SCIENCE CHINA
Information Sciences

• **LETTER** •

November 2024, Vol. 67, Iss. 11, 219204:1–219204:2
https://doi.org/10.1007/s11432-024-4141-4

Switched event-triggered control using a non-monotonic Lyapunov function

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Received 4 January 2024/Revised 22 May 2024/Accepted 30 August 2024/Published online 23 October 2024

Networked control systems (NCSs), whose feedback loops are connected via the communication networks, have a wide application in computer science, medicine, telecommunications, and so on. However, communication networks may suffer from congested network traffic due to the limited network bandwidth. One way to solve these issues is to use event-triggered sampling to alleviate network bandwidth utilization.

The event-triggered sampling can reduce sampling while maintaining the desired performance [1]. Nevertheless, there is no guarantee of how slow the triggering can be. Worse still, the event-triggered sampling may lead to an undesired Zeno behavior. In this case, the efficiency and applicability of the sampling will be degraded. To overcome the defects of the above two approaches, some attempts have been made to design event-triggered schemes (ETSs) satisfying dwell-time constraints. In [2], triggering conditions are examined only at the fixed instants. These periodic ETSs exclude Zeno behavior and are easy to realize, but the information of state is not fully used when continuous measurements are available. Indeed, the information can be used to reduce sampling and improve performance. In [3, 4], switched ETSs are devised to check the triggering condition after a dwell-time. The switched scheme is proven to tolerate a smaller number of transmissions than periodic ETS for the same triggering parameters.

Despite the aforementioned achievements in reducing the transmission number, more possibilities remain to explore. In this paper, the focus is to enlarge the triggering interval to the maximum while maintaining the desired exponential stability. To achieve this, a novel switched two-stage integral-type ETS is proposed so that the increase of the Lyapunov function (LF) during the triggering interval does not exceed the decrease of the LF in the sampling interval. Thus, the LF is allowed to increase during the triggering interval, and the ETS substantially enlarges the triggering interval. Meanwhile, an exponential deviation function is introduced in the switched ETS to obtain the assignable exponential stability.

Problem statement. Consider the NCS described as the following dynamic system:

$$\dot{\xi}(t) = \bar{A}\xi(t) + \bar{B}u(t), \tag{1}$$

where $\xi(t) \in \mathbb{R}^p$ is the state, $u(t) \in \mathbb{R}^m$ is the input, $\bar{A}$, $\bar{B}$ are known matrices with appropriate dimensions. $(\bar{A}, \bar{B})$ is assumed to be controllable, respectively.

In this paper, to reduce the network load, the ETS based controller is designed in the following form:

$$u(t) = \bar{K}\xi(t_k), \quad t \in [t_k, t_{k+1}), \tag{2}$$

where $\bar{K}$ is the control gain and has been designed such that all the eigenvalues of $\bar{A} + \bar{B}\bar{K}$ have a negative real part, and t_k denotes the triggering time. To simplify, we assume that the first triggering appears at the initial time t_0. To introduce the ETS, we define the measurement error $e(t) = \xi(t_k) - \xi(t)$.

Applying the controller (2) to (1), designing the ETS where the triggering occurs after some dwell-time h, and denoting $[t_k, t_k + h)$ as the sampling interval and $[t_k + h, t_{k+1})$ as the triggering interval, we can convert (1) into

$$\dot{\xi}(t) = \begin{cases} \bar{A}\xi(t) + \bar{B}\bar{K}\xi(t_k), t \in [t_k, t_k + h), \\ (\bar{A} + \bar{B}\bar{K})\xi(t) + \bar{B}\bar{K}e(t), t \in [t_k + h, t_{k+1}). \end{cases} \tag{3}$$

The purpose of this paper is to develop an ETS such that the transmission number is reduced, and the closed-loop augmented system (3) is exponential stable.

For the sake of clarity, we give the definition of exponentially stability.

Definition 1 ([5])**.** The equilibrium point $\xi = 0$ of the system is globally exponentially stable, if there exist constants c and λ such that for any initial condition $\xi(t_0) \in \mathbb{R}^p$ all corresponding solutions to (3) satisfy $\|\xi(t)\| \leqslant c\|\xi(t_0)\|\mathrm{e}^{-\lambda(t-t_0)}$ for all $t \geqslant t_0$.

ETS design. To guarantee a positive dwell-time while reducing sampling, this subsection will present a novel switched ETS which switches between the periodic sampling and the continuous event-trigger.

