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• **Supplementary File** •

What is the optimal inter-site distance in multi-BS cooperative sensing?

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Appendix A Derivation of the radar SINR

According to [1], the radar SINRs of BS_1 and BS_2 can be respectively expressed as

$$\mathrm{SINR}_1 = \mathrm{Tr}(|a_1|^2\mathbf{A}_1(\theta_1,\phi_1)\mathbf{w}_1\mathbf{w}_1^{\mathrm{H}}\mathbf{A}_1^{\mathrm{H}}(\theta_1,\phi_1)(\rho|b_{1,2}|^2\mathbf{G}_{1,2}(\theta_1,\phi_1,\theta_2,\phi_2)\mathbf{w}_2\mathbf{w}_2^{\mathrm{H}}\mathbf{G}_{1,2}^{\mathrm{H}}(\theta_1,\phi_1,\theta_2,\phi_2)+\sigma^2\mathbf{I}_{N_\mathrm{r}})^{-1}), \tag{A1}$$

$$\mathrm{SINR}_2 = \mathrm{Tr}(|a_2|^2\mathbf{A}_2(\theta_2,\phi_2)\mathbf{w}_2\mathbf{w}_2^{\mathrm{H}}\mathbf{A}_2^{\mathrm{H}}(\theta_2,\phi_2)(\rho|b_{2,1}|^2\mathbf{G}_{2,1}(\theta_2,\phi_2,\theta_1,\phi_1)\mathbf{w}_1\mathbf{w}_1^{\mathrm{H}}\mathbf{G}_{2,1}^{\mathrm{H}}(\theta_2,\phi_2,\theta_1,\phi_1)+\sigma^2\mathbf{I}_{N_\mathrm{r}})^{-1}). \tag{A2}$$

By adopting MRT precoding and using the equation $\mathbf{a}_\mathrm{t}^{\mathrm{H}}(\theta_1,\phi_1)\mathbf{a}_\mathrm{t}(\theta_1,\phi_1) = \mathbf{a}_\mathrm{t}^{\mathrm{H}}(\theta_2,\phi_2)\mathbf{a}_\mathrm{t}(\theta_2,\phi_2) = \frac{N_{\mathrm{t},x}\times N_{\mathrm{t},z}}{N_\mathrm{t}} = 1$, the radar SINRs can be further expressed as

$$\mathrm{SINR}_1 = \mathrm{Tr}(|a_1|^2P_1\mathbf{A}_{\mathrm{r},1}(\rho P_2|b_{1,2}|^2\mathbf{A}_{\mathrm{r},1}+\sigma^2\mathbf{I}_{N_\mathrm{r}})^{-1}), \tag{A3}$$

$$\mathrm{SINR}_2 = \mathrm{Tr}(|a_2|^2P_2\mathbf{A}_{\mathrm{r},2}(\rho P_1|b_{2,1}|^2\mathbf{A}_{\mathrm{r},2}+\sigma^2\mathbf{I}_{N_\mathrm{r}})^{-1}), \tag{A4}$$

where $\mathbf{A}_{\mathrm{r},1} = \mathbf{a}_\mathrm{r}(\theta_1,\phi_1)\mathbf{a}_\mathrm{r}^{\mathrm{H}}(\theta_1,\phi_1)$ and $\mathbf{A}_{\mathrm{r},2} = \mathbf{a}_\mathrm{r}(\theta_2,\phi_2)\mathbf{a}_\mathrm{r}^{\mathrm{H}}(\theta_2,\phi_2)$ are both rank-1 matrices. Consider the eigenvalue decomposition of $\mathbf{A}_{\mathrm{r},1}$ and $\mathbf{A}_{\mathrm{r},2}$ for $\mathbf{A}_{\mathrm{r},1} = \mathbf{U}_1^{\mathrm{H}}\mathbf{\Lambda}_1\mathbf{U}_1$ and $\mathbf{A}_{\mathrm{r},2} = \mathbf{U}_2^{\mathrm{H}}\mathbf{\Lambda}_2\mathbf{U}_2$, which satisfy $\mathbf{U}_1^{\mathrm{H}}\mathbf{U}_1 = \mathbf{U}_2^{\mathrm{H}}\mathbf{U}_2 = \mathbf{I}_{N_\mathrm{r}}$ and $\mathbf{\Lambda}_1 = \mathbf{\Lambda}_2 = \mathrm{diag}\{1,0,\cdots,0\}$. The expressions of SINR_1 and SINR_2 can be further calculated as

