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Special Topic: Large Multimodal Models

COMET: "cone of experience" enhanced large multimodal model for mathematical problem generation

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The impact of generative artificial intelligence on education is unprecedented [1]. Researchers have been exploring possibilities of combining the large multimodal model (LMM) with the teaching process. Specifically, Luo and Yang [2] have explored large model collaborative domain models to support smart education, fostering personalized and adaptive educational experiences. However, existing studies still lack in-depth research on generating educational resources, especially in mathematical problem generation.

Traditional studies of mathematical problem generation are divided into two independent subfields, namely stem generation [3] (some studies simply record as problem generation) and problem-solving. However, we believe that constructing high-quality mathematical problems requires the ability to generate both stems and solutions to form a task-closed loop. As shown in Figure 1, a high-quality mathematical problem needs to be carefully designed by domain experts and meet multiple requirements. (I) Completeness. During the teaching process, mathematical problems are aimed at teachers, students, and parents concurrently. Therefore, it should contain four logically clear parts: the mind of design, the stem, the mind of solution, and the answer. (II) Precision. The mathematical problem should accurately reflect the objectives of the curriculum, be highly related to given knowledge points, and provide the function of exercises and tests. (III) Differentiation. For certain key knowledge points under investigation, the problem should differentiate in theme, problem type, and difficulty level, to better serve complex and diverse learning needs.

LMMs offer a novel approach to mathematical problem generation. It can not only generate coherent and logical content on cross-modal data but also respond to diverse queries based on in-context learning and instruction following capabilities. However, there are still challenges in directly applying LMMs to generate math problems. On the one hand, general LMMs lack the expertise for mathematical problem generation and need to transfer training to inject domain knowledge. As training modes (such as pre-training and supervised fine-tuning) gradually solidify, the research focus of transfer training shifts towards the construction of high-quality domain datasets [4]. Previous construction methods are restricted to machine mind and target task form, resulting in issues of low data quality density and serious homogenization. We believe that the LMM training has the potential to analogy human learning. Drawing on the multi-level experiences of human learning, it can guide the design of training data at each stage with fine granularity, enabling the model to acquire richer knowledge. On the other hand, previous studies mainly focus on enhancing the individual ability of LMMs in stem generation or problem-solving. We believe that the professional knowledge and practical experience required for stem generation and problem-solving share commonalities. Integrating both abilities into a single model can mutually enhance them and is more practical for educational scenarios.

To address the above issues, we propose a "cone of experience" enhanced LMM for mathematical problem generation (COMET). Firstly, stem generation and problem-solving are unified into mathematical problem generation tasks. To the best of our knowledge, this is the first work to systematically enhance mathematical problem generation on a single LMM. Secondly, inspired by the "cone of experience" educational theory [5], we propose a three-stage fine-turning framework. The "cone of experience" divides human learning experience into three levels: symbolic, iconic, and direct experience. The different level experiences are interconnected and only by fully integrating all three levels of experiences can high-quality learning be achieved. Finally, a Chinese multimodal mathematical problem dataset (CMM12K) is formulated, filling the gap in the field of Chinese multimodal corpus and providing a high-quality benchmark for subsequent research.

Methodology. Figure 1 shows the three-stage fine-turning framework, more details can be found in Appendix A. The entire fine-tuning process is guided by the "cone of experience", injecting symbolic, iconic, and direct experience. The three-stage fine-tuning framework is expanded according to

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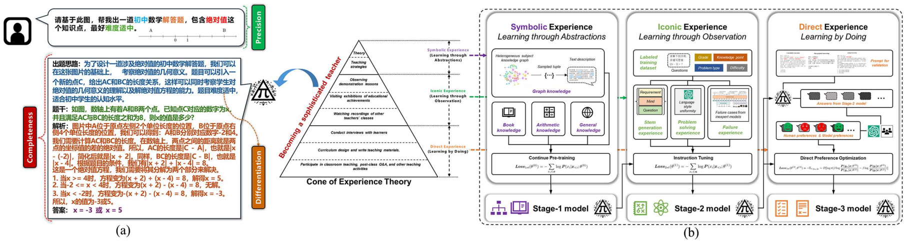

Figure 1 (Color online) Diagrams of (a) mathematical problem generation and (b) the "cone of experience" guided three-stage fine-tuning framework.

the type of injected experience, elaborating on the definitions, construction methods, and training methods.

In stage-1, we define symbolic experience as the background knowledge of the target domain, or the prerequisite knowledge for carrying out the target task. Symbolic experience does not directly help the model solve specific tasks, but it provides strong support by supplementing conceptual knowledge. For mathematical problem generation, we summarize symbolic experience into four types for production: book knowledge, graph knowledge, arithmetic knowledge, and general knowledge. All the data are associated with symbolic experience as pre-training form and are infused into the LMM for learning, i.e., no masking of data content is undertaken. The backpropagation of training computes loss from the first token of the input.

In stage-2, iconic experience is defined as the data generated by the subject in the process of performing the target task, which includes not only human experts proficient in the target task but also other LMMs. Injecting the iconic experience aims to allow LMM to learn mathematical problem generation from humans and improve upon the failed reasoning data produced by other LMMs. We summarize iconic experience into three types of production: the experience of stem generation, problem-solving, and failure. These data pertaining to the iconic experience are learned by the LMM in the form of instruction-tuning. All data are arranged in a query-response pair, and a masking process is applied to the query part. The backward propagation of training only starts calculating loss from the first token of the response.

In stage-3, direct experience is defined as the generated procedural data when the fine-tuned object carries out the target task with results feedback. Such experience aims to correct the inference preference of the LMM with higher-order domain values, allowing it to embodied evolve during the practice. The training format is {task instruction, high preference response, low preference response} and we apply direct preference optimization as the loss function.

Experiments and results. Appendix B presents the experimental setup. We verify three capabilities of LMMs on two public datasets (GSM8K and TAL-SCQ5K) and one self-built dataset (CMM12K): controllable generation (CG), analogy generation (AG), and fine-grained solving (FS). Both CG and AG reflect the ability of LMM to stem generation, FS reflects the problem-solving ability. We select six open-source LMMs and three closed-source LMMs as baselines, and perform three evaluation modes under human and GPT-4V supervision, namely scoring mode, arena mode, and objective indicators.

Appendix C presents the detailed results. Compared to baselines of the same parameter size, the proposed model consistently maintains significant advantages in CG, AG, and FS. On CMM12K, the FS accuracy of COMET leads the baseline by up to 20.67%, and the average winning rate of AG under the arena mode is about 94.33%. Compared to the open-source baselines with a parameter scale exceeding 7B, COMET still shows a significant advantage in FS accuracy on CMM12K, ranking second on GSM8K and TAL-SCQ5K. The average winning rate for CG under arena mode is about 70%. For powerful closed-source models including GPT-4o, COMET has an average winning rate of 51.0%, 55.6%, and 40.5% in the arena mode for CG, AG, and FS.

Conclusion. We propose COMET, a "cone of experience" enhanced LMM for mathematical problem generation. To explore the possibility of analogy LMM training to human learning, we define the teacher growth process into three level experiences based on the "cone of experience" educational theory and guide the construction of training data at different stages. A three-stage fine-tuning framework is designed to enhance the capabilities of stem generation and problem-solving within a single LMM to meet the requirements of educational applications. Moreover, a CMM12K is built to alleviate the scarcity of Chinese multimodal corpora in this field. Extensive experiments have demonstrated the advancement and effectiveness of the proposed model and framework.

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Supporting information Appendixes A–D. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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