

# Distributed adaptive resilient formation control for nonlinear multi-agent systems under DoS attacks

Hao WEN, Yongming LI & Shaocheng TONG\*

College of Science, Liaoning University of Technology, Jinzhou 121001, China

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The cooperative control for multi-agent systems (MASs) with fixed topology [1–3] has been extensively studied recently. However, in practice, denial-of-service (DoS) attacks become an increasingly important issue due to the widespread use of networks in information exchange. DoS attacks can often interrupt the communication network, cause the system to lose control objectives, and pose challenges for theoretical analysis [1,2]. Especially for the high-order nonlinear MASs, if the communication topology is under DoS attacks, the communication signals will be non-continuous and non-differentiable. Thus, the existing backstepping control technique cannot be adopted.

To solve the issue, Ref. [4] proposed a chainlike filter to estimate the leader's output and high-order derivatives under DoS attacks. Subsequently, a resilient consensus controller was given by using the estimated signals. Note that the proposed controller contains non-continuous communication signals, which results in the controller being discontinuous. To smooth the controller, Ref. [5] proposed an adaptive resilient consensus control scheme by utilizing an artificial time delay technique. However, the artificial time delay technique requires a priori knowledge of the minimum time interval for each edge to switch between connected and disconnected states. It means that the defender knows the model of DoS attacks. However, the defender does not know the model of DoS attacks in practice. In addition, Refs. [4,5] required that the duration of DoS attacks is only for the communication topology instead of each communication channel. Thus, they cannot deal with the resilient control issue that at least one communication channel is attacked in the total time interval.

Motivated by the work in [1–5], this study investigates the distributed resilient formation control for a class of parametric strict-feedback MASs under DoS attacks. The main contributions of this study are summarized as follows: (1) This study proposes an improved chainlike filter to estimate the leader's output and high-order derivatives, and a smooth adaptive resilient formation control scheme is proposed based on the improved chainlike filter. The proposed formation control scheme can achieve the formation control objectives and relax the limitation in [5] that the model of DoS attacks must be known. (2) Unlike the resilient consensus control methods in [4,5], the limitation on the duration of DoS attacks in this study is for each communication chan-

nel instead of the communication topology. This means we do not require the existence of time intervals where all communication channels are free from DoS attacks [4], nor the existence of time intervals where the communication topology remains connected under DoS attacks [5]. Thus, this study extends the results of [4,5].

**Problem formulation.** Consider the MASs composed of  $N$  followers. The  $i$ th agent is expressed by the following parametric strict-feedback nonlinear system:

$$\begin{cases} \dot{x}_{i,q} = x_{i,q+1} + \varphi_{i,q}^T(\bar{x}_{i,q})\theta_i, \\ \dot{x}_{i,n} = u_i + \varphi_{i,n}^T(\bar{x}_{i,n})\theta_i, \\ y_i = x_{i,1}, \end{cases} \quad (1)$$

where  $\bar{x}_{i,q} = [x_{i,1}, \dots, x_{i,q}]^T$ ,  $i = 1, \dots, N$ ,  $q = 1, \dots, n-1$ .  $x_{i,q}$  is the system state of the  $i$ th agent and is measurable.  $u_i$  and  $y_i$  are control input and output, respectively.  $\theta_i \in \mathbb{R}^m$  is an unknown parameter vector.  $\varphi_{i,q}(\bar{x}_{i,q}) \in \mathbb{R}^m$  is a known smooth nonlinear function.

The desired tracking trajectory  $y$  is generated by a leader with the following dynamics:

$$\dot{r} = Sr, \quad y = Cr, \quad (2)$$

where  $S \in \mathbb{R}^{m \times m}$  is a known matrix,  $C^T \in \mathbb{R}^m$  is a known vector, and  $r \in \mathbb{R}^m$  and  $y \in \mathbb{R}$  are the state and output of the leader. Define the formation of the  $i$ th agent as  $\xi_i(t) \in \mathbb{R}^m$ , where  $\xi_i(t)$  is a known smooth time-varying vector representing the relative distance to  $r$ . Thus the reference signal of the  $i$ th agent is  $y + C\xi_i(t)$ . For convenience,  $\xi_i(t)$  will be defined as  $\xi_i$  in the following.