$$\begin{aligned}\mathrm{SINR}_1 &= \mathrm{Tr}(|a_1|^2P_1\mathbf{A}_{\mathrm{r},1}(\rho P_2|b_{1,2}|^2\mathbf{A}_{\mathrm{r},1}+\sigma^2\mathbf{I}_{N_\mathrm{r}})^{-1}) = \mathrm{Tr}(|a_1|^2P_1\mathbf{U}_1^{\mathrm{H}}\mathbf{\Lambda}_1\mathbf{U}_1(\rho P_2|b_{1,2}|^2\mathbf{U}_1^{\mathrm{H}}\mathbf{\Lambda}_1\mathbf{U}_1+\sigma^2\mathbf{U}_1^{\mathrm{H}}\mathbf{U}_1)^{-1})\\ &= \mathrm{Tr}(|a_1|^2P_1\mathbf{U}_1^{\mathrm{H}}\mathbf{\Lambda}_1\mathbf{U}_1\mathbf{U}_1^{-1}(\rho P_2|b_{1,2}|^2\mathbf{\Lambda}_1+\sigma^2\mathbf{I}_{N_\mathrm{r}})^{-1}(\mathbf{U}_1^{-1})^{\mathrm{H}}) = |a_1|^2P_1\mathrm{Tr}(\mathbf{\Lambda}_1(\rho P_2|b_{1,2}|^2\mathbf{\Lambda}_1+\sigma^2\mathbf{I}_{N_\mathrm{r}})^{-1}),\end{aligned} \tag{A5}$$

$$\begin{aligned}\mathrm{SINR}_2 &= \mathrm{Tr}(|a_2|^2P_2\mathbf{A}_{\mathrm{r},2}(\rho P_1|b_{2,1}|^2\mathbf{A}_{\mathrm{r},2}+\sigma^2\mathbf{I}_{N_\mathrm{r}})^{-1}) = \mathrm{Tr}(|a_2|^2P_2\mathbf{U}_2^{\mathrm{H}}\mathbf{\Lambda}_2\mathbf{U}_2(\rho P_1|b_{2,1}|^2\mathbf{U}_2^{\mathrm{H}}\mathbf{\Lambda}_2\mathbf{U}_2+\sigma^2\mathbf{U}_2^{\mathrm{H}}\mathbf{U}_2)^{-1})\\ &= \mathrm{Tr}(|a_2|^2P_2\mathbf{U}_2^{\mathrm{H}}\mathbf{\Lambda}_2\mathbf{U}_2\mathbf{U}_2^{-1}(\rho P_1|b_{2,1}|^2\mathbf{\Lambda}_2+\sigma^2\mathbf{I}_{N_\mathrm{r}})^{-1}(\mathbf{U}_2^{-1})^{\mathrm{H}}) = |a_2|^2P_2\mathrm{Tr}(\mathbf{\Lambda}_2(\rho P_1|b_{2,1}|^2\mathbf{\Lambda}_2+\sigma^2\mathbf{I}_{N_\mathrm{r}})^{-1}).\end{aligned} \tag{A6}$$

Note that

$$\mathbf{\Lambda}_1(\rho P_2|b_{1,2}|^2\mathbf{\Lambda}_1+\sigma^2\mathbf{I}_{N_\mathrm{r}})^{-1} = \begin{bmatrix}1&0&\cdots&0\\0&0&\cdots&0\\\vdots&\vdots&\ddots&\vdots\\0&0&\cdots&0\end{bmatrix}\begin{bmatrix}\rho P_2|b_{1,2}|^2+\sigma^2&0&\cdots&0\\0&\sigma^2&\cdots&0\\\vdots&\vdots&\ddots&\vdots\\0&0&\cdots&\sigma^2\end{bmatrix}^{-1} = \begin{bmatrix}\frac{1}{\rho P_2|b_{1,2}|^2+\sigma^2}&0&\cdots&0\\0&0&\cdots&0\\\vdots&\vdots&\ddots&\vdots\\0&0&\cdots&0\end{bmatrix}, \tag{A7}$$

$$\mathbf{\Lambda}_2(\rho P_1|b_{2,1}|^2\mathbf{\Lambda}_2+\sigma^2\mathbf{I}_{N_\mathrm{r}})^{-1} = \begin{bmatrix}1&0&\cdots&0\\0&0&\cdots&0\\\vdots&\vdots&\ddots&\vdots\\0&0&\cdots&0\end{bmatrix}\begin{bmatrix}\rho P_1|b_{2,1}|^2+\sigma^2&0&\cdots&0\\0&\sigma^2&\cdots&0\\\vdots&\vdots&\ddots&\vdots\\0&0&\cdots&\sigma^2\end{bmatrix}^{-1} = \begin{bmatrix}\frac{1}{\rho P_1|b_{2,1}|^2+\sigma^2}&0&\cdots&0\\0&0&\cdots&0\\\vdots&\vdots&\ddots&\vdots\\0&0&\cdots&0\end{bmatrix}. \tag{A8}$$

