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Cross-subcarrier precoder design for massive MIMO-OFDM downlink with symplectic optimization

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Appendix A Proof of theorem 1

The upper bound of $\mathcal{R}_k$ is expressed as

$$\mathcal{R}_k^{\mathrm{ub}} = \sum_{c=1}^{N_v} \log(1 + r_{k,c}^{-1}\mathbb{E}\{\boldsymbol{h}_{k,c}\boldsymbol{F}_c\boldsymbol{q}_k\boldsymbol{q}_k^{\mathrm{H}}\boldsymbol{F}_c^{\mathrm{H}}\boldsymbol{h}_{k,c}^{\mathrm{H}}\}). \tag{A1}$$

Since $\boldsymbol{h}_{k,c}\boldsymbol{F}_c\boldsymbol{q}_k\boldsymbol{q}_k^{\mathrm{H}}\boldsymbol{F}_c^{\mathrm{H}}\boldsymbol{h}_{k,c}^{\mathrm{H}} = \boldsymbol{q}_k^{\mathrm{H}}\boldsymbol{F}_c^{\mathrm{H}}\boldsymbol{h}_{k,c}^{\mathrm{H}}\boldsymbol{h}_{k,c}\boldsymbol{F}_c\boldsymbol{q}_k$, we have

$$\mathcal{R}_k^{\mathrm{ub}} = \sum_{c=1}^{N_v} \log(1 + r_{k,c}^{-1}\boldsymbol{q}_k^{\mathrm{H}}\boldsymbol{F}_c^{\mathrm{H}}\mathbb{E}\{\boldsymbol{h}_{k,c}^{\mathrm{H}}\boldsymbol{h}_{k,c}\}\boldsymbol{F}_c\boldsymbol{q}_k). \tag{A2}$$

Let

$$e_{k,c} = (1 + r_{k,c}^{-1}\boldsymbol{q}_k^{\mathrm{H}}\boldsymbol{F}_c^{\mathrm{H}}\mathbb{E}\{\boldsymbol{h}_{k,c}^{\mathrm{H}}\boldsymbol{h}_{k,c}\}\boldsymbol{F}_c\boldsymbol{q}_k)^{-1}. \tag{A3}$$

Then, we rewrite the upper bound as

$$\mathcal{R}_k^{\mathrm{ub}} = \sum_{c=1}^{N_v} -\log(e_{k,c}). \tag{A4}$$

Given that $-\log(\cdot)$ is convex, we have

$$\mathcal{R}_k^{\mathrm{ub}} \geqslant \sum_{c=1}^{N_v} -\log(e_{k,c}[d]) - (e_{k,c}[d])^{-1}(e_{k,c} - e_{k,c}[d]) \tag{A5}$$

and the equality achieves at $e_{k,c} = e_{k,c}[d]$.

According to $\boldsymbol{R}_{k,c} = \mathbb{E}\{\boldsymbol{h}_{k,c}^{\mathrm{H}}\boldsymbol{h}_{k,c}\}$, we rewrite $e_{k,c}$ as

$$e_{k,c} = (1 + r_{k,c}^{-1}\boldsymbol{q}_k^{\mathrm{H}}\boldsymbol{F}_c^{\mathrm{H}}\boldsymbol{R}_{k,c}\boldsymbol{F}_c\boldsymbol{q}_k)^{-1}. \tag{A6}$$

$e_{k,c}$ is the minimum of the sum-MSE minimization problem,

$$\min_{\boldsymbol{g}_{k,c}}(1 - \boldsymbol{g}_{k,c}^{\mathrm{H}}\boldsymbol{R}_{k,c}^{1/2}\boldsymbol{F}_c\boldsymbol{q}_k)(1 - \boldsymbol{g}_{k,c}^{\mathrm{H}}\boldsymbol{R}_{k,c}^{1/2}\boldsymbol{F}_c\boldsymbol{q}_k)^* + r_{k,c}\boldsymbol{g}_{k,c}^{\mathrm{H}}\boldsymbol{g}_{k,c}. \tag{A7}$$

Based on the above derivations and [25], we obtain the minorizing function provided in (29). Since g is a concave quadratic function and $\boldsymbol{q}_k$ for different users are uncoupled in g, the problem becomes easier to solve if we transform the objective function from f to g.

