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Embedding prescribed-time adaptive control protocol unveiling distributed consensus in multirobot systems via directed topology

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Recently, multirobot systems (MRSs) have found extensive applications across various domains, including industrial manufacturing, collaborative formation of unmanned equipment, emergency disaster relief, and war scenarios [1]. These advancements are largely supported by the development of consistency control theory. However, traditional dynamics-free models may cause instability in complex robotic systems. Lagrangian dynamics offers a better approach for modeling these systems, as it facilitates controller design and optimization analysis. Despite this, challenges persist with unknown parameters and nonlinear friction within the systems.

To solve this problem, precise control of settling time in distributed systems is becoming a key focus in the consensus domain. Conventional control algorithms, such as finite-time and fixed-time control [2, 3], often fall short of meeting strict time interval requirements. By contrast, the prescribed-time control approach, noted in recent studies [4, 5], offers superior time regulation capabilities for Lagrangian systems. It addresses the consensus problem in these MRSs and allows for precise and stable control of time intervals, eliminating the need for prior information about the system.

Motivated by the above discussions, our study focuses on developing a distributed prescribed-time consensus control protocol for MRSs operating in a directed topology. We propose a novel embedded prescribed-time control approach specifically for Lagrangian systems, which consists of two components: variable-level and system-level prescribed-time embedding and error-driven adaptive laws.

Problem formulation. Consider the Lagrangian dynamics equation for robot-i in MRSs [s] of n nodes with generalized coordinate $q_i(t) = [q_{i1}(t), q_{i2}(t), \dots, q_{ip}(t)]^{\mathrm{T}}$ over a directed graph $\mathcal{G}$ as follows:

$$M_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i + g_i(q_i) = \tau_i, \tag{1}$$

where $M_i(q_i)$, $C_i(q_i, \dot{q}_i)$, and $g_i(q_i)$ are the inertia matrix, the centrifugal-Coriolis matrix, and the gravity vector, respectively. τ_i denotes the torque vector for system input, and all t values are omitted for brevity. The graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ is employed to delineate the communication topology of multirobot systems. Here, $\mathcal{V} = \{1, 2, \dots, n\}$ represents the set of nodes, and $\mathcal{E} \subseteq (\mathcal{V} \times \mathcal{V})$ indicates the edge set. Further, define the communication weights from robot-i to robot-j as a_{ij}, forming the adjacency matrix is $A = [a_{ij}] \in \mathbb{R}^{n\times n}$. This study aims to develop a prescribed-time control protocol for a multirobot system with topology graph $\mathcal{G}$ and the dynamics equations of (1), ensuring consistency is achieved over a defined time interval.

Assumption 1. $\mathcal{G}$ has at least one directed spanning tree.

Definition 1. Consider multirobot systems in graph $\mathcal{G}$ with a prescribed-time $[t_0, t_0 + t^*)$, if $(q_i - q_j) \to 0$ as $t \to t_0 + t^*$, then multirobot systems are said to from consensus in prescribed-time control.

Input and output layers. As shown in Figure 1, the input layer captures the generalized coordinate q_i of each robot, and its output is the reference velocity q_r and $\dot{q}_r$, which are embedded into a time-regulation function $f(t)$.

$$q_{ri} = -f\left(k_e \sum_{j\in\mathcal{V}} a_{ij}\left(q_i - q_j\right) - k_r \epsilon_i\right), \tag{2}$$

where k_e, k_r denote the gain coefficients, and f is denoted as

$$f(t) = \left(\frac{t^*}{t_0 + t^* - t}\right)^{\iota}, \ \iota > 1. \tag{3}$$

In addition, the virtual variable ϵ_i is defined as

$$\epsilon_i = q_i + k_e \int_{t_0}^{t} f(u) \sum_{j\in\mathcal{V}} a_{ij}\left(q_i(u) - q_j(u)\right) \mathrm{d}u. \tag{4}$$

The theoretical torque τ_i is the output layer's result. With the help of Appendix A, its linearization result can be expressed as follows:

$$\tau_i = Y_i(q_i, \dot{q}_i, \dot{q}_{ri}, q_{ri})\theta_i, \tag{5}$$

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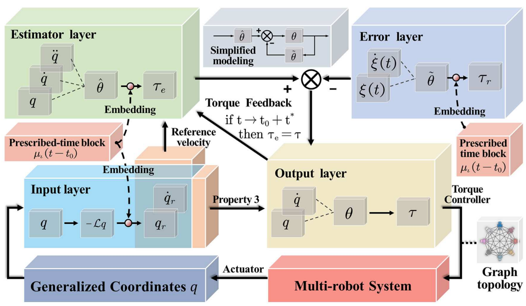

Figure 1 (Color online) Control logic and parameters conversion for prescribed-time control, where the output torque of the estimator layer is τ_e, the output torque of the error layer is τ_r, and the output layer generates the theoretical torque τ.

where θ_i is an unknown parameter vector of robot-i and matrix $Y_i(q_i, \dot{q}_i, \dot{q}_{ri}, q_{ri})$ represents the linearized regression matrix for robot-i.