Finally, we derive the expressions of the radar SINR when the UAV is located at (x,y,z) as follows

$$\mathrm{SINR}_1(x,y,z) = \frac{P_1|a_1|^2}{\rho P_2|b_{1,2}|^2+\sigma^2} = \frac{\frac{GCP_1}{(x^2+y^2+(z-h_{\mathrm{BS}})^2)^2}}{\frac{\rho GCP_2}{(x^2+(d-y)^2+(z-h_{\mathrm{BS}})^2)(x^2+y^2+(z-h_{\mathrm{BS}})^2)}+\sigma^2}, \tag{A9}$$

$$\mathrm{SINR}_2(x,y,z) = \frac{P_2|a_2|^2}{\rho P_1|b_{2,1}|^2+\sigma^2} = \frac{\frac{GCP_2}{(x^2+(d-y)^2+(z-h_{\mathrm{BS}})^2)^2}}{\frac{\rho GCP_1}{(x^2+(d-y)^2+(z-h_{\mathrm{BS}})^2)(x^2+y^2+(z-h_{\mathrm{BS}})^2)}+\sigma^2}. \tag{A10}$$

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To evaluate the sensing performance statistically, the UAV is assumed to obey uniform distribution in the cuboid region $\mathcal{D} = [-x_0, x_0] \times [\alpha d, \beta d] \times [h_{\min}, h_{\max}]$. Therefore, the probability density function (PDF) of the location of the UAV can be expressed as follows

$$f(x,y,z) = \begin{cases} \frac{1}{V_{\mathcal{D}}}, & (x,y,z) \in \mathcal{D} \\ 0, & (x,y,z) \notin \mathcal{D} \end{cases} \tag{A11}$$

where $V_{\mathcal{D}}$ denotes the volume of $\mathcal{D}$. The expectations of SINR_1 and SINR_2 are calculated as follows

$$\begin{aligned} &\overline{\mathrm{SINR}_1} = \mathbb{E}\{\mathrm{SINR}_1(x,y,z)\} = \frac{1}{V_{\mathcal{D}}} \iiint_{\mathcal{D}} \mathrm{SINR}_1(x,y,z)\mathrm{d}x\mathrm{d}y\mathrm{d}z \\ &= \frac{1}{(\beta-\alpha)x_0(h_{\max}-h_{\min})d} \int_{\alpha d}^{\beta d} \mathrm{d}y \int_0^{x_0} \mathrm{d}x \int_{h_{\min}}^{h_{\max}} \frac{\frac{GCP_1}{(x^2+y^2+(z-h_{\mathrm{BS}})^2)^2}}{\frac{\rho GCP_2}{(x^2+(d-y)^2+(z-h_{\mathrm{BS}})^2)(x^2+y^2+(z-h_{\mathrm{BS}})^2)} + \sigma^2} \mathrm{d}z, \end{aligned} \tag{A12}$$

$$\begin{aligned} &\overline{\mathrm{SINR}_2} = \mathbb{E}\{\mathrm{SINR}_2(x,y,z)\} = \frac{1}{V_{\mathcal{D}}} \iiint_{\mathcal{D}} \mathrm{SINR}_2(x,y,z)\mathrm{d}x\mathrm{d}y\mathrm{d}z \\ &= \frac{1}{(\beta-\alpha)x_0(h_{\max}-h_{\min})d} \int_{\alpha d}^{\beta d} \mathrm{d}y \int_0^{x_0} \mathrm{d}x \int_{h_{\min}}^{h_{\max}} \frac{\frac{GCP_2}{(x^2+(d-y)^2+(z-h_{\mathrm{BS}})^2)^2}}{\frac{\rho GCP_1}{(x^2+(d-y)^2+(z-h_{\mathrm{BS}})^2)(x^2+y^2+(z-h_{\mathrm{BS}})^2)} + \sigma^2} \mathrm{d}z. \end{aligned} \tag{A13}$$

Due to symmetry, we only consider the case where the UAV is closer to BS_1, i.e., $\alpha < \beta \leqslant 0.5$. Consequently, we have $\overline{\mathrm{SINR}} = \max\{\overline{\mathrm{SINR}_1}, \overline{\mathrm{SINR}_2}\} = \overline{\mathrm{SINR}_1}$. It is worth noting that the above integrals cannot be solved analytically, so we use numerical methods to calculate the above integrals.

Appendix B Derivation of the joint detection probability

We consider the likelihood ratio test (LRT) detector over one sensing block of L symbols. For BS_1, the detection problem can be formulated as a binary hypothesis testing problem [2] as follows

$$\begin{cases} \mathbf{Y}_{1,2} = \mathbf{N}, & \mathcal{H}_0 \\ \mathbf{Y}_{1,2} = a_1\mathbf{A}_1(\theta_1,\phi_1)\mathbf{w}_1\mathbf{s}_1 + \sqrt{\rho}b_{1,2}\mathbf{G}_{1,2}(\theta_1,\phi_1,\theta_2,\phi_2)\mathbf{w}_2\mathbf{s}_2 + \mathbf{N}, & \mathcal{H}_1 \end{cases} \tag{B1}$$

where $\mathbf{Y}_{1,2} = [\mathbf{y}_{1,2}(1), \cdots, \mathbf{y}_{1,2}(L)]$, $\mathbf{N} = [\mathbf{n}(1), \cdots, \mathbf{n}(L)]$, $\mathbf{s}_1 = [s_{1,1}, \cdots, s_{1,L}]$, and $\mathbf{s}_2 = [s_{2,1}, \cdots, s_{2,L}]$. In the mono-static architecture, $\sqrt{\rho}b_{1,2}\mathbf{G}_{1,2}(\theta_1,\phi_1,\theta_2,\phi_2)\mathbf{w}_2\mathbf{s}_2$ and $\mathbf{N}$ are regarded as interference/noise for BS_1. After signal vectorization, the problem can be further expressed as