Using the function g and the MM methodology, we rewrite (18) as follows:

$$\arg\max_{\boldsymbol{q}_1,\ldots,\boldsymbol{q}_K} g(\boldsymbol{q}|\boldsymbol{q}[d]) \qquad \text{s.t.} \ \sum_{k=1}^{K}\boldsymbol{q}_k^{\mathrm{H}}\boldsymbol{q}_k \leqslant P. \tag{A8}$$

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The Lagrangian function is given as

$$\mathcal{L}(\mu, \boldsymbol{q}_1, \ldots, \boldsymbol{q}_K) = -\, g(\boldsymbol{q}|\boldsymbol{q}[d]) + \mu\left(\sum_{k=1}^{K} \boldsymbol{q}_k^{\mathrm{H}} \boldsymbol{q}_k - P\right). \tag{A9}$$

The derivative of $-g(\boldsymbol{q}|\boldsymbol{q}[d])$ with respect to $\boldsymbol{q}_k$ is

$$\frac{\partial(-g)}{\partial \boldsymbol{q}_k^*} = -w_k \sum_{c=1}^{N_v} \boldsymbol{F}_c^{\mathrm{H}} \boldsymbol{A}_{k,c}[d] \boldsymbol{F}_c \boldsymbol{q}_k[d] + \sum_{c=1}^{N_v} \boldsymbol{F}_c^{\mathrm{H}} \boldsymbol{D}_c[d] \boldsymbol{F}_c \boldsymbol{q}_k. \tag{A10}$$

Then, the derivative of (A9) is

$$\frac{\partial}{\boldsymbol{q}_k^*}(\mathcal{L}(\mu, \mathbb{Q})) = -w_k \sum_{c=1}^{N_v} \boldsymbol{F}_c^{\mathrm{H}} \boldsymbol{A}_{k,c}[d] \boldsymbol{F}_c \boldsymbol{q}_k[d] + \sum_{c=1}^{N_v} \boldsymbol{F}_c^{\mathrm{H}} \boldsymbol{D}_c[d] \boldsymbol{F}_c \boldsymbol{q}_k + \mu \boldsymbol{q}_k. \tag{A11}$$

According to the first order condition, we have

$$\left(\sum_{c=1}^{N_v} \boldsymbol{F}_c^{\mathrm{H}} \boldsymbol{D}_c[d] \boldsymbol{F}_c + \mu \boldsymbol{I}_{NM}\right) \boldsymbol{q}_k = w_k \sum_{c=1}^{N_v} \boldsymbol{F}_c^{\mathrm{H}} \boldsymbol{A}_{k,c}[d] \boldsymbol{F}_c \boldsymbol{q}_k[d]. \tag{A12}$$

Appendix B Derivation of kinetic energy

We transform the complex vectors to the real vectors for simplicity. Let $\boldsymbol{q}_R = (\mathrm{Re}\{\boldsymbol{q}\}^{\mathrm{T}}\ \mathrm{Im}\{\boldsymbol{q}\}^{\mathrm{T}})^{\mathrm{T}}, \boldsymbol{r}_R = (\mathrm{Re}\{\boldsymbol{r}\}^{\mathrm{T}}\ \mathrm{Im}\{\boldsymbol{r}\}^{\mathrm{T}})^{\mathrm{T}}$, $\boldsymbol{v}_R = (\mathrm{Re}\{\boldsymbol{v}\}^{\mathrm{T}}\ \mathrm{Im}\{\boldsymbol{v}\}^{\mathrm{T}})^{\mathrm{T}}$ be the expanded real vectors of $\boldsymbol{q}, \boldsymbol{r}, \boldsymbol{v}$, respectively. The kinetic energy is constructed from a Bregman divergence $D_h(\boldsymbol{p}_R, \boldsymbol{q}_R)$ which is defined from an auxiliary smooth function $h(\boldsymbol{q}_R) = \frac{1}{2}\langle \boldsymbol{q}_R, \boldsymbol{q}_R\rangle_R$ as

$$D_h(\boldsymbol{p}_R, \boldsymbol{q}_R) = h(\boldsymbol{p}_R) - h(\boldsymbol{q}_R) - \langle \boldsymbol{q}_R, \boldsymbol{p}_R - \boldsymbol{q}_R\rangle_R = \frac{\boldsymbol{p}_R^{\mathrm{T}} \boldsymbol{p}_R}{2} + \frac{\boldsymbol{q}_R^{\mathrm{T}} \boldsymbol{q}_R}{2} - \boldsymbol{q}_R^{\mathrm{T}} \boldsymbol{p}_R. \tag{B1}$$

For a given base point $\boldsymbol{q}_R$, we obtain a new time-dependent point $\boldsymbol{q}'_R = \boldsymbol{q}_R + \mathrm{e}^{-\alpha(t)} \boldsymbol{v}_R$ in the direction of the velocity $\boldsymbol{v}_R$. Thus, the kinetic energy can be given as

$$k(\boldsymbol{q}, \boldsymbol{v}) = \frac{\mathrm{e}^{-2\alpha(t)} \langle \boldsymbol{v}_R, \boldsymbol{v}_R\rangle_R}{2}. \tag{B2}$$

