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• **Supplementary File** •

Communication-Centric Integrated Sensing and Communications With Mixed Fields

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Appendix A System Model

Fig. A1 shows the system model, where the dual-functional base station simultaneously communicates with multiple single-antenna NF and FF communication users and detects one NF target.

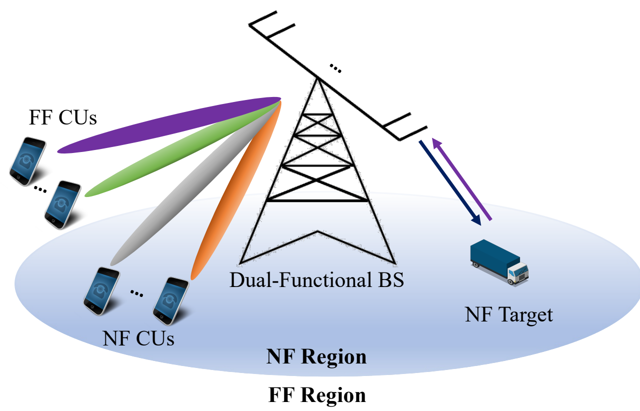

Figure A1 Mixed-ISAC system model.

Appendix B Optimal Receive Beamforming Vector

The optimal receive beamforming vector of the target can be obtained by solving the minimum variance distortionless response (MVDR) problem, which is given by [1]

$$\mathbf{f}_s = \frac{\sum_{j=1}^{M+N} \left(\mathbf{R}_c + \sigma_s^2 \mathbf{I}\right)^{-1} \mathbf{A}_s \mathbf{b}_j}{\sum_{j=1}^{M+N} \mathbf{b}_j^H \mathbf{A}_s^H \left(\mathbf{R}_c + \sigma_s^2 \mathbf{I}\right)^{-1} \mathbf{A}_s \mathbf{b}_j}, \tag{B1}$$

where $\mathbf{R}_c = \sum_{r=1}^{R} |\beta_r|^2 \mathbf{A}_r \left(\sum_{k=1}^{M+N} \mathbf{b}_k \mathbf{b}_k^H\right) \mathbf{A}_r^H$ is the sum of the covariance matrices of the clutters.

Appendix C Problem Formulation

The communications performance fairness problem can be formulated as

$$(\text{P1}) \max_{\{\mathbf{b}_j\}} \min_{j=1,\ldots,M+N} \frac{\Gamma_j}{c_j} \tag{C1a}$$

$$\text{s.t.} \sum_{j=1}^{M+N} \mathrm{tr}(\mathbf{b}_j \mathbf{b}_j^H) \leqslant P_0, \tag{C1b}$$

$$\frac{\sum_{j=1}^{M+N} |\beta_s|^2 \mathbf{f}_s^H \mathbf{A}_s \mathbf{b}_j \mathbf{b}_j^H \mathbf{A}_s^H \mathbf{f}_s}{\sum_{r=1}^{R} |\beta_r|^2 \mathbf{f}_s^H \mathbf{A}_r \left(\sum_{j=1}^{M+N} \mathbf{b}_j \mathbf{b}_j^H\right) \mathbf{A}_r^H \mathbf{f}_s + \sigma_s^2 \mathbf{f}_s^H \mathbf{f}_s} \geqslant s, \tag{C1c}$$

where c_j is the minimum SINR requirement of the j-th CU, P_0 is the total transmit power, s is the minimum required SCNR of the target. By selecting the appropriate c_j, $\Gamma_j \geqslant c_j$ for all j can be achieved in the final iteration of Algorithm D1 [2]. (P1) is

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non-convex due to the quadratic fractional objective (C1a), the quadratic constraint (C1b), and the quadratic fractional constraint (C1c). To convert the quadratic constraint into a convex one, semidefinite programming (SDP) [3] is applied by letting $\mathbf{B}_j = \mathbf{b}_j\mathbf{b}_j^H$, where $\mathbf{B}_j$ is the covariance matrix of the j-th CU. $\mathbf{B}_j$ has the properties of $\mathbf{B}_j \succeq 0$ and $\mathrm{rank}(\mathbf{B}_j) = 1$, where the rank-one property is dropped due to its non-convexity. Then, with a few simple mathematical operations, (C1c) can be converted into

$$\sum_{j=1}^{M+N}\sum_{r=1}^{R}|\beta_r|^2\,\mathbf{f}_s^H\mathbf{A}_r\mathbf{b}_j\mathbf{b}_j^H\mathbf{A}_r^H\mathbf{f}_s + \mathbf{f}_s^H\mathbf{f}_s\sigma_s^2 - \frac{\sum_{j=1}^{M+N}|\beta_s|^2\,\mathbf{f}_s^H\mathbf{A}_s\mathbf{b}_j\mathbf{b}_j^H\mathbf{A}_s^H\mathbf{f}_s}{s} \leqslant 0. \tag{C2}$$