Using the sampling state $\xi(t_k)$ at triggering time t_k, the next triggering time t_{k+1} is determined by the following switched two-stage integral-type ETS:

$$t_{k+1} = \inf\left\{ t \geqslant t_k \,\middle|\, \text{At time } t, \textbf{Events} \text{ ① and ② hold} \right\}, \tag{4}$$

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where **Event** ①: $t - t_k \geqslant h$, **Event** ②: $f(t,e,\xi) \geqslant 0$, $f(t,e,\xi) = \int_{t_k+h}^{t} e^{\mathsf{T}}(s)Qe(s)\mathrm{d}s - \varphi\int_{t_k+h}^{t}\xi^{\mathsf{T}}(s)Q\xi(s)\mathrm{d}s - (\mathrm{e}^{-\beta(t-t_k)} - \mathrm{e}^{-2\alpha h})\xi^{\mathsf{T}}(t_k)P\xi(t_k)$, φ, α, β are some positive constants with $\beta < 2\alpha$, Q and P are weighting matrices to be designed.

Remark 1. The ETS (4) is divided into two stages and consists of dual events, rather than a single event. In **Event** ①, the sensor waits for h seconds, and during this period the information transmission does not occur. In **Event** ②, the measurement error is monitored. Once the triggering condition (4) is satisfied, the current state information is sampled and transmitted to the controller and the controller is updated.

Remark 2. The designed function $f(t,e,\xi)$ in **Event** ② of the ETS (4) has two advantages. Firstly, instead of relying on the transient value $e(t)$ and $\xi(t)$ for the ETS design as in [1], the accumulation of the measurement error $\int_{t_k+h}^{t} e^{\mathsf{T}}(s)Qe(s)\mathrm{d}s$ and the accumulation of system state $\int_{t_k+h}^{t}\xi^{\mathsf{T}}(s)Q\xi(s)\mathrm{d}s$ are utilized. This approach takes into consideration the frequent fluctuations of the system state and reduces the sensitivity of ETS, which can potentially lead to a safeguarding of data transmission and conservation of communication resource.

Secondly, distinguishing from the existing important integral-type ETSs, for example [4], the exponential deviation function $(\mathrm{e}^{-\beta(t-t_k)} - \mathrm{e}^{-2\alpha h})\xi^{\mathsf{T}}(t_k)P\xi(t_k)$ is introduced to achieve the assignable exponential rate of convergence.

Stability analysis. Consider the closed-loop augmented system (3) with the ETS (4). The exponential stability condition is described by Theorem 1.

Theorem 1. Suppose that there exist $p \times p$-matrices $P > 0$, $Q > 0$, $U > 0$, X_1, X, P_2, P_3, Y_1, Y_2, T such that

$$\Psi > 0, \Phi_1 < 0, \Phi_2 < 0, \Omega < 0, \tag{5}$$

where Ψ, Φ_1, Φ_2 and Ω are symmetric matrices given in Appendix A. Then the equilibrium point $\xi = 0$ of the closed-loop augmented system (3) is exponentially stable under the ETS (4).

Proof. Define a function as follows:

$$V(t) = \begin{cases} V_1(t), & t \in [t_k, t_k + h), \\ V_2(t), & t \in [t_k + h, t_{k+1}), \end{cases} \tag{6}$$

where $V_1(t) = \xi^{\mathsf{T}}(t)P\xi(t) + (h - \tau(t))\psi^{\mathsf{T}}(t)[\frac{X+X^{\mathsf{T}}}{2}\psi(t) + (X_1 + X_1^{\mathsf{T}})\xi(t_k)] + (h - \tau(t))\int_{t_k}^{t}\mathrm{e}^{2\alpha(s-t)}\dot{\xi}^{\mathsf{T}}(s)U\dot{\xi}(s)\mathrm{d}s$ with $\tau(t) = t - t_k$, $\psi(t) = \xi(t) - \xi(t_k)$, and $V_2(t) = \xi^{\mathsf{T}}(t)P\xi(t)$. In what follows, because of the switching characteristic of the ETS (4), we will analyze the behavior of $V(t)$ in detail in two parts.

Part I: The behavior of $V(t)$ during $[t_k, t_k + h)$.

