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Mean-square consensus control of multi-agent systems driven by fractional Brownian motion

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Appendix A Related works

Here, we provide comprehensive comparisons with relevant existing research to highlight the novelty of our study.

Unlike the previous findings discussed in Refs. [1–5] which primarily focused on the controllability and stability aspects of stochastic systems with FBM, our study takes a distinctive approach to investigate asymptotic stability in the mean-square sense. This novel perspective allows us to delve deeper into the long-term behavior and performance evaluation of MASs driven by FBM, shedding light on a previously unexplored aspect of the problem.

It can be observed that the results reported in Refs. [6–11] focused on addressing the mean-square consensus problem of MASs with standard Brownian motion. By contrast, FBM is neither a semimartingale nor an independent incremental process when the Hurst parameter $H \neq 0.5$. Consequently, the approaches presented in Refs. [6–11] are not directly applicable to the problem in this study. In this study, we address these challenges by developing a novel Lyapunov function technique for stability analysis using the fractional Itô formula.

Note that [12] successfully solved the finite-time stochastic bound consensus problem for MASs under the influence of FBM. In this study, mean-square consensus control for MASs driven by FBM is considered. Notably, an innovative approach is taken in comparison to previous works, such as [13], where the containment control problem for MASs driven by FBM was solved using the properties of analytic semigroups, which increases the complexity of the theoretical analysis. In this study, the mean-square consensus control problem is addressed for MASs driven by FBM by using the fractional Itô formula and constructing the stopping time.

Appendix B Preliminaries

Notations: Throughout this paper, let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geqslant 0}, \mathbb{P})$ be a complete filtered probability space, where Ω represents the sample space, $\mathcal{F}$ represents a σ-algebra, $\{\mathcal{F}_t\}_{t\geqslant 0}$ represents a filtration, and $\mathbb{P}$ represents the probability measure. Let $\mathbb{E}$ represent mathematical expectations. Denote $L^2([0,T])$ as the family of all functions $f : [0,T] \to \mathbb{R}$ such that $\int_0^T f^2(t)dt < \infty$. Denote $\mathcal{L}^p := \mathcal{L}^p(\Omega, \mathcal{F}, \mathbb{P})$ as the space of all random variables $x : \Omega \to \mathbb{R}$ such that $\|x\|_p = (\mathbb{E}\|x\|^p)^{1/p} < +\infty$. Let $\lambda_i(M)$ be the ith eigenvalue of matrix M. $\otimes$ represents the Kronecker product,

Definition 1 ([14]). Let the Hurst parameter $H \in (0,1)$ and the standard FBM $\{B^H(t), t \geqslant 0\}$ be a continuous and centered Gaussian process defined on $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geqslant 0}, \mathbb{P})$ with the following properties:
(i) $B^H(0) = 0$, $\mathbb{E}[B^H(t)] = 0$, $\forall t \geqslant 0$.
(ii) $\mathbb{E}[B^H(t)B^H(r)] = \frac{1}{2}(|t|^{2H} + |r|^{2H} - |t-r|^{2H})$ for $t, r \geqslant 0$.
(iii) $B^H(t)$ has continuous trajectory.

In this study, the FBM $\{B^H(t), t \geqslant 0\}$ with $H \in (0.5, 1)$ is considered. Next, a Skorokhod-type stochastic integral is introduced. Most stochastic integral results can be found in monographs [14].

Let $\mathcal{E}$ be the space of step functions on $[0,T]$. For $L^2([0,T])$, the following scalar product is considered:

$$\langle f, g\rangle_H := \int_0^T \int_0^T f(u)g(s)\phi(u,s)dsdu,$$

where $\phi(u,s) = (2H^2 - H) \mid s-u \mid^{2H-2}$, Denote $\mathcal{H}_1$ as the closed subspace of $\mathcal{L}^2$, and $\mathcal{H}$ as the closure of the linear span of the indicator functions $\{\mathbf{1}_{[0,t]}, t \in [0,T]\}$ with respect to $\langle \mathbf{1}_{[0,t]}, \mathbf{1}_{[0,s]}\rangle_H \mapsto R(t,s)$. The image in $\mathcal{H}_1$ for $\psi \in \mathcal{H}$ is denoted as $B^H(\psi)$. We denote $\mathcal{S}$ the set of all polynomial functions of $B^H(\psi_j)$, $\psi_1, \ldots, \psi_n \in \mathcal{H}$. For an element $G \in \mathcal{S}$, $G = h\left(B^H(\psi_1), \ldots, B^H(\psi_n)\right)$, where $n \geqslant 1$, $h \in C_b^\infty(\mathbb{R}^n)$. The Malliavin derivative for $G \in \mathcal{S}$ is defined by $D_s^H G := \sum_{i=1}^n \frac{\partial h}{\partial x_i}\left(B^H(\psi_1), \ldots, B^H(\psi_n)\right)\psi_i(s)$.

For any $G \in \mathcal{S}$ and $p \in (0,\infty)$, define norm $\|G\|_{H,1,p} := \|G\|_p + \left[\mathbb{E}\left(\int_0^T \mid D_t^H G \mid^2 dt\right)^{p/2}\right]^{1/p}$. Denote $\mathbb{D}_{H,1,p}$ as the Banach space obtained by completing $\mathcal{S}$ with $\|\cdot\|_{H,1,p}$. Denote δ^w as the adjoint of D^H, with $Dom(\delta^w) \subseteq \mathcal{L}^2$. The divergence operator with respect to $B^H(t)$ is used as a Skorokhod-type stochastic integral. Particularly,

$$\delta^w\left(\sum_{i=1}^n a_i \mathbf{1}_{[t_i,t_{i+1})}\right) = \sum_{i=1}^n a_i\left(B^H(t_{i+1}) - B^H(t_i)\right).$$

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For $\zeta \in \mathcal{E}$, the operator K_H^* can be represented as $(K_H^*\zeta)(s) = \int_s^T \zeta(t)K_H(dt,s)$, the completion of $\mathcal{E}$ with seminorm $\|\zeta\|_{K_r} = \int_0^T(\int_s^T \mid \zeta \mid\mid K_H \mid (dt,s))^2 ds$ is denoted by $\mathcal{H}_{K_r}$. $\mathbb{D}^{1,2}(\mathcal{H}_{K_r}) \subseteq Dom(\delta^w)$ and

$$\int_0^t u_s dB^H(s) := \delta^w(u) = \int_0^T (K_H^* u)_s dW_s,$$

for any $u \in \mathbb{D}^{1,2}(\mathcal{H}_{K_r})$. Moreover, $\mathbb{E}\left[\int_0^t u_s dB^H(s)\right] = 0$.

