantaneously zero the power supplementation from the main grid. This change can be immediately reflected in the estimation of the power mismatch values of the nodes as shown in (14), ensuring that the estimated value of the power mismatch always matches the true value.

Remark 5. The fact that g_0 always appears at the same time as a_{i0} in the algorithm means that we do not use global information. Instead, g_0 is only acquired by ER's neighbors, unlike the algorithm in [26] which uses global information, our algorithm is fully distributed. Different from the algorithms in [5, 9, 23, 31, 34–37] that need to keep the power supply and demand balanced at the initial moment as well as the power mismatch value estimated to be 0, our algorithm only needs the power mismatch estimate to be the same as the true value, which is more relaxed and more applicable in practice.

Since the communication topology is randomly varying, $[\lambda(k)^{\mathrm{T}}\ y(k)^{\mathrm{T}}]^{\mathrm{T}}$ is a random variable whose convergence should also be defined in probabilistic terms. The definitions of mean square consensus and almost sure consensus can be found in [31].

Before giving our main results, some important lemmas and conditions are first presented.

Lemma 1. A Markov jump linear system $x(k+1) = \Gamma_{\theta(k)}x(k)$ has the property that if the state variable $x(k)$ can achieve the mean square consensus, then it can also realize the almost sure consensus [38].

Condition 1. The communication weight in (10) satisfies $\sigma_i \in (0, \frac{1}{\sum_{j=0}^{N} a_{ij}})$.

For the digraph $G_{ji} = (V, \{(j,i)\}, A_{ji})$, define $A_{ji} = \sigma_i f_i f_j^{\mathrm{T}}$, where f_i and f_j are unit vectors whose ith and jth elements are 1. Suppose $D_{ji} = \sigma_i f_i f_i^{\mathrm{T}}$, then L_{ji} is obtained by $D_{ji} - A_{ji}$. The row sum of L_{ji} is 0, so $I - L_{ji}$ is row stochastic. Given $S_{ji} = I - (f_j - f_i)f_j^{\mathrm{T}}$, it is obvious that s_{ji} is a column stochastic matrix.

Theorem 1. If the graph G is strongly connected and the step $\eta_i, i = 1, 2, \ldots, N$ is sufficiently small, then the proposed algorithm (10)–(15) with Condition 1 for the EDP (2) can realize the almost sure consensus in an asynchronous network.

Proof. Let us first consider the microgrid operating in the isolated mode, i.e., $g_0 = 0$. Eqs. (10) and (12) can be written in the matrix form

$$\begin{bmatrix}\lambda(k+1)\\ y(k+1)\end{bmatrix} = M(k)\begin{bmatrix}\lambda(k)\\ y(k)\end{bmatrix}, \tag{16}$$

where

$$M(k) = M_{ji} = \begin{bmatrix} I - L_{ji} & \eta D_{ji} \\ BL_{ji} & S_{ji} - \eta BD_{ji}\end{bmatrix},$$

with $B = \mathrm{diag}([\tilde{\beta}_1, \ldots, \tilde{\beta}_N])$, $\eta = \mathrm{diag}([\eta_1(k), \ldots, \eta_N(k)])$ and

$$\tilde{\beta}_i = \begin{cases} 0, & \text{if } P_i(k) \text{ is saturated,} \\ \beta_i, & \text{otherwise.}\end{cases} \tag{17}$$

$P_i(k)$ being saturated means that $P_i(k)$ equals the output power limit of the i DG. Normally, it does not happen that all DGs are saturated, i.e., at least one DG is within the power limit. If no DG exceeds the power limit, then the problem degenerates into a situation where there is no output power limit. The M_{ji} can be regarded as the matrix M_0 perturbed by ηF, where

$$M_0 = \begin{bmatrix} I - L_{ji} & 0 \\ BL_{ji} & S_{ji} \end{bmatrix}, \quad F = \begin{bmatrix} 0 & D_{ji} \\ 0 & -BD_{ji} \end{bmatrix}. \tag{18}$$

By the Gerschgorin circle theorem, M_0 has the double eigenvalue of 1, and all other eigenvalues are inside the unit circle. Then from Theorem 1 and Proposition 2 in [35], we have that there exists a positive ϵ, and as long as $\eta_i < \epsilon, i = 1, \ldots, N$, then the eigenvalues of M_{ji} satisfy

$$1 = \mu_1(\eta) > |\mu_2(\eta)| \geqslant \cdots \geqslant |\mu_{2N}(\eta)|. \tag{19}$$