$$\begin{cases} \widetilde{\mathbf{y}}_1 = -\mathbf{u}_2 + \mathbf{n}_s, & \mathcal{H}_0 \\ \widetilde{\mathbf{y}}_1 = \mathbf{u}_1 + \mathbf{n}_s, & \mathcal{H}_1 \end{cases} \tag{B2}$$

where $\widetilde{\mathbf{y}}_1 = \mathrm{vec}\{\mathbf{Y}_{1,2}\}$, $\mathbf{u}_1 = \mathrm{vec}\{a_1\mathbf{A}_1(\theta_1,\phi_1)\mathbf{w}_1\mathbf{s}_1\}$, $\mathbf{u}_2 = \mathrm{vec}\{\sqrt{\rho}b_{1,2}\mathbf{G}_{1,2}(\theta_1,\phi_1,\theta_2,\phi_2)\mathbf{w}_2\mathbf{s}_2\}$, $\mathbf{n}_s = \mathrm{vec}\{\sqrt{\rho}b_{1,2}\mathbf{G}_{1,2}(\theta_1,\phi_1,\theta_2,\phi_2)\mathbf{w}_2\mathbf{s}_2 + \mathbf{N}\} \sim \mathcal{CN}(\mathbf{0},\mathbf{R})$, and $\mathbf{R} = \mathbf{I}_L \otimes (\rho|b_{1,2}|^2\mathbf{G}_{1,2}(\theta_1,\phi_1,\theta_2,\phi_2)\mathbf{w}_2\mathbf{w}_2^{\mathrm{H}}\mathbf{G}_{1,2}^{\mathrm{H}}(\theta_1,\phi_1,\theta_2,\phi_2) + \sigma^2\mathbf{I}_{N_{\mathrm{r}}}) \triangleq \mathbf{I}_L \otimes \widetilde{\mathbf{R}}$. Therefore, the LRT detector can be expressed as

$$\Lambda(\widetilde{\mathbf{y}}_1) = \frac{f(\widetilde{\mathbf{y}}_1|\mathcal{H}_1)}{f(\widetilde{\mathbf{y}}_1|\mathcal{H}_0)} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \kappa, \tag{B3}$$

where κ is a certain threshold which is determined according to a target value of probability of false alarm P_{FA} (the probability that the system still determines the presence of a target when only noise/interference is present) for performing constant false alarm rate (CFAR) detection [3]. $f(\widetilde{\mathbf{y}}_1|\mathcal{H}_1)$ and $f(\widetilde{\mathbf{y}}_1|\mathcal{H}_0)$ denote the PDFs of $\widetilde{\mathbf{y}}_1$ under the hypotheses $\mathcal{H}_1$ and $\mathcal{H}_0$, respectively, which can be expressed as

$$f(\widetilde{\mathbf{y}}_1|\mathcal{H}_0) = \frac{1}{\pi^{N_{\mathrm{r}}L}\det(\mathbf{R})} \exp(-(\widetilde{\mathbf{y}}_1 + \mathbf{u}_2)^{\mathrm{H}}\mathbf{R}^{-1}(\widetilde{\mathbf{y}}_1 + \mathbf{u}_2)), \tag{B4}$$

$$f(\widetilde{\mathbf{y}}_1|\mathcal{H}_1) = \frac{1}{\pi^{N_{\mathrm{r}}L}\det(\mathbf{R})} \exp(-(\widetilde{\mathbf{y}}_1 - \mathbf{u}_1)^{\mathrm{H}}\mathbf{R}^{-1}(\widetilde{\mathbf{y}}_1 - \mathbf{u}_1)). \tag{B5}$$

After taking the logarithmic operation, the logarithmic LRT function can be expressed as

$$\ln\Lambda(\widetilde{\mathbf{y}}_1) = (\widetilde{\mathbf{y}}_1 + \mathbf{u}_2)^{\mathrm{H}}\mathbf{R}^{-1}(\widetilde{\mathbf{y}}_1 + \mathbf{u}_2) - (\widetilde{\mathbf{y}}_1 - \mathbf{u}_1)^{\mathrm{H}}\mathbf{R}^{-1}(\widetilde{\mathbf{y}}_1 - \mathbf{u}_1). \tag{B6}$$