Lemma B1 ([15]). Suppose that $\mathbb{U} \in C^2(\mathbb{R})$ is twice continuously differentiable and $\max\left\{|\mathbb{U}(x)|, |\frac{\partial \mathbb{U}(x)}{\partial x}|, |\frac{\partial^2 \mathbb{U}(x)}{\partial x^2}|\right\} \leqslant \varsigma_1 \exp(\varsigma_2|x|^2)$, where $\varsigma_1, \varsigma_2 > 0$ such that $\varsigma_2 < 1/4(\sup_{0\leqslant t\leqslant T} \mathbb{E}(B_t^H)^2)$. Let $a(t)$ be an adaptive bounded random process in $\mathbb{D}^{2,4}$, and $X(t) = \int_0^t a(s)dB^H(s)$, then $\frac{\partial \mathbb{U}(X(t))}{\partial X} \in \mathbb{D}^{1,2}(\mathcal{H}_{K_r})$. For each $t \in [0,T]$, we have

$$\begin{aligned} \mathbb{U}(X(t)) =\ & \mathbb{U}(0) + \int_0^t \frac{\partial \mathbb{U}(X(s))}{\partial X} dB^H(s) + \int_0^t \frac{\partial^2 \mathbb{U}(X(s))}{\partial X^2} \left(\int_0^s \frac{\partial K(s,r)}{\partial s} \left(\int_0^s D_r(K_s^* a(\theta))dw(\theta)\right) dr\right) ds \\ & + \frac{1}{2}\int_0^t \frac{\partial^2 \mathbb{U}(X(s))}{\partial X^2} \frac{\partial\left(\int_0^s (K_s^* a(r))^2 dr\right)}{\partial s} ds. \end{aligned}$$

Graph theory([16]). In this study, suppose that the MASs with N followers among the undirected communication topology graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$, with $\mathcal{V} = \{1, 2, \dots, N\}$ is the node set, $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the edge set, $\mathcal{A} = [a_{ij}]$ is the weighted adjacency matrix, where a_{ij} represents the communication quantity such that $a_{ij} = a_{ji} > 0$ if $(i,j) \in \mathcal{E}$ and $a_{ij} = 0$, otherwise. We denote $\mathcal{N}_i = \{j \in \mathcal{V} : (i,j) \in \mathcal{E}\}$ as the set of neighbors of node i. We denote $\mathcal{D} = diag\{d_1, d_2, \dots, d_n\}$ as the degree matrix, with $d_i = \sum_{j=1}^N a_{ij}$. The Laplacian of graph $\mathcal{G}$ is $\mathcal{L} = \mathcal{D} - \mathcal{A}$.

Remark 1. Malliavin analysis, introduced by Malliavin in 1976, provides a set of rules for differentiating random variables. It considers the solutions of stochastic differential equations as "smooth" Wiener functionals, thereby tackling differentiation challenges that cannot be resolved using traditional methods alone.

Remark 2. The Skorokhod-type stochastic integral derived from Malliavin analysis is utilized in this study because of its robust mathematical framework for handling complex stochastic processes and its ability to integrate random variables with respect to a broad class of adapted processes.

Appendix C The proof of Lemma 1

Because $H \in (0.5, 1)$ and $\Sigma(t) \in \mathcal{H}$, $\Upsilon(t) : \mathbb{R}_+ \to \mathbb{R}$ is continuous and well defined, that is, $\Upsilon(t) < \infty$. Note that $\overline{\Sigma} = \max_{t\geqslant 0}\{|\Sigma(t)|\}$ and $\phi_H(t-s) = (2H^2 - H)|t-s|^{2H-2}$. When $t \geqslant 1$ and $H \in (0.5, 1)$, we have

$$\begin{aligned} \Upsilon(t) &= \Sigma(t) \int_0^t \Sigma(s)\phi_H(t-s)ds \\ &= (2H^2 - H)\Sigma(t) \int_0^t \Sigma(s)(t-s)^{2H-2}ds \\ &= (2H^2 - H)\Sigma(t) \int_0^t \Sigma(t-r)r^{2H-2}dr \\ &= (2H^2 - H)\Sigma(t)\left[\int_0^1 \Sigma(t-r)r^{2H-2}dr + \int_1^t \Sigma(t-r)r^{2H-2}dr\right]. \end{aligned}$$

When $H \in (0.5, 1)$ and $r \geqslant 1$, $r^{2H-2} \leqslant 1$. Besides, $\int_0^1 r^{2H-2}dr = \frac{1}{2H-1}$. Therefore,

$$\begin{aligned} \Upsilon(t) &\leqslant (2H^2 - H)\overline{\Sigma}^2 \int_0^1 r^{2H-2}dr + (2H^2 - H)\overline{\Sigma} \int_1^t \Sigma(t-r)dr \\ &\leqslant H\overline{\Sigma}^2 + (2H^2 - H)\overline{\Sigma} \max_{t\geqslant 0}\left\{\int_0^t \Sigma(t)dt\right\} = \zeta. \end{aligned}$$

In addition, $\Upsilon(t) \leqslant \zeta$ for $t \in [0, 1)$ is clear. The proof is completed.

Appendix D The proof of Lemma 2

This proof includes two parts, showing the construction of system (5) and the system (5) has a unique solution of form $\delta(t) = e^{\varepsilon(t)}\Phi(t,0)\delta_0$, respectively.

