SCIENCE CHINA
Information Sciences

• LETTER •

December 2024, Vol. 67, Iss. 12, 229201:1–229201:2
https://doi.org/10.1007/s11432-023-4193-1

Mean-square prescribed finite-time output consensus of high-order linear multi-agent systems

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Received 9 September 2023/Revised 6 February 2024/Accepted 25 September 2024/Published online 11 November 2024

This study addresses a mean-square prescribed finite-time output consensus problem of high-order linear multi-agent systems with communication noises and thus further generalizes the results in [1–4]. The main contributions include three aspects: (i) It is challenging to analyze and display the finite-time stability due to the presence of communication noises in the sign function and the absence of communication noises in the quadratic Lyapunov function. (ii) A stochastic approximation-type protocol, based on the relative states of neighboring agents, is proposed novelly. (iii) We extend to consider the case where the noise intensities are unknown and bounded.

Problem formulation. Each agent has the following linear dynamics:

$$\dot{x}_i = Ax_i + Bu_i, \quad y_i = Cx_i, \tag{1}$$

where $x_i = \mathrm{col}(x_{i1}, \ldots, x_{in}) \in \mathbb{R}^n$, $y_i \in \mathbb{R}$ and $u_i \in \mathbb{R}$ are the state, the output, and the control input, respectively. $A = \left(\begin{smallmatrix} 0 & I_{n-1} \\ 0 & \mathbf{0} \end{smallmatrix}\right)$, $B = \mathrm{col}(0, \ldots, 0, 1)$, $C = (1\ 0\ \cdots\ 0)$.

The desired trajectory of the leader is given by

$$\dot{v} = Sv, \quad y_v = Rv, \tag{2}$$

where $v \in \mathbb{R}^{n_v}$ is the state and y_v denotes the output.

The purpose of this study is to achieve the mean-square prescribed finite-time output consensus, i.e., $\lim_{t\to T} E\|y_i(t) - y_v(t)\|^2 = 0$, where T denotes a-priori given and a user-defined finite time.

An undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ is utilized, where $\mathcal{V} = \{1, \ldots, N\}$ and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ are the sets of nodes and edges, respectively. $\mathcal{A} = [\mathcal{A}_{ij}]$ is an adjacency matrix. $L = C_r - \mathcal{A}$, $C_r = \mathrm{diag}(c_{r,11}, \ldots, c_{r,NN})$, $c_{r,ii} = \sum_{j=1}^N \mathcal{A}_{ij}$. $\bar{\mathcal{G}} = (\bar{\mathcal{V}}, \bar{\mathcal{E}})$ is used, where $\bar{\mathcal{V}} = \{0\} \bigcup \mathcal{V}$, $\bar{\mathcal{E}} \subseteq \bar{\mathcal{V}} \times \bar{\mathcal{V}}$, and node 0 is the leader. $\mathcal{B}_r = \mathrm{diag}(\mathcal{A}_{10}, \ldots, \mathcal{A}_{N0})$, $\mathcal{A}_{i0} > 0$ if node i can obtain information from node 0; otherwise, $\mathcal{A}_{i0} = 0$.

Assumption 1. The graph $\bar{\mathcal{G}}$ contains a spanning tree with the root being the leader.

Assumption 2. All the eigenvalues of S are on the imaginary axis.

Assumption 3. For any $\bar{\lambda} \in \sigma(S)$, where $\sigma(S)$ is the spectrum of S, $\mathrm{rank}\left(\begin{smallmatrix} A-\bar{\lambda}I & B \\ C & 0 \end{smallmatrix}\right) = n + 1$.

Remark 1. If Assumption 3 holds, then the following regulator equation [5] is solvable:

$$XS = AX + BU, \quad CX = R. \tag{3}$$

Let $\theta_0 = x_0 - Xv$. On the time interval $[0, T)$, a controller for the leader is designed as

$$u_0 = -K(t)\theta_0, \tag{4}$$

where $K(t)$ is designed as

$$K(t) = \left(\bar{b}_1 a^n(t)\ \cdots\ \bar{b}_n a(t)\right),$$

$\bar{b}_i \in \mathbb{R}$, $i = 1, \cdots, n$, are positive design parameters and $a(t) = \frac{1}{T-t}$.