$\ln\Lambda(\widetilde{\mathbf{y}}_1)$ under hypotheses $\mathcal{H}_0$ and $\mathcal{H}_1$, respectively, are distributed as

$$\begin{cases} \ln\Lambda(\widetilde{\mathbf{y}}_1) \sim \mathcal{N}(-\mathbf{u}_1^{\mathrm{H}}\mathbf{R}^{-1}\mathbf{u}_2 - \mathbf{u}_2^{\mathrm{H}}\mathbf{R}^{-1}\mathbf{u}_1 - \mathbf{u}_1^{\mathrm{H}}\mathbf{R}^{-1}\mathbf{u}_1 - \mathbf{u}_2^{\mathrm{H}}\mathbf{R}^{-1}\mathbf{u}_2, 2\mathbf{u}_1^{\mathrm{H}}\mathbf{R}^{-1}\mathbf{u}_1 + 2\mathbf{u}_2^{\mathrm{H}}\mathbf{R}^{-1}\mathbf{u}_2), & \mathcal{H}_0 \\ \ln\Lambda(\widetilde{\mathbf{y}}_1) \sim \mathcal{N}(\mathbf{u}_1^{\mathrm{H}}\mathbf{R}^{-1}\mathbf{u}_2 + \mathbf{u}_2^{\mathrm{H}}\mathbf{R}^{-1}\mathbf{u}_1 + \mathbf{u}_1^{\mathrm{H}}\mathbf{R}^{-1}\mathbf{u}_1 + \mathbf{u}_2^{\mathrm{H}}\mathbf{R}^{-1}\mathbf{u}_2, 2\mathbf{u}_1^{\mathrm{H}}\mathbf{R}^{-1}\mathbf{u}_1 + 2\mathbf{u}_2^{\mathrm{H}}\mathbf{R}^{-1}\mathbf{u}_2), & \mathcal{H}_1 \end{cases} \tag{B7}$$

Therefore, the probability of false alarm P_{FA} and detection probability of BS_1 interfered with by BS_2(the probability that the system correctly detects the target in the presence of noise and interference) can be respectively expressed as

$$P_{\mathrm{FA}} = \Pr\{\ln\Lambda(\widetilde{\mathbf{y}}_1) \geqslant \ln\kappa|\mathcal{H}_0\} = Q(\frac{\ln\kappa + \mathbf{u}_1^{\mathrm{H}}\mathbf{R}^{-1}\mathbf{u}_2 + \mathbf{u}_2^{\mathrm{H}}\mathbf{R}^{-1}\mathbf{u}_1 + \mathbf{u}_1^{\mathrm{H}}\mathbf{R}^{-1}\mathbf{u}_1 + \mathbf{u}_2^{\mathrm{H}}\mathbf{R}^{-1}\mathbf{u}_2}{\sqrt{2(\mathbf{u}_1^{\mathrm{H}}\mathbf{R}^{-1}\mathbf{u}_1 + \mathbf{u}_2^{\mathrm{H}}\mathbf{R}^{-1}\mathbf{u}_2)}}), \tag{B8}$$

$$P_{1,2} = \Pr\{\ln\Lambda(\widetilde{\mathbf{y}}_1) \geqslant \ln\kappa|\mathcal{H}_1\} = Q(\frac{\ln\kappa - \mathbf{u}_1^{\mathrm{H}}\mathbf{R}^{-1}\mathbf{u}_2 - \mathbf{u}_2^{\mathrm{H}}\mathbf{R}^{-1}\mathbf{u}_1 - \mathbf{u}_1^{\mathrm{H}}\mathbf{R}^{-1}\mathbf{u}_1 - \mathbf{u}_2^{\mathrm{H}}\mathbf{R}^{-1}\mathbf{u}_2}{\sqrt{2(\mathbf{u}_1^{\mathrm{H}}\mathbf{R}^{-1}\mathbf{u}_1 + \mathbf{u}_2^{\mathrm{H}}\mathbf{R}^{-1}\mathbf{u}_2)}}), \tag{B9}$$

where $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty e^{-\frac{t^2}{2}}\mathrm{d}t$ denotes the Q-function and the detection threshold κ can be determined by

$$\ln\kappa = Q^{-1}(P_{\mathrm{FA}})\sqrt{2(\mathbf{u}_1^{\mathrm{H}}\mathbf{R}^{-1}\mathbf{u}_1+\mathbf{u}_2^{\mathrm{H}}\mathbf{R}^{-1}\mathbf{u}_2)} - \mathbf{u}_1^{\mathrm{H}}\mathbf{R}^{-1}\mathbf{u}_2 - \mathbf{u}_2^{\mathrm{H}}\mathbf{R}^{-1}\mathbf{u}_1 - \mathbf{u}_1^{\mathrm{H}}\mathbf{R}^{-1}\mathbf{u}_1 - \mathbf{u}_2^{\mathrm{H}}\mathbf{R}^{-1}\mathbf{u}_2, \tag{B10}$$

where $Q^{-1}(x)$ denotes the inverse Q-function. By defining $\kappa = Q^{-1}(P_{\mathrm{FA}})$, the detection probability of BS_1 can be further expressed as

$$P_{1,2} = Q(\kappa - \frac{2\mathbf{u}_2^{\mathrm{H}}\mathbf{R}^{-1}\mathbf{u}_1 + 2\mathbf{u}_1^{\mathrm{H}}\mathbf{R}^{-1}\mathbf{u}_2 + 2\mathbf{u}_1^{\mathrm{H}}\mathbf{R}^{-1}\mathbf{u}_1 + 2\mathbf{u}_2^{\mathrm{H}}\mathbf{R}^{-1}\mathbf{u}_2}{\sqrt{2(\mathbf{u}_1^{\mathrm{H}}\mathbf{R}^{-1}\mathbf{u}_1+\mathbf{u}_2^{\mathrm{H}}\mathbf{R}^{-1}\mathbf{u}_2)}}). \tag{B11}$$