The derivative of $V_1(t)$ along the trajectories of the system (3) satisfies $\dot{V}_1(t) \leqslant -2\alpha V_1(t)$ following from $\Phi_1 < 0$, $\Phi_2 < 0$ and the detailed calculations in Appendix B. Further, one has

$$V_1(t) \leqslant \mathrm{e}^{-2\alpha(t-t_k)}V_1(t_k), \quad t \in [t_k, t_k + h). \tag{7}$$

Part II: The behavior of $V(t)$ during $t \in [t_k + h, t_{k+1})$.

By calculating the derivative of $V_2(t)$ along the trajectories of the system (3) in Appendix C, taking into account $\Omega < 0$, integrating both sides of the derivative of $V_2(t)$ from $t_k + h$ to t and using the triggering condition (4), one yields

$$\begin{aligned} &V_2(t) - V_2(t_k + h) \\ &\leqslant \int_{t_k+h}^{t}[e^{\mathsf{T}}(s)Qe(s) - \varphi\xi^{\mathsf{T}}(s)Q\xi(s)]\mathrm{d}s \\ &\leqslant (\mathrm{e}^{-\beta(t-t_k)} - \mathrm{e}^{-2\alpha h})V_2(t_k^-). \end{aligned} \tag{8}$$

Since the state of the system does not jump at $t_k + h$ and t_k, and $V(t)$ is continuous, one infers $V_1((t_k+h)^-) = V_2(t_k+h)$, $V_2(t_k^-) = V_1(t_k)$. Then, (8) can turn into

$$\begin{aligned} V_2(t) &\leqslant (\mathrm{e}^{-\beta(t-t_k)} - \mathrm{e}^{-2\alpha h})V_1(t_k) + V_2(t_k + h) \\ &\leqslant (\mathrm{e}^{-\beta(t-t_k)} - \mathrm{e}^{-2\alpha h})V_1(t_k) + \mathrm{e}^{-2\alpha(t_k+h-t_k)}V_1(t_k) \\ &\leqslant \mathrm{e}^{-\beta(t-t_k)}V_1(t_k), \quad t \in [t_k + h, t_{k+1}). \end{aligned} \tag{9}$$

Combining $\beta < 2\alpha$, (7) and (9), one obtains that $V(t) \leqslant \mathrm{e}^{-\beta(t-t_k)}V(t_k)$, $\forall t \in [t_k, t_{k+1})$. By an iterative process, $V(t)$ can be rewritten as

$$V(t) \leqslant \mathrm{e}^{-\beta(t-t_0)}V(t_0), \quad \forall t \geqslant t_0. \tag{10}$$

Through detail calculations in Appendix D, it can be obtained from (10) that $\|\xi(t)\| \leqslant c\mathrm{e}^{-\frac{\beta}{2}(t-t_0)}\|\xi(t_0)\|$, $\forall t \geqslant t_0$, where $c = \sqrt{\lambda_{\max}(P)}/\sqrt{\varsigma}$. Consequently, we can conclude that the equilibrium point $\xi = 0$ of the system (3) is exponentially stable under the ETS (4).

Remark 3. In Theorem 1, the constructed LF $V(t)$ is analyzed in two stages. During **Event** ①, covering the interval $[t_k, t_k + h)$, it can be inferred from (7) that $V(t)$ is decreasing with an exponential decay rate 2α. On the other hand, during **Event** ②, covering the interval $[t_k + h, t_{k+1})$, there is no requirement for $V(t)$ to decrease, and it only needs to satisfy (8). It is also known from (8) that the value of $V(t)$ at t_{k+1} is allowed to exceed that at $t_k + h$. This is in contrast to [4] where $V(t_{k+1})$ is smaller than $V(t_k + h)$.

In the second stage, by designing the integral-type ETS, the variation of the increase of $V(t)$ is shown. The triggering only occurs when the increase of $V(t)$ in the second stage exceeds the decrease of $V(t)$ in the first stage. This ensures stability, even though the ETS (4) allows $V(t)$ to increase during the triggering interval. Unlike the usual ETS in [1], the ETS (4) relaxes the monotonic decreasing limitation on the LF and reduces the transmission number.

Simulation example. The simulation examples are included in Appendix E.

Acknowledgements This work was supported in part by the National Natural Science Foundation of China (Grant Nos. 62103201, 62103199, 62473206, 2023T160333) and in part by the Natural Science Foundation of Jiangsu Province (Grant No. BK20210590).

Supporting information Appendixes A–E. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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