On the time interval $[0, T)$, the following controller is designed for each follower:

$$\begin{aligned} u_i = &-\mu_1 c(t)\Big(\sum_{j\in N_i} \mathcal{A}_{ij}(K_1(t)x_i - (K_1(t)x_j + \\ &\rho_{ij} n_{ij})) + \mathcal{A}_{i0}(K_1(t)x_i - (K_1(t)x_0 + \rho_{i0} n_{i0}))\Big), \end{aligned} \tag{5}$$

where

$$K_1(t) = \left(\hat{b}_1 a^n(t) c^{-1}(t)\ \cdots\ \hat{b}_n a(t) c^{-1}(t)\right),$$

$\hat{b}_i > 0$, $\mu_1 > 0$, $c(t) = \frac{1}{\ln \frac{T}{T-t} + \mu}$, $\mu > 0$, and $n_{ij}(t) \in \mathbb{R}$ and $\rho_{ij} > 0$ are independent standard white noises and the noisy intensities, respectively.

Because $L + \mathcal{B}_r > 0$, there exists a nonsingular matrix $\hat{T}$ satisfying $\hat{T}(L + \mathcal{B}_r)\hat{T}^{\mathrm{T}} = \Omega$, where $\Omega = \mathrm{diag}(\lambda_1, \ldots, \lambda_N)$ and $\lambda_i > 0$ are the eigenvalues of $L + \mathcal{B}_r$. Because all the eigenvalues of $A - BK(t)$ and $A - \mu_1\lambda_i c(t)BK_1(t)$ can be negative and have algebraic multiplicity values, by properly selecting $K(t)$, $K_1(t)$ and μ_1, there exist nonsingular matrices $U_i(t) \in \mathbb{R}^{n\times n}$ such that

$$\begin{aligned} &U_0^{-1}(t)(A - BK(t))U_0(t) = \Lambda_0(t), \\ &U_i^{-1}(t)(A - \mu_1\lambda_i c(t)BK_1(t))U_i(t) = \Lambda_i(t), \end{aligned}$$

where $\Lambda_i(t) = a(t)\Lambda_{i1}$, $\Lambda_{i1} = \mathrm{diag}(-k_1^{[i]}, \ldots, -k_n^{[i]})$, and $k_j^{[i]} > 0$.

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Let $e_i = x_i - x_0$, $e = \mathrm{col}(e_1, \ldots, e_N)$, $\bar{\xi} = \mathrm{col}(\bar{\xi}_1, \ldots, \bar{\xi}_N) = (\hat{T} \otimes I_n)e$, $\hat{\xi}_i = U_i^{-1}(t)\bar{\xi}_i$, $\bar{\theta}_0 = \mathrm{col}(\theta_0, \ldots, \theta_0)$, $\hat{\theta}_0 = (I_N \otimes U_0^{-1}(t))\bar{\theta}_0$. One has

$$\begin{aligned} \dot{\hat{\xi}}_i =& (\dot{U}_i^{-1}(t)U_i(t) + \Lambda_i(t))\hat{\xi}_i + \mu_1 c(t)U_i^{-1}(t)\times \\ & \left(\hat{T}_i \otimes B\right)D\eta + U_i^{-1}(t)(\hat{T}_i \otimes (BK(t)U_0(t)))\hat{\theta}_0, \\ \dot{\hat{\theta}}_0 =& I_N \otimes (\dot{U}_0^{-1}(t)U_0(t) + \Lambda_0(t))\hat{\theta}_0 \\ & - I_N \otimes (U_0^{-1}(t)BU)\bar{v}, \end{aligned} \tag{6}$$

where $\bar{v} = \mathrm{col}(v, \ldots, v)$, $\hat{T}_i \in \mathbb{R}^{1\times N}$ is the ith row of $\hat{T}$,