By using the transformation $\mathrm{Tr}(\mathbf{ABCD}) = \mathrm{vec}^{\mathrm{H}}\{\mathbf{D}^{\mathrm{H}}\}(\mathbf{C}^{\mathrm{T}}\otimes\mathbf{A})\mathrm{vec}\{\mathbf{B}\}$ and the assumption that $\mathbf{s}_1^{\mathrm{H}}\mathbf{s}_2 \approx \mathbb{E}\{\mathbf{s}_1^{\mathrm{H}}\mathbf{s}_2\} = 0$, $\mathbf{s}_2^{\mathrm{H}}\mathbf{s}_1 \approx \mathbb{E}\{\mathbf{s}_2^{\mathrm{H}}\mathbf{s}_1\} = 0$, $\mathbf{s}_1^{\mathrm{H}}\mathbf{s}_1 \approx \mathbb{E}\{\mathbf{s}_1^{\mathrm{H}}\mathbf{s}_1\} = L$ and $\mathbf{s}_2^{\mathrm{H}}\mathbf{s}_2 \approx \mathbb{E}\{\mathbf{s}_2^{\mathrm{H}}\mathbf{s}_2\} = L$, we can rewrite P_1 when the UAV is located at (x,y,z) into a more intractable form as follows

$$P_{1,2}(x,y,z) = Q(\kappa - \sqrt{2L(\mathrm{SINR}_1(x,y,z) + \frac{\rho P_2|b_{1,2}|^2}{\rho P_2|b_{1,2}|^2+\sigma^2})}). \tag{B12}$$

By adopting the similar derivation, the detection probability of BS_2 interfered with by BS_1 when the UAV is located at (x,y,z) can be expressed as

$$P_{2,1}(x,y,z) = Q(\kappa - \sqrt{2L(\mathrm{SINR}_2(x,y,z) + \frac{\rho P_1|b_{2,1}|^2}{\rho P_1|b_{2,1}|^2+\sigma^2})}). \tag{B13}$$

Eventually, according to the equation $1-Q(x) = Q(-x)$, the joint detection probability of the two BSs when the UAV is located at (x,y,z) can be calculated as

$$\begin{aligned} P_{\mathrm{D}}(x,y,z) &= 1-(1-P_{1,2}(x,y,z))\cdot(1-P_{2,1}(x,y,z)) \\ &= 1 - Q(\sqrt{2L(\mathrm{SINR}_1(x,y,z)+\frac{\rho P_2|b_{1,2}|^2}{\rho P_2|b_{1,2}|^2+\sigma^2})}-\kappa)\cdot Q(\sqrt{2L(\mathrm{SINR}_2(x,y,z)+\frac{\rho P_1|b_{2,1}|^2}{\rho P_1|b_{2,1}|^2+\sigma^2})}-\kappa). \end{aligned} \tag{B14}$$

Similarly, the expectation of the joint detection probability is calculated as follows

$$\begin{aligned} \overline{P_{\mathrm{D}}} &= \mathbb{E}\{P_{\mathrm{D}}(x,y,z)\} = \frac{1}{V_{\mathcal{D}}}\iiint_{\mathcal{D}} P_{\mathrm{D}}(x,y,z)\mathrm{d}x\mathrm{d}y\mathrm{d}z \\ &= \frac{1}{(\beta-\alpha)x_0(h_{\max}-h_{\min})d}\int_{\alpha d}^{\beta d}\mathrm{d}y\int_0^{x_0}\mathrm{d}x\int_{h_{\min}}^{h_{\max}} P_{\mathrm{D}}(x,y,z)\mathrm{d}z. \end{aligned} \tag{B15}$$

It is worth noting that the above integral cannot be solved analytically, so we use numerical methods to calculate the above integral.

Appendix C Derivation of partial derivatives of the average radar SINR and the average joint detection probability with respect to inter-site distance d