Suppose $e(k) = [(\lambda(k) - \lambda^* \mathbf{1})^{\mathrm{T}}, y(k)^{\mathrm{T}}]^{\mathrm{T}}$, where the λ^* is the optimal value of the incremental cost, we have $e(k+1) = M(k)e(k)$. Define

$$\tilde{e}(k+1) = (M(k)e(k)) \otimes (M(k)e(k)) = (M(k) \otimes M(k))\tilde{e}(k). \tag{20}$$

We can then obtain that $[\lambda^{\mathrm{T}}, y^{\mathrm{T}}]^{\mathrm{T}}$ achieves mean-square consensus by showing that $\tilde{e}$ converges to zero. Since $M(k)$ is i.i.d, so does $(M(k) \otimes M(k))$. Then we have

$$\begin{aligned} E[\tilde{e}(k+1)|\tilde{e}(k)] &= E[M(k) \otimes M(k)]\tilde{e}(k) \\ &= E[M \otimes M]\tilde{e}(k) \\ &= E[M \otimes M]^{k+1}\tilde{e}(0). \end{aligned} \tag{21}$$

From (19) and Lemma 1 in [31], we know that the eigenvalue of $E[M \otimes M]$ satisfy

$$1 = \mu_1 > |\mu_2| \geqslant \cdots \geqslant |\mu_{4N^2}|. \tag{22}$$

Therefore there exists a non-singular matrix Q satisfying

$$Q^{-1}E(M \otimes M)Q = \begin{bmatrix} 1 & 0 \\ 0 & J \end{bmatrix}, \tag{23}$$

where J is a Jordan block matrix. It is easy to varify that $v_1^{\mathrm{T}} = ([\mathbf{1}^{\mathrm{T}}B, \mathbf{1}^{\mathrm{T}}] \otimes [\mathbf{1}^{\mathrm{T}}B, \mathbf{1}^{\mathrm{T}}])$ and $u_1 = (\frac{1}{a}[\mathbf{1}^{\mathrm{T}}, \mathbf{0}^{\mathrm{T}}]^{\mathrm{T}} \otimes \frac{1}{a}[\mathbf{1}^{\mathrm{T}}, \mathbf{0}^{\mathrm{T}}]^{\mathrm{T}})$ are left and right eigenvectors of $E[M \otimes M]$ corresponding to eigenvalue 1. From

$$E(M \otimes M)^k = Q \begin{bmatrix} 1 & 0 \\ 0 & J^k \end{bmatrix} Q^{-1}, \tag{24}$$

we have

$$E(\tilde{e}(k+1)|\tilde{e}(k)) \to u_1 v_1^{\mathrm{T}} \tilde{e}(0) = \left(\frac{1}{a} \begin{bmatrix} \mathbf{1}\mathbf{1}^{\mathrm{T}}B & \mathbf{1}\mathbf{1}^{\mathrm{T}} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \lambda(0) - \mathbf{1}\lambda^* \\ y(0) \end{bmatrix} \right) \otimes \left(\frac{1}{a} \begin{bmatrix} \mathbf{1}\mathbf{1}^{\mathrm{T}}B & \mathbf{1}\mathbf{1}^{\mathrm{T}} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \lambda(0) - \mathbf{1}\lambda^* \\ y(0) \end{bmatrix} \right). \tag{25}$$

Since

$$\begin{bmatrix} \mathbf{1}\mathbf{1}^{\mathrm{T}}B & \mathbf{1}\mathbf{1}^{\mathrm{T}} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \lambda(0) - \mathbf{1}\lambda^* \\ y(0) \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^N P_i(0) - \sum_{i=1}^N \alpha_i + \sum_{i=1}^N y_i(0) - a\lambda^* \\ \vdots \\ \sum_{i=1}^N P_i(0) - \sum_{i=1}^N \alpha_i + \sum_{i=1}^N y_i(0) - a\lambda^* \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{26}$$

from the initial condition $y_i(0) = P_{Di} - P_i(0)$ and $a\lambda^* = \sum_{i=1}^N P_i^* - \sum_{i=1}^N \alpha_i$, we can obtain that the above equation is a 0-vector. Then we have $E[\tilde{e}(k+1)|\tilde{e}(k)] \to 0_{4N^2}$ as $k \to \infty$. That is, the proposed algorithm realized the mean square consensus. Immediately following Lemma 1, we can see that the algorithm (10)–(15) can achieve the almost sure consensus.