$$\begin{aligned} &D = \mathrm{diag}(d_1, \ldots, d_N) \in \mathbb{R}^{N\times N(N+1)}, \\ &d_i = (\mathcal{A}_{i0}, \mathcal{A}_{i1}, \ldots, \mathcal{A}_{iN}) \in \mathbb{R}^{1\times(N+1)}, \\ &n_i = \mathrm{col}(n_{i0}, n_{i1}, \ldots, n_{iN}) \in \mathbb{R}^{N+1}, \\ &\eta = \mathrm{col}(n_1, \ldots, n_N) \in \mathbb{R}^{N(N+1)}. \end{aligned}$$

Lemma 1. $\dot{U}_i^{-1}(t)U_i(t) + \Lambda_i(t) = a(t)\bar{\Lambda}_i$, where $\bar{\Lambda}_i \in \mathbb{R}^{n\times n}$ is a constant matrix and can be Hurwitz stable for $i = 0, 1, \ldots, N$.

Proof. See the Appendix A.

The matrices $U_i^{-1}(t)(\hat{T}_i \otimes (BK(t)U_0(t)))$ and $U_0^{-1}(t)BU$ can be written as $a(t)\Delta_i$ and $a^{-(n-1)}(t)\Pi$, respectively. In addition, we obtain that $U_i^{-1}(t)(\hat{T}_i \otimes B)D = a(t)D_i(t)$, where

$$D_i = a^{-n}(t)\left(D_{i1} \ \cdots \ D_{i,N(N+1)}\right).$$

Next, we introduce a time transformation method. Let $\theta(s) = T(1 - e^{-s})$, $t = \theta(s)$, and $\nu(t)$ is a solution of system (2). Let $\zeta_i(t)$ and $\zeta_0(t)$ represent the solutions of system (6). Define $v_s(s) = \nu(t)$, $\bar{\psi}_i(s) = \zeta_i(t)$, $\psi_0(s) = \zeta_0(t)$, $\bar{v}_s(s) = \mathrm{col}(v_s(s), \ldots, v_s(s))$, and $\bar{\psi}_0(s) = \mathrm{col}(\psi_0(s), \ldots, \psi_0(s))$. Based on Lemma 1, one has

$$\begin{aligned} \bar{\psi}_i'(s) =& \bar{\Lambda}_i\bar{\psi}_i(s) + \Delta_i\bar{\psi}_0(s) \\ & + \mu_1 c(\theta(s))D_i(\theta(s))\eta(\theta(s)), \\ \bar{\psi}_0'(s) =& I_N \otimes \bar{\Lambda}_0\bar{\psi}_0(s) \\ & - \theta'(s)a^{-(n-1)}(\theta(s))(I_N \otimes \Pi)\bar{v}_s(s), \\ v_s'(s) =& Te^{-s}Sv_s(s), \end{aligned} \tag{7}$$

where

$$\begin{aligned} &c(\theta(s)) = \frac{1}{s+\mu}, \ v_s'(s) = \frac{dv_s(s)}{ds}, \\ &\bar{\psi}_i'(s) = \frac{d\bar{\psi}_i(s)}{ds}, \ \bar{\psi}_0'(s) = \frac{d\bar{\psi}_0(s)}{ds}, \\ &\bar{v}_s(s) = \mathrm{col}(v_s(s), \ldots, v_s(s)) \in \mathbb{R}^{Nn_v}. \end{aligned} \tag{8}$$

Theorem 1. Consider the multi-agent system (1)–(2). Suppose that Assumptions 1–3 hold. Under the controllers (4) and (5), all the followers achieve the mean-square prescribed finite-time output consensus.

Proof. See Appendix B.

Acknowledgements This work was supported in part by the National Natural Science Foundation of China (Grant Nos. 62103341, 62473322) and the Natural Science Foundation of Sichuan Province (Grant No. 2024NSFSC0512).

Supporting information Appendixes A–C. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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