Then, (P1) can be reformulated as

$$\begin{aligned}
\text{(P1.1)}\ \max_{\{\mathbf{B}_j\}}\ & \min_{j=1,\dots,M+N}\frac{\Gamma_j}{c_j} && \text{(C3a)}\\
\text{s.t.}\ & \sum_{j=1}^{M+N}\mathrm{tr}(\mathbf{B}_j)\leqslant P_0, && \text{(C3b)}\\
& \sum_{j=1}^{M+N}\sum_{r=1}^{R}|\beta_r|^2\,\mathbf{f}_s^H\mathbf{A}_r\mathbf{B}_j\mathbf{A}_r^H\mathbf{f}_s + \mathbf{f}_s^H\mathbf{f}_s\sigma_s^2 \\
& - \frac{\sum_{k=1}^{M+N}|\beta_i|^2\,\mathbf{f}_s^H\mathbf{A}_s\mathbf{B}_j\mathbf{A}_s^H\mathbf{f}_s}{s}\leqslant 0. && \text{(C3c)}\\
& \mathbf{B}_j\succeq 0, j=1,\dots,M+N. && \text{(C3d)}
\end{aligned}$$

(P1.1) is still non-convex due to the fractional objective (C3a). By introducing an auxiliary variable t, (P1.1) can be reformulated as

$$\begin{aligned}
\text{(P1.2)}\ \min_{\{\mathbf{B}_j\},t}\ & -t && \text{(C4a)}\\
\text{s.t.}\ & \sum_{j=1}^{M+N}\mathrm{tr}(\mathbf{B}_j)\leqslant P_0, && \text{(C4b)}\\
& \sum_{j=1}^{M+N}\sum_{r=1}^{R}|\beta_r|^2\,\mathbf{f}_s^H\mathbf{A}_r\mathbf{B}_j\mathbf{A}_r^H\mathbf{f}_s + \mathbf{f}_s^H\mathbf{f}_s\sigma_s^2 \\
& - \frac{\sum_{j=1}^{M+N}|\beta_s|^2\,\mathbf{f}_s^H\mathbf{A}_s\mathbf{B}_j\mathbf{A}_s^H\mathbf{f}_s}{s}\leqslant 0, && \text{(C4c)}\\
& f_j\left(\{\mathbf{B}_j\}\right) - \omega c_j g_j\left(\{\mathbf{B}_j\}\right)\geqslant t, \forall j, && \text{(C4d)}\\
& \mathbf{B}_j\succeq 0, j=1,\dots,M+N, && \text{(C4e)}
\end{aligned}$$

where ω is obtained from the previous iteration of the Dinkelbach-type algorithm [4], and it can be expressed as

$$\omega_{(ite2)} = \min_{j=1,\dots,M+N}\frac{f_j\left(\{\mathbf{B}_j\}_{(ite2-1)}\right)}{c_j g_j\left(\{\mathbf{B}_j\}_{(ite2-1)}\right)}. \tag{C5}$$

(P1.2) is convex and can be solved by cvx toolbox [5]. After acquiring the optimized $\{\mathbf{B}_j\}$ from problem (P1.2) in the final iteration, the solution to (P1) can be approximated as [6]

$$\mathbf{b}_j^\dagger = \left(\mathbf{h}_j^T\mathbf{B}_j\mathbf{h}_j^*\right)^{-\frac{1}{2}}\mathbf{B}_j\mathbf{h}_j^*, \forall j. \tag{C6}$$

The optimized SINR is non-decreasing with each iteration and has an upper bound, which means the algorithm converges.

Appendix D Dinkelbach-type SCA algorithm for Beamforming Design

We propose a successive convex approximation (SCA) algorithm based on Dinkelbach's method, as detailed in Algorithm D1.

Algorithm D1 Dinkelbach-type SCA algorithm for Beamforming Design

Initialize the convergence precision ϵ_1 and ϵ_2, and the number of iterations $ite1 = 1$.
Initialize the transmit BF vectors $\{\mathbf{b}_j\}_0$.
Initialize the optimal receive BF vector $\mathbf{f}_s$, the auxiliary variable t_0, and ω_0 using (B1) and $\{\mathbf{b}_j\}_0$.
while $\left|\omega_{(ite1)} - \omega_0\right| \leqslant \epsilon_1$ **do**
    Set $ite2 = 1$.
    while $\left|t_{(ite2)} - t_0\right| \leqslant \epsilon_2$ **do**
        Obtain $\{\mathbf{B}_j\}_{(ite2)}$ and $t_{(ite2)}$ by solving (P1.2).
        Update $\omega_{(ite2)}$ using (C5).
        Set $t_0 = t_{(ite2)}$, $ite2 = ite2 + 1$.
    end while
    Set $\omega_0 = \omega_{(ite2)}$.
    Obtain $\{\mathbf{b}_j\}_{(ite1)}$ using (C6).
    Calculate and Update $\mathbf{f}_s^{(ite1)}$ using (B1).
    Update $\omega_{(ite1)}$ with updated $\mathbf{f}_s^{(ite1)}$.
    $ite1 = ite1 + 1$.
end while
return $\{\mathbf{b}_j\}_{(ite1)}$, the solution to (P1).

Appendix E The coordinates of CUs, targets, and clutters

There are two NF CUs located at $(10m, -10^\circ)$ and $(10m, -20^\circ)$, respectively, and two FF CUs located at $(45m, -11^\circ)$ and $(45m, -21^\circ)$, respectively, and one NF target located at $(10m, 30^\circ)$. There are two clutter located at $(6.5m, 29.6^\circ)$ and $(6.5m, 30.4^\circ)$, respectively. Note that the closer the clutters are to the target, the greater their impact on the SCNR will be. Hence, we only consider the case where the clutters are close to the target and therefore, they are in the same field as the target.

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