When the microgrid switches to the grid-connected mode, i.e., $g_0 = 1$. The following matrix equation can be obtained from the algorithm:

$$e(k+1) = M(k)e(k), \tag{27}$$

where

$$M(k) = M_{ji} = \begin{bmatrix} I - L_{ji} - \sigma A_{*0} & 0_{N\times N} \\ BL_{ji} + \sigma BA_{*0} & S_{ji}(I - A_{*0}A_{0*}) \end{bmatrix},$$

and $\sigma = \mathrm{diag}([\sigma_1, \ldots, \sigma_N])$, $A_{*0} = \mathrm{diag}([a_{10}, \ldots, a_{N0}])$, $A_{0*} = \mathrm{diag}([a_{01}, \ldots, a_{0N}])$. The eigenvalues of $E[I - L_{ji} - \sigma A_{*0}]$ and $E[S_{ji}(I - A_{*0}A_{0*})]$ are all within the unit circle (the proof can be found in our conference paper in [39]). Then by the same line of proof as in the isolated mode, we get $E[\tilde{e}(k+1)|\tilde{e}(k)] \to 0_{4N^2}$ as $k \to \infty$. That is the algorithm (10)–(15) can achieve the mean square consensus and almost sure consensus in the grid-connected mode.

Through the above analysis, we know that the algorithm (10)–(15) can achieve the optimal value of the EDP in both isolated and grid-connected. The remaining question is whether the algorithm can realize the smooth mode switch, that is automatically recognise the mode switch and redistribute the output power of each node to offset power fluctuations caused by main grid access or disconnection. Fortunately, as we mentioned in Remark 2, the clever design of algorithm (12)–(14) allows the sudden change in power supply caused by mode switching to be keenly captured by the ER's neighboring nodes, and this sudden change in power is eliminated through distributed collaboration. A specific proof can be found in our previous work and related work [32] and will not be given in this paper.

3.2 Gossip-based ED algorithm of microgrids integrating isolated and grid-connected modes with synchronous network

The conclusions under asynchronous networks can be directly generalized to synchronous networks. Suppose that at each moment k, a total of n directed edges are activated, forming a subgraph $G'(k)$. We use $L(k)$ to represent the Laplace matrix at time k and $L(k) = D(k) - A(k)$. Define the communication coefficient matrix $W(k)$ of $y(k)$ as follows:

$$w_{ij}(k) = \begin{cases} \dfrac{1}{d_j^{\mathrm{out}}(k) + 1}, & j \in N_i^{\mathrm{in}} \cup \{i\}, \\ 0, & \text{otherwise}. \end{cases} \tag{28}$$

It is easy to verify that the matrix $W(k)$ is column stochastic. Then we give the gossip-based ED algorithm in a synchronous network

$$\lambda_i(k+1) = \lambda_i(k) + \sigma_i \left[\sum_{j=1}^{N} a_{ij}(k)(\lambda_j(k) - \lambda_i(k)) + g_0 a_{i0}(\lambda_0 - \lambda_i(k)) \right] + (1 - g_0 a_{i0})\eta_i(k)\Delta\hat{P}_{ai}(k), \tag{29}$$

$$y_i(k) = \sum_{i=1}^{N} w_{ij}(k) y_j(k) + \Delta P_i(k+1) - \Delta P_i(k). \tag{30}$$

The rest of the state variables have the same status update form of (11), (13)–(15) in the above asynchronous network.

Remark 6. Under an asynchronous communication network, just one edge is activated at each moment, so by the Gershgorin Circle Theorem one can compute an upper bound on σ_i that guarantees convergence of the algorithm to be 1/2. Under a synchronous communication network, the upper bound becomes $1/\sum_{j=0}^{N} a_{ij}(k)$, which is no less than $1/\sum_{j=0}^{N} a_{ij}$. So Condition 1 is relatively conservative, and σ_i can be adjusted by dynamically adapting it to the in-degree of node i at time k.

Theorem 2. If the graph G is strongly connected and the step $\eta_i, i = 1, 2, \ldots, N$ is sufficiently small, then the proposed algorithm (10)–(15) with Condition 1 for the EDP (2) can realize the almost sure consensus in a synchronous network.