Using the relation between $\boldsymbol{v}_R$ and $\boldsymbol{r}_R$, the kinetic energy is reformulated as

$$k(\boldsymbol{q}, \boldsymbol{r}) = D_{h^*}(\mathrm{e}^{-\gamma(t)} \boldsymbol{r}_R + \nabla h(\boldsymbol{q}_R), \nabla h(\boldsymbol{q}_R)) = \frac{\mathrm{e}^{-2\gamma(t)} \langle \boldsymbol{r}_R, \boldsymbol{r}_R\rangle_R}{2}, \tag{B3}$$

where $h^l = \frac{1}{2}\langle \boldsymbol{r}_R, \boldsymbol{r}_R\rangle_R$ is the Legendre conjugate of h, and

$$D_{h^l}(\boldsymbol{r}_R, \boldsymbol{s}_R) = h^l(\boldsymbol{r}_R) - h^l(\boldsymbol{s}_R) - \frac{\partial h^l}{\partial \boldsymbol{s}_R}(\boldsymbol{s}_R)(\boldsymbol{r}_R - \boldsymbol{s}_R) = \frac{\boldsymbol{r}_R^{\mathrm{T}} \boldsymbol{r}_R}{2} + \frac{\boldsymbol{s}_R^{\mathrm{T}} \boldsymbol{s}_R}{2} - \boldsymbol{s}_R^{\mathrm{T}} \boldsymbol{r}_R. \tag{B4}$$

We obtain (37) using the relation $\langle \boldsymbol{r}_R, \boldsymbol{r}_R\rangle_R = \langle \boldsymbol{r}, \boldsymbol{r}\rangle$.

Appendix C Proof of theorem 2

Rewrite $r_{k,c}$ and $\tilde{r}_{k,c}$ as

$$r_{k,c} = \sigma_z^2 + \sum_{l\neq k}^{K} \bar{\boldsymbol{h}}_{k,c} \boldsymbol{F}_c \boldsymbol{q}_l \boldsymbol{q}_l^{\mathrm{H}} \boldsymbol{F}_c^{\mathrm{H}} \bar{\boldsymbol{h}}_{k,c}^{\mathrm{H}} + \sum_{l\neq k}^{K} \eta_k (\boldsymbol{F}_c \boldsymbol{q}_l \boldsymbol{q}_l^{\mathrm{H}} \boldsymbol{F}_c^{\mathrm{H}}), \tag{C1}$$

and

$$\tilde{r}_{k,c} = \sigma_z^2 + \sum_{l=1}^{K} \bar{\boldsymbol{h}}_{k,c} \boldsymbol{F}_c \boldsymbol{q}_l \boldsymbol{q}_l^{\mathrm{H}} \boldsymbol{F}_c^{\mathrm{H}} \bar{\boldsymbol{h}}_{k,c}^{\mathrm{H}} \sum_{l=1}^{K} \eta_k (\boldsymbol{F}_c \boldsymbol{q}_l \boldsymbol{q}_l^{\mathrm{H}} \boldsymbol{F}_c^{\mathrm{H}}). \tag{C2}$$

The matrix $\boldsymbol{R}_{k,c} = \mathbb{E}\{\boldsymbol{h}_{k,c}^{\mathrm{H}} \boldsymbol{h}_{k,c}\}$ is given as

$$\boldsymbol{R}_{k,c} = \bar{\boldsymbol{h}}_{k,c}^{\mathrm{H}} \bar{\boldsymbol{h}}_{k,c} + \mathbb{E}\{\tilde{\boldsymbol{h}}_{k,c}^{\mathrm{H}} \tilde{\boldsymbol{h}}_{k,c}\} = \bar{\boldsymbol{h}}_{k,c}^{\mathrm{H}} \bar{\boldsymbol{h}}_{k,c} + \boldsymbol{\Xi}_k. \tag{C3}$$

Substituting $\boldsymbol{F}_c = \boldsymbol{E}_c(\boldsymbol{F} \otimes \boldsymbol{I}_M)$ into (31) and (32), we have

$$\sum_{c=1}^{N_v} \boldsymbol{F}_c^{\mathrm{H}} \boldsymbol{D}_c[d] \boldsymbol{F}_c = (\boldsymbol{F} \otimes \boldsymbol{I}_M)^{\mathrm{H}} \sum_{c=1}^{N_v} \boldsymbol{E}_c^{\mathrm{H}} \boldsymbol{D}_c[d] \boldsymbol{E}_c (\boldsymbol{F} \otimes \boldsymbol{I}_M) = (\boldsymbol{F}^{\mathrm{H}} \otimes \boldsymbol{I}_M) \boldsymbol{\Lambda}[d] (\boldsymbol{F} \otimes \boldsymbol{I}_M), \tag{C4}$$

and

$$\sum_{c=1}^{N_v} \boldsymbol{F}_c^{\mathrm{H}} \boldsymbol{A}_{k,c}[d] \boldsymbol{F}_c = (\boldsymbol{F} \otimes \boldsymbol{I}_M)^{\mathrm{H}} \sum_{c=1}^{N_v} \boldsymbol{E}_c^{\mathrm{H}} \boldsymbol{A}_{k,c}[d] \boldsymbol{E}_c (\boldsymbol{F} \otimes \boldsymbol{I}_M) = (\boldsymbol{F}^{\mathrm{H}} \otimes \boldsymbol{I}_M) \boldsymbol{\Gamma}_k[d] (\boldsymbol{F} \otimes \boldsymbol{I}_M). \tag{C5}$$