Part 1: First, we describe the construction of system (5). We denote the measurement error as

$$e_i(t) = \xi_i(t) - \xi_0(t), \quad i = 1, 2, \dots, N,$$

then

$$\begin{aligned} de_i(t) &= Ae_i(t)dt + \Sigma(t)e_i(t)dB^H(t) + BG_1\left[\sum_{j\in\mathcal{N}_i} a_{ij}(\xi_i(t) - \xi_j(t)) + p_i e_i(t)\right]dt \\ &= Ae_i(t)dt + \Sigma(t)e_i(t)dB^H(t) + BG_1\left[\sum_{j\in\mathcal{N}_i} a_{ij}(e_i(t) - e_j(t)) + p_i e_i(t)\right]dt. \end{aligned} \tag{D1}$$

Let $e(t) = \left[e_1(t)^T, e_2(t)^T, \dots, e_N(t)^T\right]^T$ and $\mathbb{P} = diag\{p_1, p_2, \dots, p_N\}$ then, the compact form of (D1) can be written as

$$de(t) = [I_N \otimes A + (\mathcal{L} + \mathbb{P}) \otimes BG_1]\, e(t)dt + \Sigma(t)e(t)dB^H(t).$$

When Assumption 2 is satisfied, from Ref. [17], then there exist a nonsingular transformation matrix $\mathbb{Q} \in \mathbb{R}^{N\times N}$ such that

$$\mathbb{Q}^{-1}(\mathcal{L}+\mathbb{P})\mathbb{Q} = diag\{\lambda_1(\mathcal{L}+\mathbb{P}), \lambda_2(\mathcal{L}+\mathbb{P}), \ldots, \lambda_N(\mathcal{L}+\mathbb{P})\} \triangleq \Lambda,$$

where $0 < \lambda_1(\mathcal{L}+\mathbb{P}) \leqslant \lambda_2(\mathcal{L}+\mathbb{P}) \leqslant \cdots \lambda_N(\mathcal{L}+\mathbb{P})$. Denote $\delta(t) = (\mathbb{Q}^{-1}\otimes I_n)e(t)$, then

$$\begin{aligned} d\delta(t) &= (\mathbb{Q}^{-1}\otimes I_n)de(t) \\ &= (\mathbb{Q}^{-1}\otimes I_n)(I_N\otimes A)(\mathbb{Q}\otimes I_n)\delta(t)dt + (\mathbb{Q}^{-1}\otimes I_n)((\mathcal{L}+\mathcal{Q})\otimes BG_1)(\mathbb{Q}\otimes I_n)\delta(t)dt \\ &\quad + (\mathbb{Q}^{-1}\otimes I_n)\Sigma(t)(\mathbb{Q}\otimes I_n)\delta(t)dB^H(t) \\ &= (I_N\otimes A + \Lambda\otimes BG_1)\,\delta(t)dt + \Sigma(t)\delta(t)dB^H(t). \end{aligned}$$

Part 2: In the following, we prove that system (5) has a unique solution of the form $\delta(t) = e^{\varepsilon(t)}\Phi(t,0)\delta_0$. Set $g(t,\varepsilon(t)) = e^{\varepsilon(t)}\Phi(t,0)\delta_0$. From Lemma B1, we obtain

$$\begin{aligned} \delta(t) &= g(t,\varepsilon(t)) \\ &= \delta_0 + \int_0^t \frac{\partial g(u,\varepsilon(u))}{\partial u}du + \int_0^t \Sigma(u)\frac{\partial g(u,\varepsilon(u))}{\partial \varepsilon}dB^H(u) \\ &\quad + \int_0^t \Sigma(u)\frac{\partial^2 g(u,\varepsilon(u))}{\partial \varepsilon^2}\Big[\int_0^u \frac{\partial K(u,s)}{\partial u}\Big(\int_0^u D_s(K_u^*\Sigma(\theta))d\omega(\theta)\Big)ds\Big]du \\ &\quad + \frac{1}{2}\int_0^t \frac{\partial^2 g(u,\varepsilon(u))}{\partial\varepsilon^2}\frac{\partial(\int_0^u (K_u^*\Sigma(s))^2ds)}{\partial u}du. \end{aligned}$$

According to Ref. [2] and the matrix equation (4), then

$$\begin{aligned} &\int_0^u \frac{\partial K(u,s)}{\partial u}\Big(\int_0^u D_s(K_u^*\Sigma(\theta))d\omega(\theta)\Big)ds = 0, \\ &\frac{\partial(\int_0^u (K_u^*\Sigma(s))^2ds)}{\partial u} = \Sigma(u)\int_0^u \Sigma(s)\phi_H(u-s)ds = \Upsilon(u), \\ &\frac{\partial g(u,\varepsilon(u))}{\partial \kappa} = \frac{\partial^2 g(u,\kappa(u))}{\partial \varepsilon^2} = e^{\varepsilon(u)}\Phi(u,0)\delta_0, \\ &\frac{\partial g(u,\varepsilon(u))}{\partial u} = e^{\varepsilon(u)}M(u)\Phi(u,0)\delta_0. \end{aligned}$$

From the analysis above and $M(u) = I_N\otimes A + \Lambda\otimes BG_1 - \frac{1}{2}\Upsilon(u)I_p$, we have

$$\begin{aligned} \delta(t) &= g(t,\varepsilon(t)) \\ &= \delta_0 + \int_0^t e^{\varepsilon(u)}\left[M(u)\Phi(u,0)\delta_0 + \tfrac{1}{2}\Upsilon(u)\Phi(u,0)\delta_0\right]du + \int_0^t \Sigma(u)e^{\varepsilon(u)}\Phi(u,0)\delta_0 dB^H(u) \\ &= \delta_0 + \int_0^t e^{\varepsilon(u)}\left[\left(I_N\otimes A + \Lambda\otimes BG_1 - \tfrac{1}{2}\Upsilon(u)I_p\right)\Phi(u,0)\delta_0 + \tfrac{1}{2}\Upsilon(u)\Phi(u,0)\delta_0\right]du + \int_0^t \Sigma(u)\delta(u)dB^H(u) \\ &= \delta_0 + \int_0^t (I_N\otimes A + \Lambda\otimes BG_1)\delta(u)du + \int_0^t \Sigma(u)\delta(u)dB^H(u). \end{aligned}$$

From Definition 1, the stochastic process $\delta(t) = e^{\varepsilon(t)}\Phi(t,0)\delta_0$ is a unique continuous strong solution to stochastic system (5). The proof is completed.

