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Mean-square consensus control of multi-agent systems driven by fractional Brownian motion

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In the 1940s, the stochastic process with long memory was first proposed by Kolmogorov. Then, Mandelbrot et al. [1] named it fractional Brownian motion (FBM) and studied its properties. The nonlocality, non-Markov property, and long-memory property of FBM allow for a better description of the complex dynamic behavior observed in the real world. In recent decades, the rapid development of FBM has resulted in clear applications in various research fields such as hydrology, climate, wavelet analysis, and finance. However, because FBM is neither a semimartingale nor an independent incremental process when the Hurst parameter $H \neq 0.5$, stochastic analysis of FBM is very complex and difficult. Duncan et al. [2] investigated the linear-quadratic problem of stochastic differential equations by using the Gauss-Volterra process.

The coordination control of multi-agent systems (MASs) has always been a popular research topic in biology, physics, and control. A system composed of many autonomous or semiautonomous subsystem agents in a distributed configuration interconnected through a network is called an MAS. The distributed coordination of networked fractional order systems over a directed interaction graph was studied in [3]. Furthermore, the stability of the system may be affected by various random factors, such as stochastic noise, changes in physical environmental factors, unmodeled dynamics within the system, and structural mutations. Therefore, significant efforts have been made in recent years to coordinate the control of stochastic MASs [4, 5]. However, all the aforementioned studies assumed stochastic terms in the state dynamics modeled by Brownian motion, which has a Markov property. It should be noted that FBM can provide more accurate models to help predict and optimize the behavior and performance of MASs, such that MASs driven by FBM have some critical significance in research and application. However, unlike the traditional Brownian motion, FBM is not a semi-martingale, and its autocorrelation and non-Markov properties increase the complexity of the dynamic analysis of a system with FBM. Therefore, there are currently only a few results on the coordination control of MASs using FBM.

Motivated by the above discussion, in this study, we developed a novel technique to analyze the dynamic behavior of MASs driven by FBM. First, a robust solution for a closed-loop stochastic system is obtained. Subsequently, the mean-square consensus control problem is solved for MASs driven by FBM using the fractional Itô formula. Furthermore, a simulation was conducted to verify the accuracy of the obtained theoretical results.

Problem formulation. Suppose that the dynamics of the MASs with N followers over a communication topology $\mathcal{G}$ are modeled by

$$d\xi_i(t) = A\xi_i(t)dt + Bu_i(t)dt + \Sigma(t)\xi_i(t)dB^H(t), \qquad (1)$$

where $i = 1, 2, \ldots, N$, $\xi_i(t) \in \mathbb{R}^n$ and $u_i(t) \in \mathbb{R}^m$ represent the state and control inputs of the ith follower, respectively. $A \in \mathbb{R}^{n\times n}$ is the system matrix, and $B \in \mathbb{R}^{n\times m}$ is the input matrix. $\Sigma(t) \in \mathcal{H}$ is a continuous real-valued function that represents noise intensity. $B^H(t) \in \mathbb{R}$ is a real-valued FBM with Hurst parameter $H \in (0.5, 1)$. The initial state is denoted as $\xi_i(0) = \xi_{i0}$.

Consider a leader with the following form:

$$d\xi_0(t) = A\xi_0(t)dt + \Sigma(t)\xi_0(t)dB^H(t), \qquad (2)$$

where $\xi_0(t) \in \mathbb{R}^n$ denotes the leader state. The initial state is denoted as $\xi_0(0) = \xi_0$.

The objective of this study is to design a controller such that the leader-following consensus of the MASs (1) driven by FBM can be achieved in the mean-square sense; that is, $\lim_{t\to+\infty} \mathbb{E}\|\xi_i(t) - \xi_0(t)\|^2 = 0$, $\forall i = 1, 2, \ldots, N$.

Remark 1. In many practical applications, we focus more on persistence and long-term relevance phenomena. The FBM with $H \in (0.5, 1)$ is considered because within this range, FBM exhibits long-term memory properties.