Firstly, the partial derivatives of $\mathrm{SINR}_1(x,y,z)$ and $\mathrm{SINR}_2(x,y,z)$ with respect to inter-site distance d are calculated as follows

$$\begin{aligned} \frac{\partial\mathrm{SINR}_1(x,y,z)}{\partial d} &= \frac{-2B_1(d-y)(x^2+(d-y)^2+(z-h_{\mathrm{BS}})^2)(x^2+y^2+(z-h_{\mathrm{BS}})^2)^2}{(A_1(x^2+y^2+(z-h_{\mathrm{BS}})^2)+B_1(x^2+y^2+(z-h_{\mathrm{BS}})^2)^2(x^2+(d-y)^2+(z-h_{\mathrm{BS}})^2))^2} \\ &+ \frac{2(d-y)}{A_1(x^2+y^2+(z-h_{\mathrm{BS}})^2)+B_1(x^2+y^2+(z-h_{\mathrm{BS}})^2)^2(x^2+(d-y)^2+(z-h_{\mathrm{BS}})^2)}, \end{aligned} \tag{C1}$$

$$\frac{\partial\mathrm{SINR}_2(x,y,z)}{\partial d} = \frac{-(x^2+y^2+(z-h_{\mathrm{BS}})^2)(2A_2(d-y)+4B_2(d-y)(x^2+y^2+(z-h_{\mathrm{BS}})^2)(x^2+(d-y)^2+(z-h_{\mathrm{BS}})^2))}{(A_2(x^2+(d-y)^2+(z-h_{\mathrm{BS}})^2)+B_2(x^2+(d-y)^2+(z-h_{\mathrm{BS}})^2)^2(x^2+y^2+(z-h_{\mathrm{BS}})^2))^2}, \tag{C2}$$

where $A_1 = \frac{\rho P_2}{P_1}$, $B_1 = \frac{\sigma^2}{GCP_1}$, $A_2 = \frac{\rho P_1}{P_2}$ and $B_2 = \frac{\sigma^2}{GCP_2}$.

Then, by using the derivation formula for integrals with parameters, the partial derivatives of the average radar SINR with respect to the inter-site distance d can be expressed as

$$\begin{aligned} \frac{\partial\overline{\mathrm{SINR}_1}}{\partial d} &= \frac{1}{(\beta-\alpha)x_0(h_{\max}-h_{\min})}\frac{\partial\int_{\alpha d}^{\beta d}\mathrm{d}y\int_0^{x_0}\mathrm{d}x\int_{h_{\min}}^{h_{\max}}\frac{1}{d}\mathrm{SINR}_1(x,y,z)\mathrm{d}z}{\partial d} \\ &= \frac{1}{(\beta-\alpha)x_0(h_{\max}-h_{\min})}\int_{\alpha d}^{\beta d}\mathrm{d}y\int_0^{x_0}\mathrm{d}x\int_{h_{\min}}^{h_{\max}}(-\frac{1}{d^2}\mathrm{SINR}_1(x,y,z)+\frac{1}{d}\frac{\partial\mathrm{SINR}_1(x,y,z)}{\partial d})\mathrm{d}z \\ &+ \frac{\beta}{(\beta-\alpha)x_0(h_{\max}-h_{\min})}\int_0^{x_0}\mathrm{d}x\int_{h_{\min}}^{h_{\max}}\frac{1}{d}\mathrm{SINR}_1(x,\beta d,z)\mathrm{d}z - \frac{\alpha}{(\beta-\alpha)x_0(h_{\max}-h_{\min})}\int_0^{x_0}\mathrm{d}x\int_{h_{\min}}^{h_{\max}}\frac{1}{d}\mathrm{SINR}_1(x,\alpha d,z)\mathrm{d}z, \end{aligned} \tag{C3}$$

$$\begin{aligned} \frac{\partial\overline{\mathrm{SINR}_2}}{\partial d} &= \frac{1}{(\beta-\alpha)x_0(h_{\max}-h_{\min})}\frac{\partial\int_{\alpha d}^{\beta d}\mathrm{d}y\int_0^{x_0}\mathrm{d}x\int_{h_{\min}}^{h_{\max}}\frac{1}{d}\mathrm{SINR}_2(x,y,z)\mathrm{d}z}{\partial d} \\ &= \frac{1}{(\beta-\alpha)x_0(h_{\max}-h_{\min})}\int_{\alpha d}^{\beta d}\mathrm{d}y\int_0^{x_0}\mathrm{d}x\int_{h_{\min}}^{h_{\max}}(-\frac{1}{d^2}\mathrm{SINR}_2(x,y,z)+\frac{1}{d}\frac{\partial\mathrm{SINR}_2(x,y,z)}{\partial d})\mathrm{d}z \\ &+ \frac{\beta}{(\beta-\alpha)x_0(h_{\max}-h_{\min})}\int_0^{x_0}\mathrm{d}x\int_{h_{\min}}^{h_{\max}}\frac{1}{d}\mathrm{SINR}_2(x,\beta d,z)\mathrm{d}z - \frac{\alpha}{(\beta-\alpha)x_0(h_{\max}-h_{\min})}\int_0^{x_0}\mathrm{d}x\int_{h_{\min}}^{h_{\max}}\frac{1}{d}\mathrm{SINR}_2(x,\alpha d,z)\mathrm{d}z. \end{aligned} \tag{C4}$$

Due to symmetry, we only consider the case where the UAV is closer to BS_1, i.e., $\alpha < \beta \leqslant 0.5$. Consequently, we have $\frac{\partial\overline{\mathrm{SINR}}}{\partial d} = \frac{\partial(\max\{\overline{\mathrm{SINR}_1},\overline{\mathrm{SINR}_2}\})}{\partial d} = \frac{\partial\overline{\mathrm{SINR}_1}}{\partial d}$.