Appendix E The proof of Theorem 1

Consider the auxiliary function of the form

$$\mathbb{U}(\delta(t)) = \delta(t)^T(I_N\otimes P)\delta(t) = \left(e^{\varepsilon(t)}\Phi(t,0)\delta_0\right)^T(I_N\otimes P)\left(e^{\varepsilon(t)}\Phi(t,0)\delta_0\right).$$

Clearly, $\mathbb{U}(\delta(t))$ is a symmetric positive definite and $\mathbb{U}(0) = 0$. For each $\aleph \geqslant \|\delta_0\|$, we define the stopping time as follows: $\tau_\aleph = \inf\{t \geqslant 0 : \|\delta(t)\| \geqslant \aleph\}$. Note that $\tau_\aleph \to \infty$ is $\aleph \to \infty$ a.s. By Lemma B1 and the matrix equation (4), we obtain

$$\begin{aligned} &e^{\ell(t\wedge\tau_\aleph)}\mathbb{U}(t\wedge\tau_\aleph, \varepsilon(t\wedge\tau_\aleph)) - \mathbb{U}(0,\varepsilon(0)) \\ &= \int_0^{t\wedge\tau_\aleph} \ell e^{\ell s}\mathbb{U}(s,\varepsilon(s))ds + \int_0^{t\wedge\tau_\aleph} e^{\ell s}\frac{\partial\mathbb{U}(s,\varepsilon(s))}{\partial s}ds + \frac{1}{2}\int_0^{t\wedge\tau_\aleph} e^{\ell s}\frac{\partial^2\mathbb{U}(s,\varepsilon(s))}{\partial\varepsilon^2}\frac{\partial(\int_0^s (K_s^*\Sigma(u))^2du)}{\partial s}ds \\ &\quad + \int_0^{t\wedge\tau_\aleph} e^{\ell s}\Sigma(s)\frac{\partial^2\mathbb{U}(s,\kappa(s))}{\partial\kappa^2}\Big(\int_0^s \frac{\partial K(s,u)}{\partial s}\Big(\int_0^s D_u(K_s^*\Sigma(\theta))d\omega(\theta)\Big)du\Big)ds + \int_0^{t\wedge\tau_\aleph} e^{\ell s}\frac{\partial\mathbb{U}(s,\varepsilon(s))}{\partial\varepsilon}\Sigma(s)dB^H(s), \end{aligned} \tag{E1}$$

where

$$\int_0^s \frac{\partial K(s,u)}{\partial s}\left(\int_0^s D_u(K_s^*\Sigma(\theta))d\omega(\theta)\right)du = 0,$$

$$\frac{\partial(\int_0^s (K_s^*\Sigma(u))^2 du)}{\partial s} = \Sigma(s)\int_0^s \Sigma(u)\phi_H(s-u)du = \Upsilon(s),$$

$$\frac{\partial \mathbb{U}(s,\varepsilon(s))}{\partial \varepsilon} = 2[e^{\varepsilon(s)}M(s,0)\delta_0]^T(I_N\otimes P)[e^{\varepsilon(s)}M(s,0)\delta_0] = 2\delta(s)^T(I_N\otimes P)\delta(s),$$

$$\frac{\partial^2 \mathbb{U}(s,\varepsilon(s))}{\partial \varepsilon^2} = 4[e^{\varepsilon(s)}M(s,0)\delta_0]^T(I_N\otimes P)[e^{\varepsilon(s)}M(s,0)\delta_0] = 4\delta(s)^T(I_N\otimes P)\delta(s),$$

$$\begin{aligned}
\frac{\partial \mathbb{U}(s,\varepsilon(s))}{\partial s} &= e^{\varepsilon(s)}\delta_0^T\frac{\partial M(s,0)^T}{\partial s}(I_N\otimes P)[e^{\varepsilon(s)}M(s,0)\delta_0] + [e^{\varepsilon(s)}M(s,0)\delta_0]^T(I_N\otimes P)e^{\varepsilon(s)}\frac{\partial M(s,0)}{\partial s}\delta_0\\
&= e^{\varepsilon(s)}\delta_0^T M(s,0)^T\left(I_N\otimes A + \Lambda\otimes BG_1 - \tfrac{1}{2}\Upsilon(s)I_p\right)^T(I_N\otimes P)\left[e^{\varepsilon(s)}M(s,0)\delta_0\right]\\
&\quad + [e^{\varepsilon(s)}M(s,0)\delta_0]^T(I_N\otimes P)e^{\varepsilon(s)}\left(I_N\otimes A + \Lambda\otimes BG_1 - \tfrac{1}{2}\Upsilon(s)I_p\right)M(s,0)\delta_0\\
&= \delta(s)^T\left(I_N\otimes A^TP + \Lambda\otimes G_1^TB^TP - \tfrac{1}{2}\Upsilon(s)I_N\otimes P\right)\delta(s)\\
&\quad + \delta(s)^T\left(I_N\otimes PA + \Lambda\otimes PBG_1 - \tfrac{1}{2}\Upsilon(s)I_N\otimes P\right)\delta(s).
\end{aligned}$$

Noted that $\mathbb{E}\left[\int_0^{t\wedge\tau_\aleph} e^{\ell s}\frac{\partial \mathbb{U}}{\partial \varepsilon}(s,\varepsilon(s))\Sigma(s)dB^H(s)\right] = 0$, then taking mathematical expectation on Eq. (E1) together with Lemmas 1-2, one obtain

$$\begin{aligned}
&\mathbb{E}\left[e^{\ell(t\wedge\tau_\aleph)}\mathbb{U}(t\wedge\tau_\aleph,\varepsilon(t\wedge\tau_\aleph))\right]\\
&= \mathbb{U}(0,\varepsilon(0)) + \mathbb{E}\Big[\int_0^{t\wedge\tau_\aleph}\Big(\ell e^{\ell s}\delta(s)^T(I_N\otimes P)\delta(s) + e^{\ell s}\delta(s)^T(I_N\otimes A^TP + I_N\otimes PA\\
&\quad + \Lambda\otimes G_1^TB^TP + \Lambda\otimes PBG_1)\delta(s) + e^{\ell s}\Upsilon(s)\delta(s)^T(I_N\otimes P)\delta(s)\Big)ds\Big]\\
&\leqslant \mathbb{U}(0,\varepsilon(0)) + \mathbb{E}\Big[\int_0^{t\wedge\tau_\aleph}\Big(e^{\ell s}\delta(s)^T((\ell+\zeta)I_N\otimes P + I_N\otimes A^TP + I_N\otimes PA\\
&\quad + \Lambda\otimes G_1^TB^TP + \Lambda\otimes PBG_1)\delta(s)\Big)ds\Big]\\
&\triangleq \mathbb{U}(0,\varepsilon(0)) + \mathbb{E}\Big[\int_0^{t\wedge\tau_\aleph}\Big(e^{\ell s}\delta(s)^T\mathbb{J}\delta(s)\Big)ds\Big],
\end{aligned}$$

where $\mathbb{J} = I_N\otimes A^TP + I_N\otimes PA + \Lambda\otimes G_1^TB^TP + \Lambda\otimes PBG_1 + (\ell+\zeta)I_N\otimes P$. Denote

$$\mathbb{J}_i = A^TP + PA + \lambda_i(\mathcal{L}+\mathbb{P})G_1^TB^TP + \lambda_i(\mathcal{L}+\mathbb{P})PBG_1 + (\ell+\zeta)P, \quad i=1,2,\ldots,N.$$