Protocol design. For each follower, we design the following distributed controller:

$$u_i(t) = G_1\Big[\sum_{j\in\mathcal{N}_i} a_{ij}(\xi_i(t) - \xi_j(t)) + p_i(\xi_i(t) - \xi_0(t))\Big], \qquad (3)$$

where $i = 1, 2, \ldots, N$, $G_1 \in \mathbb{R}^{m\times n}$ represents the control gain matrix, $\mathcal{N}_i$ represents the set of neighbors of agent i, a_{ij} represents the communication quantity, $p_i > 0$ if the ith follower could receive the leader's information, and $p_i = 0$, otherwise. Furthermore, let $\mathcal{L}$ represent the Laplacian of graph $\mathcal{G}$. Let $\mathbb{P} = diag\{p_1, p_2, \ldots, p_N\}$, $\overline{\Sigma} = \max_{t\geqslant 0}\{|\Sigma(t)|\}$, $\zeta = H\overline{\Sigma}^2 + (2H^2 - H)\overline{\Sigma}\max_{t\geqslant 0}\left\{\int_0^t \Sigma(t)dt\right\}$, $\Theta = A + \frac{\zeta}{2}I_n$, and $B_i = -\lambda_i(\mathcal{L} + \mathbb{P})B$, $\forall i = 1, 2, \ldots, N$. To continue, the following assumptions are necessary.

Assumption 1. The pair (Θ, B_i) is stabilizable.

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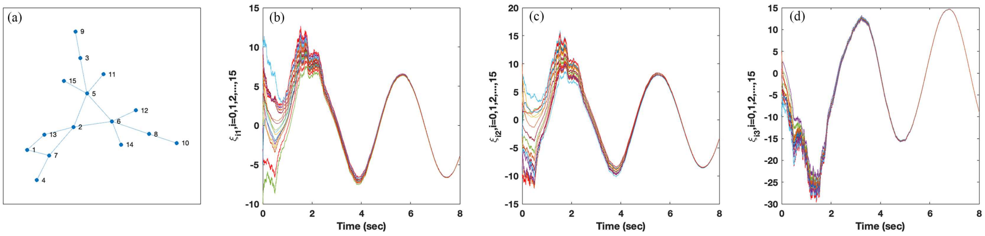

Figure 1 (Color online) Simulation results. (a) Communication topology; (b) the first component; (c) the second component; (d) the third component.

Assumption 2. Graph $\mathcal{G}$ is connected, and there exists at least one follower that can receive information from the leader, but not vice versa.

Main results. Before presenting the main results of this study, we first present the following two useful lemmas. Let $\varepsilon(t) = \int_0^t \Sigma(s)dB^H(s)$, $\Upsilon(t) = \Sigma(t)\int_0^t \Sigma(s)\phi_H(t-s)ds$, with $\phi_H(t-s) = (2H^2 - H)|t-s|^{2H-2}$. Let $p = n \times N$, $\Lambda = diag\{\lambda_1(\mathcal{L}+\mathbb{P}), \lambda_2(\mathcal{L}+\mathbb{P}), \ldots, \lambda_N(\mathcal{L}+\mathbb{P})\}$, and $M(t) = I_N \otimes A + \Lambda \otimes BG_1 - \frac{1}{2}\Upsilon(t)I_p$. Suppose that $\Phi(t,0) \in \mathbb{R}^{p\times p}$ is the solution matrix of the following matrix equation:

$$\begin{cases} \dot{\Phi}(t,0) = M(t)\Phi(t,0), \quad t > 0, \\ \Phi(0,0) = I_{p\times p}. \end{cases} \tag{4}$$

Lemma 1. The stochastic process $\delta(t) = e^{\varepsilon(t)}\Phi(t,0)\delta_0$ is the unique continuous strong solution to the following stochastic system:

$$\begin{cases} d\delta(t) = & (I_N \otimes A + \Lambda \otimes BG_1)\,\delta(t)dt \\ & +\Sigma(t)\delta(t)dB^H(t), \quad t > 0, \\ \delta(0) = & \delta_0. \end{cases} \tag{5}$$

Lemma 2. For $H \in (0.5, 1)$, $\Upsilon(t) \leqslant \zeta$.