Proof. Since $I - L(k)$ and $W(k)$ are row stochastic and column stochastic respectively, we can replace L_{ji} with $L(k)$, D_{ji} with $D(k)$, and S_{ji} with $W(k)$ in the above proof under asynchronous networks. Then Theorem 2 can be derived and a similar process is omitted here.

Remark 7. The activation of one or more edges at each moment can be considered as a communication failure of other edges at each moment. For example, under the synchronous communication network, each edge is set to fail at each moment with a certain probability, which is just in line with the design of the gossip algorithm, so the proposed algorithm can handle the communication link failure well. Under the asynchronous network, if a particular DG fails or loses communication, the rest of the system can still function and make real-time adjustments to the ED without being significantly affected by the failure. This redundant propagation mechanism greatly improves the robustness of the system.

Remark 8. The random communication link activation under a synchronous fixed network constitutes a time-varying communication network, which means the proposed algorithm can be used under a time-varying communication network. Unlike many ED algorithms under switched networks that require the communication topology to be B-strongly connected, the algorithm proposed in this paper eliminates this condition, i.e., the algorithm can achieve ED of microgrids with probability 1 under randomly switched communication topologies as long as the expectation of the time-varying communication graphs is strongly connected. This is a less stringent condition and will have more applications, such as collusion prevention, which many scholars have studied. The performance of the algorithm on collusion prevention will be validated and analyzed in the following simulation case study.

Remark 9. Whether under synchronous communication networks or in asynchronous networks, we study the case of a directed communication graph, which is different from the algorithms in [31, 35] that consider only undirected graphs. Directed graphs are more pervasive and easier to implement, while our conclusions can be directly generalized to the case of undirected communication. The inability to construct the double stochastic coefficient matrix under a directed communication network makes the design of the algorithm difficult. A common solution is to use the Push-Sum framework or the Push-Pull framework. In this paper, the Push-Pull architecture is used, where the marginal costs λ are 'pushed' to the neighbors and the power mismatch estimates y are 'pulled' from the neighbors. Push-Pull structured algorithms can better handle ill-conditioned problems or situations where the communication coefficient matrices are relatively unbalanced [40].

Remark 10. In contrast to algorithms that use a decreasing step size [7, 9, 32] that has a sub-linear convergence rate, our algorithm uses a fixed step size and can converge linearly, where the specific linear convergence rate can be found in [40]. The decaying step size can not run for a long time as it will eventually decay to 0. In the face of changes such as mode switching, node failures, and load changes, the algorithm with diminishing step size will fail due to the step size being too small. Our proposed fixed step size algorithm solves these problems and enjoys faster convergence. Besides, the use of a non-consistent variable step size enjoys greater degrees of freedom.

4 Numerical results

In this section, several case studies are carried out to validate the effectiveness of the proposed algorithm. Consider a microgrid based on the energy internet framework containing six DGs, four loads, and one ER. Its electrical structure and communication topology network are shown in Figures 2(a) and (b), respectively. The parameters and output limits of DGs are written in Table 1. Consider this microgrid with a total power demand of 460 MW for 4 loads, where the demand for each load is 150, 150, 110, and 50 MW. The main electricity price obtained by the ER is 68, i.e., $\lambda_0 = 68$.

4.1 Case 1: feasibility of the algorithm with asynchronous network

To demonstrate the feasibility of the algorithm under an asynchronous communication network, the following scenario is considered: the 6-bus microgrid undergoes a first mode switch from grid-connected to isolated mode at $k = 550$ and a second mode switch from isolated mode to grid-connected operation mode at $k = 2200$. The communication weight δ is set as $\delta_1 = \cdots = \delta_N = 0.4$. The uncoordinated step size η is set as $\eta_1 = \eta_2 = \eta_3 = 0.001, \eta_4 = \eta_5 = \eta_6 = 0.008$. From the simulation results in Figure 3, we can see that despite only one directed edge being activated at each moment, the algorithm can achieve ED. From Figures 3(c) and (d), we know that switching from grid-connected to isolated mode produces larger fluctuations in power mismatch than switching from isolated to grid-connected mode. This is because the former instantly results in a large power supply deficit, whereas the latter is a much smaller power mismatch arising from a shift in marginal cost.