**Assumption 1** ([5]). All the eigenvalues of matrix  $S$  are semi-simple with zero real parts. Without loss of generality, we assume  $S$  is a skew-symmetric matrix.

**Control objective.** For MASs (1) with DoS attacks, this study is to design an adaptive distributed resilient control scheme such that

- (1) all signals of the controlled MASs are bounded; and
- (2) the follower's output  $y_i$ ,  $i = 1, \dots, N$  can track the leader's signal  $y$  and maintain a desired formation form, i.e.,  $\lim_{t \rightarrow \infty} y_i - y - C\xi_i = 0$ .

**Assumption 2** ([4]). The undirected communication graph  $G$  is connected, and at least one agent can obtain the information from the leader when the network is not under attacks.

\* Corresponding author (email: jztongsc@163.com)

**Assumption 3** ([1, 2]). There exist constants  $T_D^{ij} > 1, \zeta_D^{ij} > 0, T_F^{ij} > 0, \zeta_F^{ij} > 0$  such that for  $\forall 0 < t_1 < t_2$ , the following inequalities hold:

$$|\Lambda^{i,j}(t_1, t_2)| < \zeta_D^{ij} + \frac{t_2 - t_1}{T_D^{ij}}, n^{i,j}(t_1, t_2) < \zeta_F^{ij} + \frac{t_2 - t_1}{T_F^{ij}}. \quad (3)$$

The definitions of communication network topology, DoS attacks, and symbols in Assumption 3 can be found in Appendix A.

**Definition 1** ([3]). For a switching graph  $\mathcal{G}_{\sigma(t)}$  and a time interval  $[t_a, t_b], t_b - t_a \geq M, M > 0$ , let  $t^1, \dots, t^{m-1}, t^i < t^{i+1}$  denote the switching moment of  $\mathcal{G}_{\sigma(t)}$  in  $[t_a, t_b]$ . If the union of graphs  $\cup_{i=0}^m \mathcal{G}_{\sigma(t^i)}, t^0 = t_a, t^m = t_b$  is connected, then the graph  $\mathcal{G}_{\sigma(t)}$  is said to be jointly connected in  $[t_a, t_b]$ .

**Lemma 1.** Under Assumption 3, the graph  $\bar{G}_{\sigma(t)}$  is jointly connected under DoS attacks in  $[t_k, t_{k+1}], t_k = k \cdot \max_{i,j} \left( \frac{2T_D^{ij} \zeta_D^{ij}}{T_D^{ij} - 1} \right), k = 0, 1, \dots$ , and there exist  $t_k^i \in \mathbb{R}, t_k^{i+1} - t_k^i > 0, i = 0, \dots, m_k, t_k^0 = t_k, t_k^{m_k} = t_{k+1}$  such that the topology is fixed in  $[t_k^i, t_k^{i+1}]$ .

*Proof.* Please see Appendix B.

**Lemma 2** ([3]). If the graph  $\bar{G}_{\sigma(t)}$  is jointly connected in  $[t_k, t_{k+1}]$ , then  $\cup_{i=0}^{m_k} l(\sigma(t_k^i)) = \{1, \dots, N\}$ , where  $l(\sigma) = \{k | \lambda_k^\sigma \neq 0\}$  and  $\lambda_k^\sigma$  is the eigenvalue of  $H_\sigma$ . The labeling rule for  $\lambda_k^\sigma$  is the same as the rule in [3].