Since $0<\lambda_1(\mathcal{L}+\mathbb{P})\leqslant\lambda_2(\mathcal{L}+\mathbb{P})\leqslant\cdots\lambda_N(\mathcal{L}+\mathbb{P})$ under Assumption 2, then $\gamma_1\lambda_i(\mathcal{L}+\mathbb{P}) > \frac{\lambda_i(\mathcal{L}+\mathbb{P})}{\lambda_1(\mathcal{L}+\mathbb{P})}\geqslant 1$ for all $i=1,2,\ldots,N$. When condition (6) is satisfied, take $G_1 = -\gamma_1B^TP$ with $\gamma_1 > \frac{1}{\lambda_1(\mathcal{L}+\mathbb{P})}$, then

$$\mathbb{J}_i < A^TP + PA - 2PBB^TP + (\ell+\zeta)P < 0,\ \forall i=1,2,\ldots,N,$$

such that $\mathbb{J}<0$. Therefore,

$$e^{\ell(t\wedge\tau_\aleph)}\mathbb{E}\left[\mathbb{U}(t\wedge\tau_\aleph,\varepsilon(t\wedge\tau_\aleph))\right]\leqslant\mathbb{U}(0,\varepsilon(0)).$$

We denote $\lambda_{\min}(P)=\min\{\lambda_i(P), i=1,\ldots,n\}$ and $\lambda_{\max}(P)=\max\{\lambda_i(P), i=1,\ldots,n\}$. Noted that

$$\lambda_{\min}(P)\|\delta(t)\|^2\leqslant\mathbb{U}(t,\varepsilon(t))\leqslant\lambda_{\max}(P)\|\delta(t)\|^2,$$

such that

$$e^{\ell(t\wedge\tau_\aleph)}\lambda_{\min}(P)\mathbb{E}\|\delta(t\wedge\tau_\aleph)\|^2\leqslant\lambda_{\max}(P)\|\delta_0\|^2.$$

By allowing $\aleph\to+\infty$, we obtain

$$\mathbb{E}\|\delta(t)\|^2\leqslant\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}\|\delta_0\|^2e^{-\ell t}.$$

Therefore, $\lim\limits_{t\to+\infty}\mathbb{E}\|\delta(t)\|^2=0$, such that $\lim\limits_{t\to+\infty}\mathbb{E}\|e(t)\|^2=\lim\limits_{t\to+\infty}\mathbb{E}\|(\mathbb{Q}\otimes I_n)\delta(t)\|^2=0$. Therefore,

$$\lim_{t\to+\infty}\mathbb{E}\|\xi_i(t)-\xi_0(t)\|^2=0,\quad\forall i=1,2,\ldots,N.$$

The proof is completed.

Remark 3. Suppose that $Q\in\mathbf{R}^{n\times n}$ is a positive definite symmetric matrix. From Ref. [18], when the pair (A,B) is stabilizable and the pair (A,Q) is detectable, then there must exists a unique positive definite symmetric matrix $P\in\mathbf{R}^{n\times n}$ to the following equation:

$$A^TP+PA-2PBB^TP+Q=0.$$

such that when pair (A,B) is stabilizable, there must exists at least one positive definite symmetric matrix $P\in\mathbf{R}^{n\times n}$ satisfying

$$A^TP+PA-2PBB^TP<0.$$

In this study, the influence of FBM is reflected in $\zeta = H\overline{\Sigma}^2 + (2H^2 - H)\overline{\Sigma}\max_{t\geqslant 0}\left\{\int_0^t \Sigma(t)dt\right\}$, which depends on the Hurst parameter H and noise intensity $\Sigma(t)$. To design a controller to counteract the impact of noise on the system stability, it is necessary to guarantee the existence of matrix P in the proof of Theorem 1.

$$A^TP + PA + \lambda_i(\mathcal{L}+\mathbb{P})G_1^TB^TP + \lambda_i(\mathcal{L}+\mathbb{P})PBG_1 + (\ell+\zeta)P$$

$$= \mathbb{J}_i < A^TP + PA - 2PBB^TP + (\ell+\zeta)P < 0, \ \forall i = 1, 2, \ldots, N,$$

which is guaranteed by the assumption that pair (Θ, B_i) is stabilizable. Therefore, under Assumption 1, there exists a symmetric positive definite matrix $P \in \mathbb{R}^{n\times n}$ that satisfies Eq. (6).

Appendix F The proof of Corollary 2

Let $\xi(t) = [\xi_1(t)^T, \ldots, \xi_N(t)^T]^T$ then,

$$d\xi(t) = (I_N \otimes A)\xi(t)dt + (\mathcal{L}\otimes BG_2)\xi(t)dt + \Sigma(t)\xi(t)dB^H(t). \tag{F1}$$

When Assumption 4 is satisfied, from Ref. [19], there exists an orthogonal matrix $\tilde{\mathbb{Q}} = \left[\begin{array}{cc}\frac{1}{N}\mathbf{1}_N & \bar{\mathbb{Q}}\end{array}\right] \in \mathbb{R}^{N\times N}$, with $\bar{\mathbb{Q}} \in \mathbb{R}^{N\times(N-1)}$ and $\tilde{\mathbb{Q}}^T = \tilde{\mathbb{Q}}^{-1}$, such that $\tilde{\mathbb{Q}}^T\mathcal{L}\tilde{\mathbb{Q}} = diag\{\lambda_1(\mathcal{L}), \lambda_2(\mathcal{L}), \ldots, \lambda_N(\mathcal{L})\} \triangleq \Lambda$, with $0 = \lambda_1(\mathcal{L}) < \lambda_2(\mathcal{L}) \leqslant \cdots \leqslant \lambda_N(\mathcal{L})$. Denote

$$\bar{\xi}(t) = \left(\tilde{\mathbb{Q}}^T \otimes I_n\right)\xi(t), \tag{F2}$$

then

$$d\bar{\xi}(t) = (I_N\otimes A + \Lambda\otimes BG_2)\,\bar{\xi}(t)dt + \Sigma(t)\bar{\xi}(t)dB^H(t). \tag{F3}$$