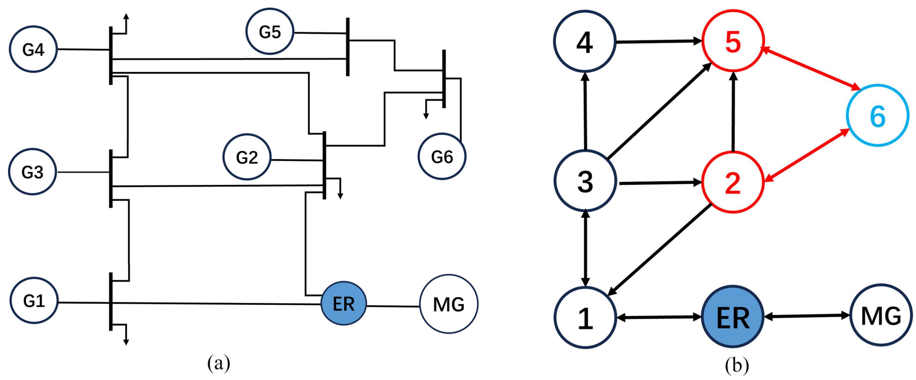

Figure 2 (Color online) Microgrid model considered in the simulation. (a) The electrical structure of the microgrid; (b) the communication topology of the microgrid.

Table 1 Parameters of the DGs.

DG	α_i	β_i	γ_i	$\underline{P}_i$	$\overline{P}_i$
DG1	−4780.11	70.81	−126572	40	200
DG2	−4638.77	69.24	−112750	20	170
DG3	−4337.61	64.52	−120578	0	100
DG4	−5027.20	73.93	−107705	0	150
DG5	−4498.96	67.48	−121390	45	110
DG6	−4520.92	66.73	−132298	10	300

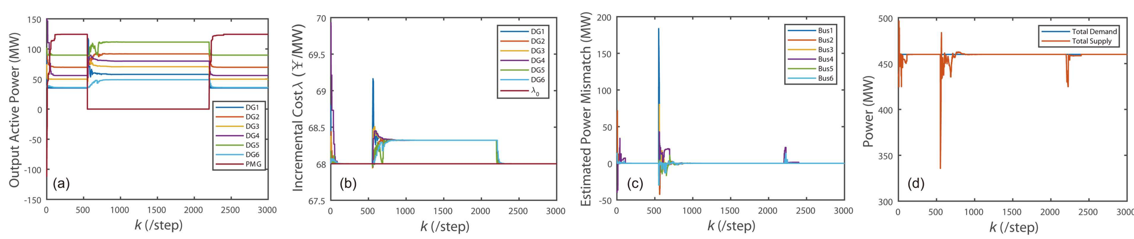

Figure 3 (Color online) Simulation results of Case 1. (a) The output active power of each DG; (b) the incremental cost of each DG; (c) the estimated average power mismatch of each ICU; (d) the total demand and supply of the microgrid.

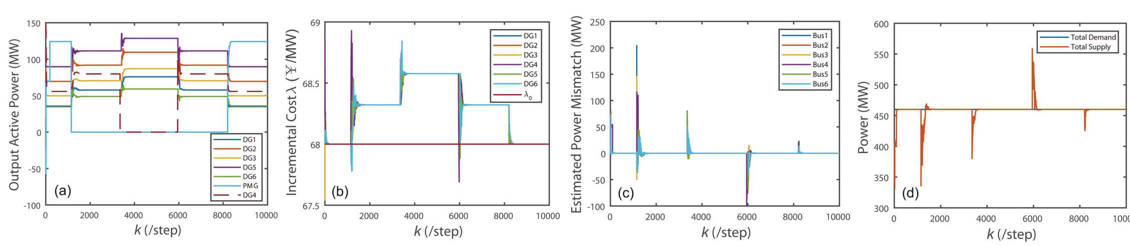

Figure 4 (Color online) Simulation results of Case 2. (a) The output active power of each DG; (b) the incremental cost of each DG; (c) the estimated average power mismatch of each ICU; (d) the total demand and supply of the microgrid.

4.2 Case 2: capability of the plug and play under an asynchronous network

In this case study, the plug-and-play ability of the algorithm is verified under an asynchronous communication network. In this case study, the operation mode switching moments of the microgrid are $k = 1150$ and $k = 8200$, respectively. Suppose that after the microgrid switches to the isolated mode, the 4th DG breaks down at $k = 3350$ and recovers at $k = 5950$. The communication weight δ is set as $\delta_1 = \cdots = \delta_N = 0.4$. The uncoordinated step size η is set as $\eta_1 = \eta_2 = \eta_3 = 0.0001, \eta_4 = \eta_5 = \eta_6 = 0.0006$. As Figure 4 shows, the power mismatch caused by the failure and recovery of the 4th DG is automatically eliminated by other DGs. This means the algorithm can achieve the plug-and-play capability for microgrids.