Let $\bar{r}_{k,c}^{-1}[d] = r_{k,c}^{-1}[d] - \tilde{r}_{k,c}^{-1}[d]$. Using the decompositions in (41) and (42), we further obtain

$$\hat{\boldsymbol{D}}[d] = (\boldsymbol{F}^{\mathrm{H}} \otimes \boldsymbol{I}_M)(\bar{\boldsymbol{\Lambda}}[d] + \tilde{\boldsymbol{\Lambda}}[d])(\boldsymbol{F} \otimes \boldsymbol{I}_M), \tag{C6}$$

$$\hat{\boldsymbol{A}}_k[d] = (\boldsymbol{F}^{\mathrm{H}} \otimes \boldsymbol{I}_M)(\bar{\boldsymbol{\Gamma}}_k[d] + \tilde{\boldsymbol{\Gamma}}_k[d])(\boldsymbol{F} \otimes \boldsymbol{I}_M). \tag{C7}$$

The matrices $\tilde{\boldsymbol{\Lambda}}[d] = \mathrm{Bdiag}\{\tilde{\boldsymbol{D}}_1[d], \ldots, \tilde{\boldsymbol{D}}_{N_v}[d]\}$ and $\tilde{\boldsymbol{\Gamma}}_k[d] = \mathrm{Bdiag}\{\tilde{\boldsymbol{A}}_{k,1}[d], \ldots, \tilde{\boldsymbol{A}}_{k,N_v}[d]\}$ are written as

$$\tilde{\boldsymbol{\Lambda}}[d] = \sum_{k'=1}^{K} \boldsymbol{\Phi}_{k'}[d] \otimes \boldsymbol{\Xi}_{k'}, \tag{C8}$$

$$\tilde{\boldsymbol{\Gamma}}_k[d] = \boldsymbol{\Theta}_k[d] \otimes \boldsymbol{\Xi}_k, \tag{C9}$$

where $\boldsymbol{\Phi}_{k'}[d] = \mathrm{Diag}\{w_k \bar{r}_{k',1}^{-1}[d], \ldots, w_k \bar{r}_{k',N_v}^{-1}[d]\}$ and $\boldsymbol{\Theta}_k[d] = \mathrm{Diag}\{r_{k,1}^{-1}[d], \ldots, r_{k,N_v}^{-1}[d]\}$ are diagonal.

Then, the stochastic channel related parts of $\hat{\boldsymbol{D}}[\mathrm{d}]$ and $\hat{\boldsymbol{A}}_k[d]$ can be computed as

$$(\boldsymbol{F}^{\mathrm{H}} \otimes \boldsymbol{I}_M)\tilde{\boldsymbol{\Lambda}}[d](\boldsymbol{F} \otimes \boldsymbol{I}_M) = (\boldsymbol{F}^{\mathrm{H}} \otimes \boldsymbol{I}_M)\left(\sum_{k'=1}^{K} \tilde{\boldsymbol{\Lambda}}_{k'}[d] \otimes \boldsymbol{\Xi}_{k'}\right)(\boldsymbol{F} \otimes \boldsymbol{I}_M) = \sum_{k'=1}^{K} (\boldsymbol{F}^{\mathrm{H}} \tilde{\boldsymbol{\Lambda}}_{k'}[d] \boldsymbol{F}) \otimes \boldsymbol{\Xi}_{k'}, \tag{C10}$$

and

$$(\boldsymbol{F}^{\mathrm{H}} \otimes \boldsymbol{I}_M)\tilde{\boldsymbol{\Gamma}}_k[d](\boldsymbol{F} \otimes \boldsymbol{I}_M) = (\boldsymbol{F}^{\mathrm{H}} \otimes \boldsymbol{I}_M)(\boldsymbol{\Theta}_k[d] \otimes \boldsymbol{\Xi}_k)(\boldsymbol{F} \otimes \boldsymbol{I}_M) = (\boldsymbol{F}^{\mathrm{H}} \boldsymbol{\Theta}_k[d] \boldsymbol{F}) \otimes \boldsymbol{\Xi}_k. \tag{C11}$$

Finally, we can obtain the results provided in (43).