Let $\eta(t) = [\bar{\xi}_2(t)^T, \ldots, \bar{\xi}_N(t)^T]^T$ and $\bar{\Lambda} = diag\{\lambda_2(\mathcal{L}), \ldots, \lambda_N(\mathcal{L})\}$, then

$$d\bar{\xi}_1(t) = A\bar{\xi}_1(t)dt + \Sigma(t)\bar{\xi}_1(t)dB^H(t), \tag{F4}$$

$$d\eta(t) = \left(I_{N-1}\otimes A + \bar{\Lambda}\otimes BG_2\right)\eta(t)dt + \Sigma(t)\eta(t)dB^H(t), \tag{F5}$$

with $\eta(0) = \eta_0$. Let $\bar{M}(t) = I_{N-1}\otimes A + \bar{\Lambda}\otimes BG_2 - \frac{1}{2}\Upsilon(t)I_{\bar{p}}$ where $\bar{p} = n\times(N-1)$. Suppose that $\bar{\Phi}(t,0) \in \mathbb{R}^{\bar{p}\times\bar{p}}$ is the solution matrix for the following matrix equation:

$$\begin{cases} \dot{\bar{\Phi}}(t,0) = \bar{M}(t)\Phi(t,0), & t > 0, \\ \bar{\Phi}(0,0) = I_{\bar{p}\times\bar{p}}. \end{cases} \tag{F6}$$

Set

$$\eta(t) = e^{\varepsilon(t)}\bar{\Phi}(t,0)\eta_0, \quad \text{with} \quad \varepsilon(t) = \int_0^t \Sigma(s)dB^H(s), \quad t \geqslant 0. \tag{F7}$$

Similar to the proof of Lemma 2, the stochastic process $\{\eta(t), t\geqslant 0\}$ in form (F7) is a unique continuous strong solution to the stochastic system (F5). Consider the auxiliary function of the form

$$\mathbb{U}(t,\varepsilon(t)) = \left(e^{\varepsilon(t)}\bar{\Phi}(t,0)\eta_0\right)^T(I_{N-1}\otimes\bar{P})\left(e^{\varepsilon(t)}\bar{\Phi}(t,0)\eta_0\right).$$

For each $\aleph \geqslant \|\eta_0\|$, we define the stopping time as follows: $\tau_\aleph = \inf\{t\geqslant 0 : \|\eta(t)\| \geqslant \aleph\}$. Note that $\tau_\aleph \to \infty$ as $\aleph\to\infty$ a.s. By Lemma B1 and the matrix equation (F6),

$$\begin{aligned}
&e^{\bar{\ell}(t\wedge\tau_\aleph)}\mathbb{U}(t\wedge\tau_\aleph, \varepsilon(t\wedge\tau_\aleph)) - \mathbb{U}(0,\varepsilon(0)) \\
&= \int_0^{t\wedge\tau_\aleph}\bar{\ell}e^{\bar{\ell}s}\mathbb{U}(s,\varepsilon(s))ds + \int_0^{t\wedge\tau_\aleph}e^{\bar{\ell}s}\tfrac{\partial\mathbb{U}(s,\varepsilon(s))}{\partial s}ds + \tfrac{1}{2}\int_0^{t\wedge\tau_\aleph}e^{\bar{\ell}s}\tfrac{\partial^2\mathbb{U}(s,\varepsilon(s))}{\partial\varepsilon^2}\tfrac{\partial(\int_0^s(K_s^*\Sigma(u))^2du)}{\partial s}ds \\
&\quad + \int_0^{t\wedge\tau_\aleph}e^{\bar{\ell}s}\Sigma(s)\tfrac{\partial^2\mathbb{U}(s,\kappa(s))}{\partial\kappa^2}(\int_0^s\tfrac{\partial K(s,u)}{\partial s}(\int_0^s D_u(K_s^*\Sigma(\theta))d\omega(\theta))du)ds + \int_0^{t\wedge\tau_\aleph}e^{\bar{\ell}s}\tfrac{\partial\mathbb{U}(s,\varepsilon(s))}{\partial\varepsilon}\Sigma(s)dB^H(s),
\end{aligned}$$

where

$$\int_0^s \tfrac{\partial K(s,u)}{\partial s}(\int_0^s D_u(K_s^*\Sigma(\theta))d\omega(\theta))du = 0,$$

$$\tfrac{\partial(\int_0^s(K_s^*\Sigma(u))^2du)}{\partial s} = \Sigma(s)\int_0^s\Sigma(u)\bar{\Phi}_H(s-u)du = \Upsilon(s),$$

$$\tfrac{\partial\mathbb{U}(s,\varepsilon(s))}{\partial\varepsilon} = 2[e^{\varepsilon(s)}\bar{M}(s,0)\eta_0]^T(I_{N-1}\otimes\bar{P})[e^{\varepsilon(s)}\bar{M}(s,0)\eta_0] = 2\eta(s)^T(I_{N-1}\otimes\bar{P})\eta(s),$$

$$\tfrac{\partial^2\mathbb{U}(s,\varepsilon(s))}{\partial\varepsilon^2} = 4[e^{\varepsilon(s)}\bar{M}(s,0)\eta_0]^T(I_{N-1}\otimes\bar{P})[e^{\varepsilon(s)}\bar{M}(s,0)\eta_0] = 4\eta(s)^T(I_{N-1}\otimes\bar{P})\eta(s),$$

$$\begin{aligned}
\tfrac{\partial\mathbb{U}(s,\varepsilon(s))}{\partial s} &= e^{\varepsilon(s)}\eta_0^T\tfrac{\partial\bar{M}(s,0)^T}{\partial s}(I_{N-1}\otimes\bar{P})[e^{\varepsilon(s)}\bar{M}(s,0)\eta_0] + [e^{\varepsilon(s)}\bar{M}(s,0)\eta_0]^T(I_{N-1}\otimes\bar{P})e^{\varepsilon(s)}\tfrac{\partial\bar{M}(s,0)}{\partial s}\eta_0 \\
&= e^{\varepsilon(s)}\eta_0^T\bar{M}(s,0)^T\left(I_{N-1}\otimes A + \bar{\Lambda}\otimes BG_2 - \tfrac{1}{2}\Upsilon(s)I_{\bar{p}}\right)^T(I_{N-1}\otimes\bar{P})\left[e^{\varepsilon(s)}\bar{M}(s,0)\eta_0\right] \\
&\quad + [e^{\varepsilon(s)}\bar{M}(s,0)\eta_0]^T(I_{N-1}\otimes\bar{P})e^{\varepsilon(s)}\left(I_{N-1}\otimes A + \bar{\Lambda}\otimes BG_2 - \tfrac{1}{2}\Upsilon(s)I_{\bar{p}}\right)\bar{M}(s,0)\eta_0 \\
&= \eta(s)^T\left(I_{N-1}\otimes A^T\bar{P} + \bar{\Lambda}\otimes G_2^TB^T\bar{P} - \tfrac{1}{2}\Upsilon(s)I_{N-1}\otimes\bar{P}\right)\eta(s) \\
&\quad + \eta(s)^T\left(I_{N-1}\otimes\bar{P}A + \bar{\Lambda}\otimes\bar{P}BG_2 - \tfrac{1}{2}\Upsilon(s)I_{N-1}\otimes\bar{P}\right)\eta(s).
\end{aligned}$$