By defining

$$D_1 \triangleq \frac{\rho P_2|b_{1,2}|^2}{\rho P_2|b_{1,2}|^2+\sigma^2} = \frac{\rho GCP_2}{\rho GCP_2+\sigma^2(x^2+y^2+(z-h_{\mathrm{BS}})^2)(x^2+(d-y)^2+(z-h_{\mathrm{BS}})^2)}, \tag{C5}$$

$$D_2 \triangleq \frac{\rho P_1|b_{2,1}|^2}{\rho P_1|b_{2,1}|^2+\sigma^2} = \frac{\rho GCP_1}{\rho GCP_1+\sigma^2(x^2+y^2+(z-h_{\mathrm{BS}})^2)(x^2+(d-y)^2+(z-h_{\mathrm{BS}})^2)}, \tag{C6}$$

$$E_1 \triangleq \frac{\partial D_1}{\partial d} = \frac{-2\rho\sigma^2 GCP_2(d-y)(x^2+y^2+(z-h_{\mathrm{BS}})^2)}{(\rho GCP_2+\sigma^2(x^2+y^2+(z-h_{\mathrm{BS}})^2)(x^2+(d-y)^2+(z-h_{\mathrm{BS}})^2))^2}, \tag{C7}$$

$$E_2 \triangleq \frac{\partial D_2}{\partial d} = \frac{-2\rho\sigma^2 GCP_1(d-y)(x^2+y^2+(z-h_{\mathrm{BS}})^2)}{(\rho GCP_1+\sigma^2(x^2+y^2+(z-h_{\mathrm{BS}})^2)(x^2+(d-y)^2+(z-h_{\mathrm{BS}})^2))^2}, \tag{C8}$$

and using the chain rule for derivation, the partial derivative of $P_{\mathrm{D}}(x,y,z)$ with respect to inter-site distance d are first calculated as follows

$$\begin{aligned}\frac{\partial P_{\mathrm{D}}(x,y,z)}{\partial d} &= \sqrt{\frac{L}{4\pi}}\cdot(\frac{\partial \mathrm{SINR}_1(x,y,z)}{\partial d}+E_1)\cdot\frac{\exp(\frac{-(\sqrt{2L(\mathrm{SINR}_1(x,y,z)+D_1)}-\kappa)^2}{2})}{\sqrt{\mathrm{SINR}_1(x,y,z)+D_1}}\cdot Q(\sqrt{2L(\mathrm{SINR}_2(x,y,z)+D_2)}-\kappa)\\ &+\sqrt{\frac{L}{4\pi}}\cdot(\frac{\partial \mathrm{SINR}_2(x,y,z)}{\partial d}+E_2)\cdot\frac{\exp(\frac{-(\sqrt{2L(\mathrm{SINR}_2(x,y,z)+D_2)}-\kappa)^2}{2})}{\sqrt{\mathrm{SINR}_2(x,y,z)+D_2}}\cdot Q(\sqrt{2L(\mathrm{SINR}_1(x,y,z)+D_1)}-\kappa).\end{aligned} \tag{C9}$$

Similarly, by using the derivation formula for integrals with parameters, the partial derivative of the average joint detection probability with respect to the inter-site distance d can be expressed as

$$\begin{aligned}\frac{\partial\overline{P_{\mathrm{D}}}}{\partial d} &= \frac{1}{(\beta-\alpha)x_0(h_{\max}-h_{\min})}\frac{\partial\int_{\alpha d}^{\beta d}\mathrm{d}y\int_0^{x_0}\mathrm{d}x\int_{h_{\min}}^{h_{\max}}\frac{1}{d}P_{\mathrm{D}}(x,y,z)\mathrm{d}z}{\partial d}\\ &= \frac{1}{(\beta-\alpha)x_0(h_{\max}-h_{\min})}\int_{\alpha d}^{\beta d}\mathrm{d}y\int_0^{x_0}\mathrm{d}x\int_{h_{\min}}^{h_{\max}}(-\frac{1}{d^2}P_{\mathrm{D}}(x,y,z)+\frac{1}{d}\frac{\partial P_{\mathrm{D}}(x,y,z)}{\partial d})\mathrm{d}z\\ &+\frac{\beta}{(\beta-\alpha)x_0(h_{\max}-h_{\min})}\int_0^{x_0}\mathrm{d}x\int_{h_{\min}}^{h_{\max}}\frac{1}{d}P_{\mathrm{D}}(x,\beta d,z)\mathrm{d}z-\frac{\alpha}{(\beta-\alpha)x_0(h_{\max}-h_{\min})}\int_0^{x_0}\mathrm{d}x\int_{h_{\min}}^{h_{\max}}\frac{1}{d}P_{\mathrm{D}}(x,\alpha d,z)\mathrm{d}z.\end{aligned} \tag{C10}$$

It is worth noting that the above integrals cannot be solved analytically, so we use numerical methods to calculate the above integrals.

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