The main results of this study are as follows.

Theorem 1. Under Assumptions 1 and 2, if there exists a symmetric positive definite matrix $P \in \mathbb{R}^{n\times n}$ satisfying

$$A^TP + PA + (\ell + \zeta)P - 2PBB^TP < 0, \tag{6}$$

for some $\ell > 0$, then with the distributed controller (3) and $G_1 = -\gamma_1 B^TP$, $\gamma_1 > 1/\lambda_1(\mathcal{L}+\mathbb{P})$, the leader-following consensus control of the MASs (1) can be achieved in the mean-square sense.

Generalization. Consider the following MASs with N agents over a communication topology $\mathcal{G}$:

$$d\xi_i(t) = A\xi_i(t)dt + Bu_i(t)dt + \Sigma(t)\xi_i(t)dB^H(t), \tag{7}$$

where $i = 1, 2, \ldots, N$. The objective of this generalization is to design a controller, such that $\lim_{t\to+\infty} \mathbb{E}\|\xi_i(t) - \xi_j(t)\|^2 = 0$, $\forall i, j = 1, 2, \ldots, N$. In this study, we designed the following distributed controller

$$u_i(t) = G_2 \sum_{j\in\mathcal{N}_i} a_{ij}\left(\xi_i(t) - \xi_j(t)\right), \tag{8}$$

where $i = 1, 2, \ldots, N$, $G_2 \in \mathbb{R}^{m\times n}$ is the control gain matrix. Define $\Theta = A + \frac{\zeta}{2}I_n$ and $\tilde{B}_i = -\lambda_i(\mathcal{L})B$, $\forall i = 2, \ldots, N$. The following assumptions are made.

Assumption 3. The pair $(\Theta, \tilde{B}_i)$ is stabilizable.

Assumption 4. The graph $\mathcal{G}$ is connected.

Corollary 1. Under Assumptions 3 and 4, if there exists a symmetric positive definite matrix $\bar{P} \in \mathbb{R}^{n\times n}$ satisfying

$$A^T\bar{P} + \bar{P}A + (\bar{\ell} + \zeta)\bar{P} - 2\bar{P}BB^T\bar{P} < 0, \tag{9}$$

for some $\bar{\ell} > 0$, then with the distributed controller (8) and $G_2 = -\gamma_2 B^T\bar{P}$, $\gamma_2 > 1/\lambda_2(\mathcal{L})$, the leaderless consensus control of the MASs (7) driven by FBM can be achieved in the mean-square sense.

Remark 2. We can find that the results of [4, 5] only addressed the mean-square consensus problem of MASs with standard Brownian motion, which has Markov property. By contrast, FBM is neither a semimartingale nor an independent incremental process when the Hurst parameter $H \neq 0.5$. Thus, the approaches proposed in [4, 5] were unsuitable for this study, and we developed a novel Lyapunov function technique for stability analysis using the fractional Itô formula.

Simulation. Consider the MASs (1) on $\mathbb{R}^3$ with $N = 15$ among the communication topology depicted in Figure 1(a). Suppose that agents 1, 5, 10 can obtain the state information of the leader, so that Assumption 2 is satisfied. Appendix G presents the parameter choices for the simulation. Then, under controller (3), the state trajectories of all agents are depicted in Figure 1(b)–(d), which show that the leader-following consensus control of MASs (1) is achieved in the mean-square sense.

Conclusion. This study solved the leader-following and leaderless consensus control problems of MASs driven by FBM in the mean-square sense. Distributed controllers were designed, and specific inequality conditions were derived to determine the control gain matrix in the controller. Furthermore, the effectiveness of the proposed approach was verified through a simulation. In future research, we will explore the effects of controller delays and packet loss on the consensus control of directed networks.

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Supporting information Appendixes A–G. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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