SCIENCE CHINA
Information Sciences

• LETTER •

July 2024, Vol. 67, Iss. 7, 179305:1–179305:2
https://doi.org/10.1007/s11432-024-4074-y

Design of double protograph LDPC codes based JSCC systems via the ACE-PEG algorithm

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Received 9 April 2024/Revised 20 May 2024/Accepted 12 June 2024/Published online 27 June 2024

In this study, we present a state-of-the-art design of double protograph low-density parity-check (DP-LDPC) codes based on progressive edge-growth (PEG) algorithm with approximated cycle extrinsic message degree (ACE) properties, tailored for joint source-channel coding (JSCC) systems. Compared with the conventional optimization schemes, such a novel-designed base matrix contains more higher-degree variable nodes. These higher-degree variable nodes successfully increase connectivity between cycles or stopping sets and the remaining part of the Tanner graph after the 'copy and permute' operation. The parallel differential evolutionary (PDE) algorithm is also used to tackle high-dimensional global optimization problems with high complexity brought on by higher-degree variable nodes. Simulation results demonstrate significant performance improvement in both the error-floor and waterfall regions for the proposed source and channel codes compared to the conventional DP-LDPC codes, achieved by concurrently optimizing both the source and channel codes.

JSCC preliminaries. Due to the merit, two protograph LDPC codes [1, 2] can be used as the source and channel codes for the JSCC system [3]. The parity-check matrix for DP-LDPC codes has the following base matrix,

$$\boldsymbol{B}_{\mathrm{J}} = \left[\begin{array}{c|c} \boldsymbol{B}_{\mathrm{S}} & \boldsymbol{B}_{\mathrm{L}} \\ \hline \boldsymbol{0} & \boldsymbol{B}_{\mathrm{C}} \end{array}\right], \quad (1)$$

where the base submatrix of source $\boldsymbol{B}_{\mathrm{S}}$ and that of channel $\boldsymbol{B}_{\mathrm{C}}$ have dimensions $r_{\mathrm{S}} \times c_{\mathrm{S}}$ and $r_{\mathrm{C}} \times c_{\mathrm{C}}$, respectively. The submatrix $\boldsymbol{B}_{\mathrm{L}}$ with dimension $r_{\mathrm{S}} \times c_{\mathrm{C}}$ represents the edge connection between two tandem protograph LDPC codes. Thus, the overall dimension of $\boldsymbol{B}_{\mathrm{J}}$ is $(r_{\mathrm{S}} + r_{\mathrm{C}}) \times (c_{\mathrm{S}} + c_{\mathrm{C}})$. The two tandem protograph LDPC codes are derived from the 'copy and permute' operation based on the matrix in (1). The resultant parity-check matrix is defined as

$$\boldsymbol{H}_{\mathrm{J}} = \left[\begin{array}{c|c} \boldsymbol{H}_{\mathrm{S}} & \boldsymbol{H}_{\mathrm{L}} \\ \hline \boldsymbol{0} & \boldsymbol{H}_{\mathrm{C}} \end{array}\right], \quad (2)$$

where the dimensions of all the submatrices are integer-fold of these of the corresponding ones in (1). The matrix $\boldsymbol{H}_{\mathrm{J}}$ has the dimension $(R_{\mathrm{S}} + R_{\mathrm{C}}) \times (C_{\mathrm{S}} + C_{\mathrm{C}})$.

Introduction to higher-degree variable nodes. In the conventional optimization schemes, the strategy in favor of low-degree variable nodes (VNs) is employed to avoid short cycles and stopping sets. However, these nodes are vulnerable to channel noise due to inadequate message interaction with other nodes. It is noted that the higher-degree VNs, while more likely to cause short cycles and stopping sets, do not necessarily impose an equal influence on the decoding process. Therefore, more careful treatment of harmful short cycles and stopping sets are expected to improve the overall decoding performance [4]. To improve performance, it is critical to take the full potential of the higher-degree VNs while mitigating the negative impact of harmful short cycles and stopping sets. Such a balance can be achieved by developing efficient yet effective methods to identify and treat these cycles and stopping sets in the Tanner graph, ultimately leading to improved decoding performance. One essential metric for measuring the connection between a cycle and the remaining part of the Tanner graph is extrinsic message degree (EMD), which is also known as ACE [5]. Such a metric provides valuable information about the interconnectivity among cycles, aiding in the identification and treatment of harmful short cycles and stopping sets.