From the above analysis, we obtain

$$\begin{aligned}&\mathbb{E}\left[e^{\bar{\ell}(t\wedge\tau_{\aleph})}\mathbb{U}(t\wedge\tau_{\aleph},\varepsilon(t\wedge\tau_{\aleph}))\right]\\&\leqslant\mathbb{U}(0,\varepsilon(0))+\mathbb{E}\bigg[\int_0^{t\wedge\tau_{\aleph}}\Big(e^{\bar{\ell}s}\eta(s)^T((\bar{\ell}+\zeta)I_{N-1}\otimes\bar{P}+I_{N-1}\otimes A^T\bar{P}+I_{N-1}\otimes\bar{P}A\\&\quad+\bar{\Lambda}\otimes G_2^TB^T\bar{P}+\bar{\Lambda}\otimes\bar{P}BG_2)\eta(s)\Big)ds\bigg]\\&\triangleq\mathbb{U}(0,\varepsilon(0))+\mathbb{E}\bigg[\int_0^{t\wedge\tau_{\aleph}}\Big(e^{\bar{\ell}s}\eta(s)^T\bar{\mathbb{J}}\eta(s)\Big)ds\bigg],\end{aligned}$$

where $\bar{\mathbb{J}}=I_{N-1}\otimes A^T\bar{P}+I_{N-1}\otimes\bar{P}A+\bar{\Lambda}\otimes G_2^TB^T\bar{P}+\bar{\Lambda}\otimes\bar{P}BG_2+(\bar{\ell}+\zeta)I_{N-1}\otimes\bar{P}$. Denote

$$\bar{\mathbb{J}}_i=A^T\bar{P}+\bar{P}A+\lambda_i(\mathcal{L})G_2^TB^T\bar{P}+\lambda_i(\mathcal{L})\bar{P}BG_2+(\bar{\ell}+\zeta)\bar{P},\quad i=2,3,\ldots,N.$$

Since $0=\lambda_1(\mathcal{L})<\lambda_2(\mathcal{L})\leqslant\cdots\lambda_N(\mathcal{L})$ under Assumption 4, then $\gamma_2\lambda_i(\mathcal{L})>\frac{\lambda_i(\mathcal{L})}{\lambda_2(\mathcal{L})}\geqslant 1$, for all $i=2,3,\ldots,N$. When condition (9) is satisfied, take $G_2=-\gamma_2B^T\bar{P}$ with $\gamma_2>\frac{1}{\lambda_2(\mathcal{L})}$, then

$$\bar{\mathbb{J}}_i<A^T\bar{P}+\bar{P}A-2\bar{P}BB^T\bar{P}+(\bar{\ell}+\zeta)\bar{P}<0,\ \forall i=2,3,\ldots,N,$$

such that $\bar{\mathbb{J}}<0$. Therefore,

$$e^{\bar{\ell}(t\wedge\tau_{\aleph})}\mathbb{E}\left[\mathbb{U}(t\wedge\tau_{\aleph},\varepsilon(t\wedge\tau_{\aleph}))\right]\leqslant\mathbb{U}(0,\varepsilon(0)).$$

By allowing $\aleph\to+\infty$, we obtain

$$\mathbb{E}\|\eta(t)\|^2\leqslant\frac{\lambda_{\max}(\bar{P})}{\lambda_{\min}(\bar{P})}\|\eta_0\|^2e^{-\bar{\ell}t},$$

such that $\lim_{t\to+\infty}\mathbb{E}\|\eta(t)\|^2=0$. From the inverses of Eq. (**F2**), then

$$\xi(t)=(\tilde{\mathbb{Q}}\otimes I_n)\begin{bmatrix}\bar{\xi}_1(t)\\\eta(t)\end{bmatrix}=\begin{bmatrix}\frac{1}{N}(\mathbf{1}_N\otimes I_n) & \bar{\mathbb{Q}}\otimes I_n\end{bmatrix}\begin{bmatrix}\bar{\xi}_1(t)\\\eta(t)\end{bmatrix}=\frac{1}{N}(\mathbf{1}_N\otimes I_n)\bar{\xi}_1(t)+(\bar{\mathbb{Q}}\otimes I_n)\eta(t).$$

Furthermore, one has

$$\lim_{t\to+\infty}\mathbb{E}\left\|\xi(t)-\frac{1}{N}(\mathbf{1}_N\otimes I_n)\bar{\xi}_1(t)\right\|^2=\lim_{t\to+\infty}\mathbb{E}\left\|(\bar{\mathbb{Q}}\otimes I_n)\eta(t)\right\|^2=0,$$

such that

$$\lim_{t\to+\infty}\mathbb{E}\|\xi_i(t)-\xi_j(t)\|^2=0,\quad\forall i,j=1,2,\ldots,N.$$

The proof is completed.

