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• Supplementary File •

On Optimal Streaming Kernelization Algorithms

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Appendix A The proof of Lemma 1

For an instance (x, z) of the INDEX problem, where $x = x_1 x_2 \cdots x_n \in \{0,1\}^n$ and $z \in \{1, 2, \ldots, n\}$, let $h = \lceil \sqrt[d]{n} \rceil$, and fix an injection π from $\{1, 2, \ldots, n\}$ to the set $\{\langle b_1, b_2, \ldots, b_d\rangle \mid 1 \leqslant b_i \leqslant h, 1 \leqslant i \leqslant d\}$ of ordered d-tuples. Suppose $\pi(z) = \langle a_1, a_2, \ldots, a_d\rangle$.

Let $U = \{v_{i,b}, v_{i,b,t} \mid 1 \leqslant i \leqslant d, 1 \leqslant b \leqslant h, 1 \leqslant t \leqslant d\}$. Define a collection $\mathcal{C}_{x,z}$ of d-sets of U as follows:

(G1) For each $1 \leqslant i \leqslant d$ and each $b_i \neq a_i$, there is a d-set $\{v_{i,b_i,1}, \ldots, v_{i,b_i,i-1}, v_{i,b_i}, v_{i,b_i,i+1}, \ldots, v_{i,b_i,d}\}$ in $\mathcal{C}_{x,z}$. There are totally $d(h-1)$ d-sets in group (G1).

(G2) For each bit x_y of x such that $x_y = 1$, where $\pi(y) = \langle b_1, b_2, \ldots, b_d\rangle$, there is a d-set $\{v_{1,b_1}, v_{2,b_2}, \ldots, v_{d,b_d}\}$ in $\mathcal{C}_{x,z}$. The total number of d-sets in group (G2) is equal to the number of 1-bits in x.

Lemma 1. A sufficient and necessary condition for the collection $\mathcal{C}_{x,z}$ to have a hitting set with exactly $d(h-1)$ elements is that $x_z = 0$.

Proof. Recall $\pi(z) = \langle a_1, a_2, \ldots, a_d\rangle$. We say that a set H *hits* another set S if $H \cap S \neq \emptyset$.

Case 1. $x_z = 0$. Consider the set $H_0 = \{v_{i,b_i} \mid 1 \leqslant i \leqslant d, b_i \neq a_i\}$ of $d(h-1)$ elements.

It is obvious that the set H_0 hits every d-set in group (G1): by definition, each d-set in group (G1) contains an element v_{i,b_i} for some index i such that $b_i \neq a_i$, which is in H_0.

For each d-set $S = \{v_{1,b_1}, v_{2,b_2}, \ldots, v_{d,b_d}\}$ in group (G2), where $\pi(y) = \langle b_1, b_2, \ldots, b_d\rangle$ and $x_y = 1$, since $x_z = 0$, $\pi(z) = \langle a_1, a_2, \ldots, a_d\rangle$, there must be an index j such that $a_j \neq b_j$. Thus, the element v_{j,b_j} in H_0 is in the set S. As a result, the set H_0 hits every d-set in group (G2).

Thus, under the condition $x_z = 0$, the collection $\mathcal{C}_{x,z}$ has a hitting set H_0 with exactly $d(h-1)$ elements.

Case 2. $x_z = 1$. Let H_1 be a hitting set with the minimum number of elements for the collection $\mathcal{C}_{x,z}$.

Since all d-sets in group (G1) are pairwise disjoint, the set H_1 must have $d(h-1)$ distinct elements to hit the $d(h-1)$ d-sets in group (G1).

Since $x_z = 1$ and $\pi(z) = \langle a_1, a_2, \ldots, a_d\rangle$, there is a d-set $S = \{v_{1,a_1}, v_{2,a_2}, \ldots, v_{d,a_d}\}$ in group (G2). None of the elements v_{1,a_1}, v_{2,a_2}, …, v_{d,a_d} in the set S can appear in any d-set in group (G1): by definition, all elements of form v_{i,b_i} in d-sets in group (G1) must satisfy $b_i \neq a_i$. Thus, none of the $d(h-1)$ elements that are used by H_1 to hit the d-sets in group (G1) can be used by H_1 to hit the set S, so H_1 must need an additional element to hit the set S. As a result, the smallest hitting set H_1 for the collection $\mathcal{C}_{x,z}$ contains at least $d(h-1)+1$ elements, i.e., the collection $\mathcal{C}_{x,z}$ has no hitting set with $d(h-1)$ elements.

This completes the proof of the lemma.

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