Estimator and error layers. Consider the error $(\dot{q}_i - q_{ri})$ caused by the reference vector q_{ri}, also referred to as the sliding vector ξ_i, and the error developed by the virtual vector ϵ_i. The prescribed time control protocol at the estimator layer is finally designed as follows:

$$\tau_{ei} = Y_i\left(q_i, \dot{q}_i, \dot{q}_{ri}, q_{ri}\right)\hat{\theta}_i - \epsilon_i - K_{si}\xi_i - fK_{pi}\xi_i, \tag{6}$$

where f is embedded similarly, K_{si} and K_{pi} are diagonal matrices with positive elements, and $\hat{\theta}_i$ is the estimate of θ_i. The estimate error $\tilde{\theta}_i$ is defined by $(\hat{\theta}_i - \theta_i)$. Thus, the torque τ_{ri} of the error layer can be denoted as follows:

$$\tau_{ri} = Y_i\left(q_i, \dot{q}_i, \dot{q}_{ri}, q_{ri}\right)\tilde{\theta}_i - \epsilon_i - K_{si}\xi_i - fK_{pi}\xi_i. \tag{7}$$

To self-adapt parameters, we design a direct adaptive controller; in other words, the parameter adaptation is driven by the error ξ_i and can be represented as follows:

$$\dot{\hat{\theta}}_i = -\Gamma_i Y_i^{\mathrm{T}}\left(q_i, \dot{q}_i, \dot{q}_{ri}, q_{ri}\right)\xi_i, \tag{8}$$

where Γ_i is symmetric and positive definite constant matrix.

Remark 1. The prescribed-time controller in this paper is governed by the distributed control protocol in (6) and the error-driven adaptive law in (8) for dynamic parameter tuning.

Remark 2. Prescribed-time embedding can be interpreted as two different concepts. (i) At the variable level, it means that the time module is embedded as a component, e.g., the time function $f(t)$ is embedded in q_r, ϵ_i, and τ_{ei}, making the variables time-regulable. (ii) At the system level, it refers to the changed temporal characteristics of the original system. Specifically, it involves redefining the settling- time in our controller. This embedded approach is particularly applicable to robotic systems with Lagrangian dynamics.

Control objective. The purpose of this study is to design a distributed adaptive protocol so that (i) the starting time t_0 and settling time t^* can be regulated, while the convergence of the system is guaranteed; (ii) for Lagrangian dynamical systems with unknown parameters, the controller needs to ensure system stability; (iii) multirobot systems achieve velocity and position consensus within a prescribed time regulation.

Main results. The following results offer the conditions necessary for the stability of our protocols and adaptive laws, as well as the conditions required for reaching consensus.

Theorem 1. If the control protocols and adaptation laws satisfy $K_{pi}/k_r \geqslant k^i_{\max} I_p$, then the multi-robot systems will be prescribed-time input-state stable (PT-ISS).

Proof. $k^i_{\max}$ is a constant parameter (see Appendix A for details), and the proof can be found in Appendix B.

Theorem 2. With the topology $\mathcal{G}$ and Theorem 1, the distributed multi-robot systems will reach consensus within the prescribed time via our protocol and adaptive law.

Proof. The proof can be found in Appendix C.

Remark 3. The Lagrangian system dynamics of each robot exhibit parameter linearization. Therefore, we estimate the unknown parameter θ_i as $\hat{\theta}_i$ at the estimator layer while updating $Y_i(q_i, \dot{q}_i, \dot{q}_{ri}, q_{ri})$ with the generalized coordinate q_i and its reference velocity q_{ri}, enabling the controller to estimate the nonlinear system. The introduction of the virtual vector ϵ_i and the sliding vector ξ_i requires the inclusion of error control in the protocol, resulting in the final control protocol as shown in (6). The proposed adaptive law (8) dynamically adapts to the parameters to eliminate the effect of parameter uncertainty on the system. Furthermore, in the Lyapunov function, we introduce a quadratic term for the estimation error of these parameters and prove its stability, as shown in Appendix B.

Simulation. Considering eight robots satisfying (1), we designed three sets of comparison experiments to investigate the effects of unknown parameters, different initial states, and different stabilization times on the prescribed-time controller. The simulation results demonstrate that the controller with prescribed-time embedding is time-regulable and highly robust. The particular design and detailed results of this simulation can be observed in Appendix D.

Conclusion. This study proposes an embedded prescribed-time distributed control algorithm for Lagrangian multi-robot systems with unknown parameters. The algorithm regulates time by embedding a prescribed-time function while utilizing sliding mode control and an error-driven adaptive law to eliminate parameter uncertainties. Numerical experiments are provided to verify the initial state irrelevance, parameter uncertainty, and settling time regulation of the proposed control method.

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Supporting information Appendixes A–D. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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