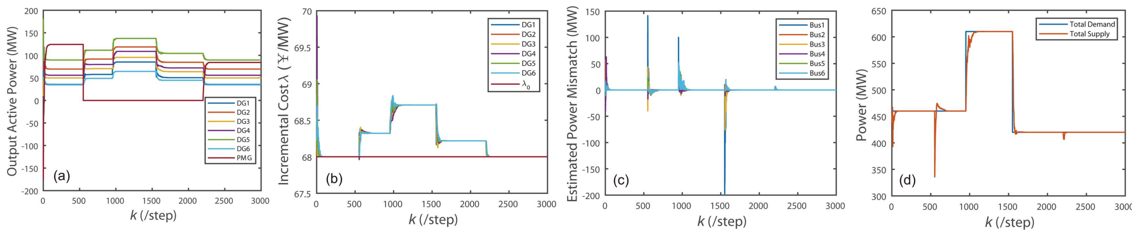

Figure 5 (Color online) Simulation results of Case 3. (a) The output active power of each DG; (b) the incremental cost of each DG; (c) the estimated average power mismatch of each ICU; (d) the total demand and supply of the microgrid.

4.3 Case 3: feasibility of the algorithm under a synchronous network with time-varying loads

In this case study, the feasibility of the proposed algorithm under a synchronous network with time-varying loads is validated. The mode-switching moments are the same as in Case 1. The loads become $[250, 150, 0, 11, 0, 100]$ when $k = 950$ and change to $[50, 160, 0, 110, 0, 100]$ when $k = 1550$. Suppose that the 12 directed edges in Figure 2(b) have a probability of 0.3 to be activated at each moment. The communication weight δ is set as $\delta_1 = \cdots = \delta_N = 0.2$. The uncoordinated step size η is set as $\eta_1 = \eta_2 = \eta_3 = 0.001, \eta_4 = \eta_5 = \eta_6 = 0.005$. Figure 5 demonstrates that the incremental cost λ and the power mismatch estimate y converge to the optimum. The fluctuations at moments such as mode switching or load changes are smaller due to the improvement of communication efficiency compared to those under asynchronous communication. Each edge is activated with a probability of 0.3, which can also be interpreted as each communication link fails with a probability of 0.7, so the proposed algorithm can handle the case of communication link failure, which has to be taken into account in practical applications.

4.4 Case 4: application in collusion prevention

In a fixed topology, some malicious nodes can lock onto a target and thus conspire to mislead the victim to suffer losses. However, when the communication topology is randomly switched, the malicious nodes cannot target the victim nodes, and the great uncertainty of the communication link makes collusion no longer possible. Specifically, nodes 2 and 5 in Figure 2(b) can conspire to jointly transmit misleading information to node 6 so that its marginal cost becomes 65.7, which is less than the optimal marginal cost. A smaller marginal cost means less power used in exchange for a profit, and while node 6 suffers a loss, the remaining nodes gain a greater benefit. We perform simulations with the proposed algorithm, the conventional optimization algorithm with a fixed topology, and the centralized algorithm, respectively, in the isolated mode of operation. The communication weight δ is set as $\delta_1 = \cdots = \delta_N = 0.2$. The uncoordinated step size η is set as $\eta_1 = \eta_2 = \eta_3 = 0.004, \eta_4 = \eta_5 = \eta_6 = 0.008$. The centralized algorithm does not require iteration and is used here as a comparison group. The Euclidean distance of the marginal cost λ to the optimum λ^* is shown in Figure 6(a). Although the conventional algorithm can ensure the balance between power supply and demand in the presence of victim nodes (as shown in Figure 6(b)), the result is not globally optimal. This is also reflected in Figure 6(c). The proposed algorithm achieves the convergence of all marginal costs to the optimal value, but the conventional algorithm makes the victim node's marginal cost smaller than the rest of the nodes and suffers a loss. The overall cost increases from 2.4457×10^5 to 2.4485×10^5 even though the other nodes gain more benefits.