*Main results.* Due to the effect of DoS attacks, the leader's output is intermittently known by the followers, and the communication signals are non-continuous. Therefore, we design the following chainlike filter to estimate the leader's output and high-order derivatives:

$$\begin{cases} \dot{\eta}_{i,1} = S(\eta_{i,2} - \xi_i) + \dot{\xi}_i - \delta_i(\eta_{i,1} - \eta_{i,2}), \\ \dot{\eta}_{i,2} = S(\eta_{i,3} - \xi_i) + \dot{\xi}_i - \delta_i(\eta_{i,2} - \eta_{i,3}), \\ \vdots \\ \dot{\eta}_{i,n+q} = S(\eta_{i,n+q} - \xi_i) + \dot{\xi}_i - c\mu_i^\sigma(\eta_{i,n+q} - \xi_i - r) \\ \quad - c \sum_{j \in N_i} a_{ij}^\sigma [(\eta_{i,n+q} - \xi_i) - (\eta_{j,n+q} - \xi_j)], \\ \hat{y}_i = C\eta_{i,1}, \end{cases} \quad (4)$$

where  $\eta_{i,1}, \eta_{i,2}, \dots, \eta_{i,n+q}$  are estimates of  $r + \xi_i$ , and  $\delta_i, c \in \mathbb{R}^+, q \in \mathbb{N}^+$  are design parameters.

**Remark 1.** Note that since the  $n$ th-order derivative in the chainlike filter in [4] for nonlinear MASs (1) is non-continuous, the controller designed based on the chainlike filter is also non-continuous. While the  $n$ th-order derivative in the improved chainlike filter (4) is continuous, the designed controller is continuous. Although a circular filter with an artificial time delay technique proposed in [5] can smooth the controller, it requires that the model of DoS attacks is known.

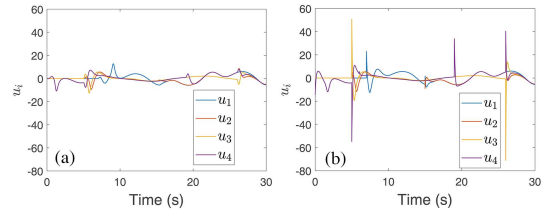
**Theorem 1.** Consider the MASs (1) under Assumptions 1–3. The output of the designed distributed resilient formation estimator (4) converges to  $r + \xi_i$ .

*Proof.* Please see Appendix C.

**Theorem 2.** Consider the MASs (1) under Assumptions 1–3. The controller (D12) with estimator (4) and parameter adaptive law (D13) can guarantee that the following properties hold: (1) The designed distributed adaptive control method can ensure the stability of MASs. (2) All followers' outputs can track the leader's output  $y$  with a desired formation form, i.e.,  $\lim_{t \rightarrow \infty} y_i = y + C\xi_i, i = 1, \dots, N$ .

*Proof.* Please see Appendix D.

*Simulation.* We consider MASs composed of four followers and one leader. The simulation results indicate that under controller (D12), parameter adaptive law (D13), and estimator (4), the formation tracking errors converge to zero asymptotically. To show the smoothness of the proposed controller, we give a comparison with the controller in [4]. The comparison results are shown in Figure 1. Additionally, the convergence performance of the estimator (4) is insensitive to the choice of the parameter  $q$  of estimator (4). For the detailed results of the simulation please refer to Appendix E.



**Figure 1** (Color online) Comparison results of the controllers. (a) Curves of the proposed controller; (b) curves of controller in [4].

*Conclusion.* This article has investigated the adaptive resilient formation control of parametric strict-feedback MASs when all channels are subjected to DoS attacks. To remove the constraint that there exist time intervals at which all the communication channels do not suffer from DoS attacks, we use the concept of the jointly connected network to describe the switching topology under DoS attacks. Then, an improved chainlike filter is proposed to estimate the leader's output and high-order derivatives. Based on the chainlike filter and backstepping control technique, a distributed adaptive resilient formation control scheme with a smooth controller has been developed. The proposed adaptive resilient control scheme can ensure that all the closed-loop signals are bounded and the followers can track the leader. Note that the results of this article rely on the symmetry of the Laplacian matrix. Therefore, how to extend these results to the case of a directed graph is the direction of our future effort.

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**Supporting information** Appendixes A–E. The supporting information is available online at [info.scichina.com](http://info.scichina.com) and [link.springer.com](http://link.springer.com). The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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