Appendix G Parameter choices for simulation

Appendix G.1

Consider MASs (1) on $\mathbb{R}^3$ with $N=15$ among the communication topologies depicted in Figure 1 (a). Suppose that agents 1, 5, 10 can obtain the state information of the leader such that Assumption 2 is satisfied. The initial values are selected as follows: $\xi_1(0)=(4,-2,8)^T$, $\xi_2(0)=(-6,0,-5)^T$, $\xi_3(0)=(-7,-10,2)^T$, $\xi_4(0)=(-7,7,-3)^T$, $\xi_5(0)=(10,7,3)^T$, $\xi_6(0)=(-7,1,-3)^T$, $\xi_7(0)=(-2,2,8)^T$, $\xi_8(0)=(4,5,4)^T$, $\xi_9(0)=(-2,2,-5)^T$, $\xi_{10}(0)=(-10,3,7)^T$, $\xi_{11}(0)=(-2,9,-7)^T$, $\xi_{12}(0)=(9,-3,2)^T$, $\xi_{13}(0)=(9,7,-6)^T$, $\xi_{14}(0)=(-5,10,-4)^T$, $\xi_{15}(0)=(-8,6,-4)^T$, $\xi_0(0)=(9,1,7)^T$. Suppose that $H=0.6$, $\Sigma(t)=0.8e^{-0.5t}$, $\ell=0.2$, $A=\begin{pmatrix}-1.5&0.8&-0.7\\-1.2&0&-1\\1.7&2.1&0.8\end{pmatrix}$, $B=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}$. Then $\overline{\Sigma}=0.8$ and $\zeta=H\overline{\Sigma}^2+(2H^2-H)\overline{\Sigma}\int_0^{\infty}0.8e^{-0.5t}dt=0.5376$. Take $P=\begin{pmatrix}3.126&-2.3185&0.3806\\-2.3185&3.1191&0.1758\\0.3806&0.1758&1.2722\end{pmatrix}$, such that inequality condition (6) is satisfied. By calculations, we obtain $\lambda_1(\mathcal{L}+\mathbb{P})=0.1317$. We consider $\gamma_1=1/\lambda_1(\mathcal{L}+\mathbb{P})+0.01=7.603$, such that the feedback control gain matrix is taken as $G_1=(-9.0326,-7.4236,-13.9027)$. The state trajectories of all followers with dynamics (1) and the leader with dynamics (2) without the controller are depicted in Figure G1. Under controller (3), the state trajectories of all agents are depicted in Figure 1 (b)?(d), which show that the leader-following consensus control of MASs (1) is achieved in a mean-square sense.

Appendix G.2

Consider MASs (7) on $\mathbb{R}^2$ with $N=20$ in the communication topology depicted in Figure G2 (a) such that Assumption 4 is satisfied. The initial states are selected as follows: $\xi_1(0)=(10,-7)^T$, $\xi_2(0)=(1,-3)^T$, $\xi_3(0)=(6,3)^T$, $\xi_4(0)=(-5,7)^T$, $\xi_5(0)=(8,-3)^T$, $\xi_6(0)=(-3,6)^T$, $\xi_7(0)=(2,-5)^T$, $\xi_8(0)=(1,2)^T$, $\xi_9(0)=(-4,5)^T$, $\xi_{10}(0)=(-1,-1)^T$, $\xi_{11}(0)=(1,0)^T$, $\xi_{12}(0)=(9,4)^T$, $\xi_{13}(0)=(0,7)^T$, $\xi_{14}(0)=(-5,-1)^T$, $\xi_{15}(0)=(5,7)^T$, $\xi_{16}(0)=(10,-1)^T$, $\xi_{17}(0)=(6,-4)^T$, $\xi_{18}(0)=(-6,4)^T$, $\xi_{19}(0)=(2,-5)^T$, $\xi_{20}(0)=(-3,5)^T$. Suppose that $H=0.7$, $\Sigma(t)=0.8e^{-0.5t}$, $\bar{\ell}=0.2$, $A=\begin{pmatrix}0&1\\-1&0\end{pmatrix}$, $B=(1,1)^T$. Then, $\overline{\Sigma}=0.8$ and $\zeta=H\overline{\Sigma}^2+(2H^2-H)\overline{\Sigma}\int_0^{\infty}0.8\ e^{-0.5t}dt=0.8064$. Take $\bar{P}=\begin{pmatrix}4.749&-1.5985\\-1.5985&1.8929\end{pmatrix}$, such that the inequality condition (9) is satisfied. By calculating, one has $\lambda_2(\mathcal{L})=0.1162$. Taking $\gamma_2=1/\lambda_2(\mathcal{L})+0.01=8.6159$, such that the feedback control gain matrix is $G_2=(-27.1449,-2.5371)$. The state trajectories of all agents with dynamics (7) and controller (8) are depicted in Figure G2 (b)?(c), which shows that the leaderless consensus control of MASs (7) is achieved in the mean-square sense.

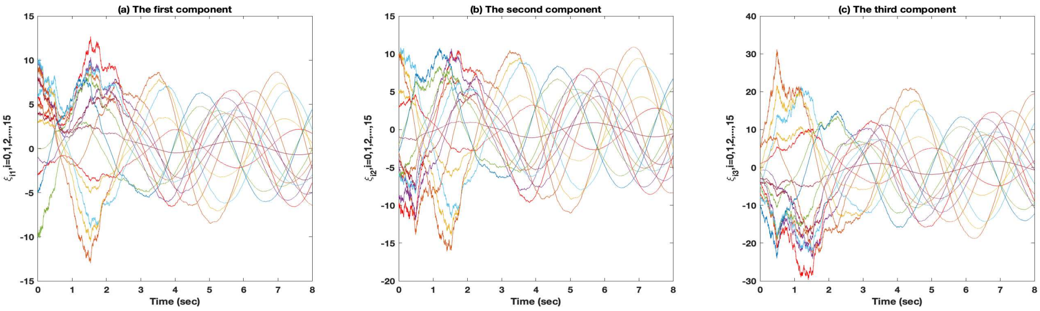

Figure G1 State trajectories of all agents without controller.

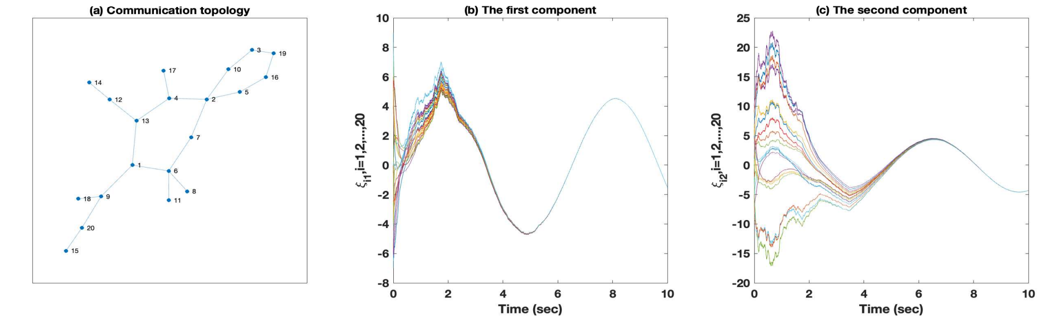

Figure G2 Simulation results in Appendix G.2.

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