4.5 Case 5: comparison with existing algorithms

The speed of convergence of an algorithm is an important performance metric of an algorithm. Many algorithms use a fixed step size and achieve linear convergence speed, for example, the Push-DIGing algorithm [41] for directed communication networks was obtained by using the Push-Sum protocol in the DIGing algorithm, which uses only column stochastic coefficient matrices. In contrast, Xi et al. [42] proposed the Xi-Row algorithm applicable to directed graphs, using only a row stochastic matrix. Our proposed algorithm uses the Push-Pull protocol to handle the directed communication, which needs both row stochastic and column stochastic matrices. Benefiting from the fixed step size, these algorithms can achieve the optimal consensus with a linear rate. However, the simultaneous use of row-stochastic and column-stochastic matrices removes the communication bottleneck caused by a very unbalanced communication topology graph, and thus our proposed algorithm will outperform under a very unbalanced

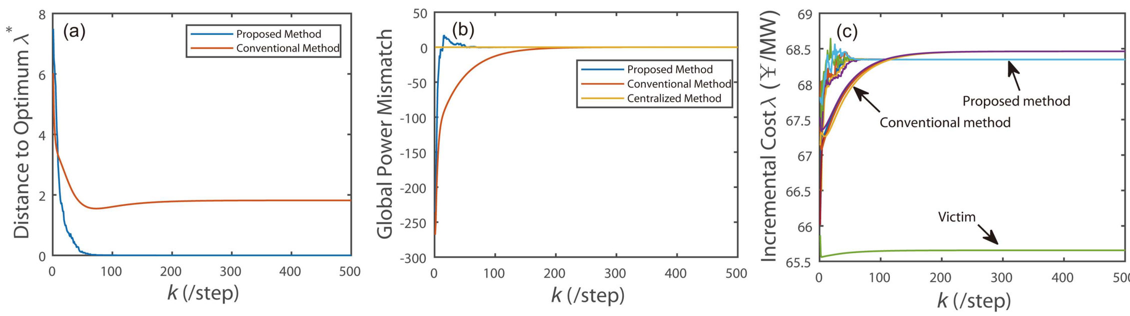

Figure 6 (Color online) Simulation results of Case 4. (a) The Euclidean distance to the optimum; (b) global power mismatch; (c) incremental cost.

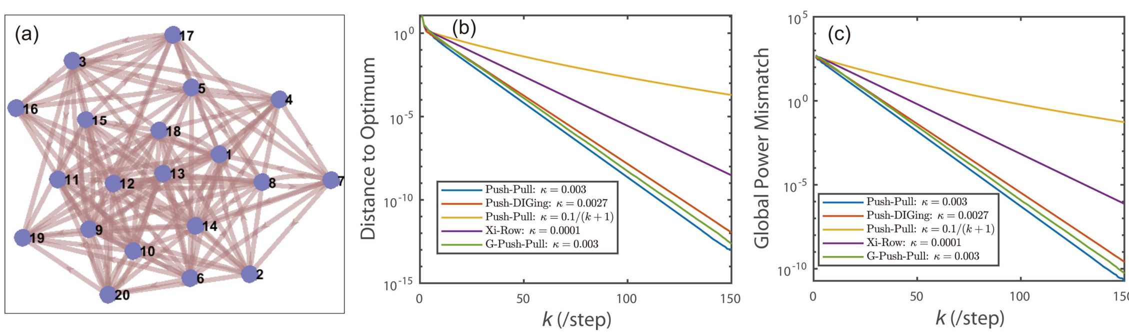

Figure 7 (Color online) Simulation results of Case 5. (a) The communication topology of the microgrid; (b) the distance to optimal incremental cost; (c) the global power mismatch.

communication network. This case study gives the performance of these algorithms for ED in a large-scale microgrid system operating in isolated mode only. A 51-bus microgrid system in [34] containing 20 DGs is considered, and the communication topology between the DGs is a strongly connected but not vary balanced graph which is generated randomly and can be seen in Figure 7(a). By manual optimization, we give the step size of each algorithm: 0.003 for the Push-Pull algorithm over a fixed topology, 0.0027 for the Push-DIGing algorithm, $0.1/(k+1)$ for the Push-Pull algorithm with diminishing step, 0.0001 for the Xi-Row algorithm and 0.003 for our proposed synchronous gossip-base Push-Pull algorithm. The activation probability of each communication link at each moment is set to 0.3, which also means that each communication link has a 0.7 probability of failure at each moment.

From Figure 7, we can see that the algorithms with the fixed step all have a linear convergence rate, while the Push-Pull algorithm with a diminishing step in [32] can only converge sublinearly. The proposed synchronous gossip-base Push-Pull algorithm achieves linear convergence even though the communication topology is randomly generated based on the original communication graph at each simulation moment, which means that the proposed algorithm can deal with communication link failure well. It also enjoys faster convergence than Xi-Row and Push-DIGing thanks to the fact that the Push-Pull structure is better able to deal with graphs that are not well-balanced. At the same time, the proposed algorithm uses fewer communication links per moment than the Push-Pull algorithm with fixed topology but achieves similar convergence rates. As mentioned before, the generated communication subgraph constitutes a time-varying network which means that the proposed algorithm can achieve optimal consensus as long as the the expectation of the time-varying communication network is strongly connected.

4.6 Case 6: 51-bus system implementation

The large-scale simulation was performed on the 51-bus microgrid containing 20 DGs mentioned in case study 5. The communication topology which is strongly connected and DG parameters are the same as those in case study 5. The switching time of the microgrid operation mode is the same as in the previous case studies. The communication weight δ is set as $\delta_1 = \cdots = \delta_{20} = 0.2$. The optimization step size η is set as $\eta_1 = \cdots = \eta_{20} = 0.01$. The activation probability of each communication link is set to 0.3, which means that each link fails with a probability of 0.7 at each moment. The nodes that have bi-directed communication with the ER are labeled as 1, 2, 8, 10, i.e., $a_{i0} = a_{0i} = 1, i = 1, 2, 8, 10$. The

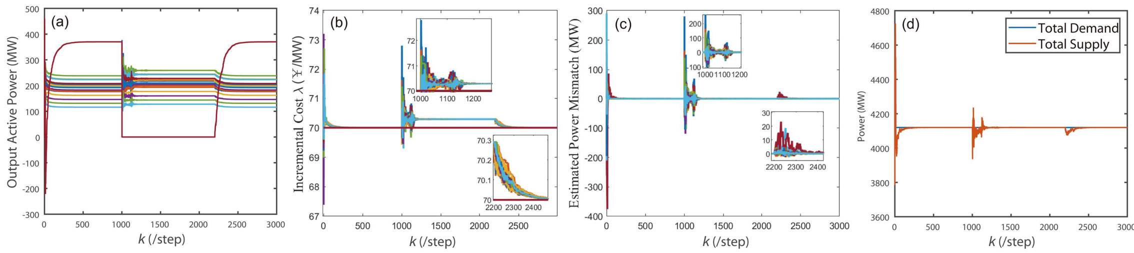

Figure 8 (Color online) Simulation results of Case 6. (a) The output active power of each DG; (b) the incremental cost of each DG; (c) the estimated average power mismatch of each ICU; (d) the total demand and supply of the microgrid.

electricity price of the main grid was set at 70, i.e., $\lambda_0 = 70$. The demand of each DG node was set as $[250; 350; 100; 110; 100; 150; 100; 300; 300; 100; 150; 150; 200; 110; 200; 150; 100; 300; 500; 400]$ MW, and the total demand is $P_D = 4120$ MW.

Figure 8 depicts that each DG achieves optimal power output before and after mode switching, showing the algorithm's effectiveness on large-scale microgrid systems. At the same time, the system remains stable and has a fast convergence rate despite the relatively high probability of failure of each communication link.

5 Conclusion

This paper proposes a gossip-based ED algorithm, which can cope with communication link failure and achieve smooth switching of microgrid operation modes. The algorithm under asynchronous directed communication networks is first given and proved to achieve almost sure consensus if the graph G is strongly connected. Then the algorithm is extended to synchronous communication networks and shown to achieve almost sure consensus under any randomly switched directed communication topologies as long as the expectation of the communication topology graphs is strongly connected. This is different from many algorithms that require the switched communication topology graph to be B-strongly connected and thus has a greater scope for applications such as collusion prevention. The algorithms using a non-consistent step size are fully distributed and consider directed communication structures under both synchronous and asynchronous communication networks. Compared to many existing algorithms, our algorithm is less demanding regarding initial conditions and easier to implement in practice. Finally, we demonstrate the effectiveness of the algorithm and its application to collusion prevention through several simulation cases. In the future, consideration can be given to further improve the convergence speed to superlinearity by using the idea of optimal control. Imperfect communication, such as communication delay and network attack, should be considered to further improve the practicality of the algorithm.

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