Construction of DP-LDPC codes via ACE-PEG algorithm. The ACE-PEG algorithm is an improved algorithm of the PEG algorithm. In the PEG algorithm, if the complement of the set of check nodes (CNs) that are connected to the root node is non-empty at level ℓ but empty at level $(\ell + 1)$, this indicates that in the Tanner graph, all CNs can be reached from the root node, and therefore a cycle of length $2(\ell + 2)$ is unavoidable. In contrast to the PEG algorithm that randomly selects the CN, the ACE-PEG algorithm selects the CN with the highest connectivity to the remaining part of the graph among the candidate CNs. Each candidate CN is sequentially added to the cycle of the Tanner graph. We calculate its ACE value and select the CN with the largest ACE value. The PDE algorithm is significantly more effective than the differential evolutionary algorithm in solving high-dimensional high-complexity global optimization problems. The decoding threshold given by joint protograph extrinsic information transfer (JPEXIT) analysis is used as the foundation for the selection of the PDE algorithm in the optimization of DP-LDPC codes. This algorithm is more effective for the optimization of higher-degree VNs, which benefits from the improved per-

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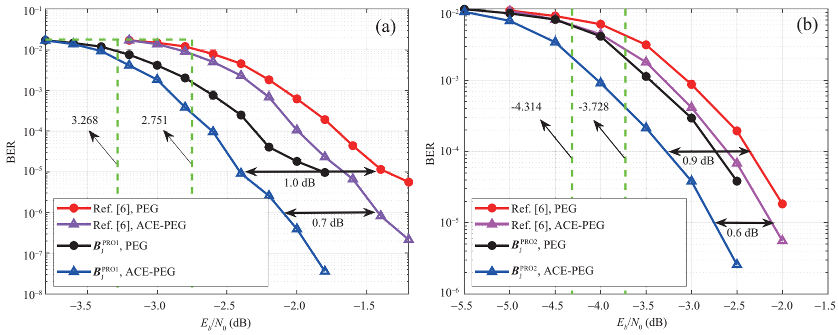

Figure 1 (Color online) Comparison of BER simulation results. (a) $p = 0.04$; (b) $p = 0.01$.

formance of high-dimensional global optimization.

The higher-degree VNs that have been 'copied and permuted' through the ACE-PEG algorithm will bring significant performance gains to the JSCC system. Thus, the design criteria of DP-LDPC codes based on ACE-PEG and PDE algorithms are summarized below:

(i) The number of VNs with a degree higher than 3 is greater than the number of VNs with a degree lower than or equal to 3.

(ii) The components have a maximum value of 5 to avoid excessive complexity and numerous cycles or stopping sets.

(iii) The maximum degree of all the VNs in DP-LDPC codes is 15.

(iv) The minimum number of VNs with a degree higher than 3 is $r_{\rm S} + r_{\rm C} + 1$.

(v) The minimum number of VNs with a degree higher than 3 in the channel code is $r_{\rm S} + 1$.

Different from the conventional approaches to JSCC code design, the proposed design brings irregular code structures in the base matrix. However, the proposed design relies on the high-dimensional global search of the PDE algorithm customized for finding irregular codes with better performance.

The design objective of the PDE algorithm for the DP-LDPC codes is formulated as below:

$$\min \mathbf{JTH}\left(\boldsymbol{B}_{\rm J}\right), \ \max \mathbf{STH}\left(\boldsymbol{B}_{\rm J}\right), \ \text{s.t.} \ \boldsymbol{\Omega}\left(\boldsymbol{B}_{\rm J}\right) = 1, \quad (3)$$

where $\mathbf{JTH}\left(\boldsymbol{B}_{\rm J}\right)$ denotes the joint source-channel decoding threshold, $\mathbf{STH}\left(\boldsymbol{B}_{\rm J}\right)$ denotes the source decoding threshold, and $\boldsymbol{\Omega}\left(\boldsymbol{B}_{\rm J}\right) = 1$ indicates that conditions (i)–(iii) are satisfied.

The optimization solution of the base matrix satisfying conditions (i)–(v) for the case where the source probability is $p = 0.04$ and the parameters are $r_{\rm S} = 2$, $c_{\rm S} = 4$ for source matrix (1/2 code rate) and $r_{\rm C} = 3$, $c_{\rm C} = 5$ for channel matrix with one punctured VN (1/2 code rate) is given by

$$\boldsymbol{B}_{\rm J}^{\rm PRO1} = \left[\begin{array}{cccc:ccccc} 4 & 2 & 4 & 2 & 0 & 1 & 0 & 0 & 0 \\ 4 & 1 & 1 & 5 & 0 & 0 & 1 & 0 & 0 \\ \hdashline 0 & 0 & 0 & 0 & 0 & 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 & 2 & 5 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 4 & 5 & 3 \end{array}\right]. \quad (4)$$

For another case of $p = 0.01$, when $r_{\rm S} = 2$, $c_{\rm S} = 8$, $r_{\rm C} = 3$, and $c_{\rm C} = 5$ with one punctured VN, the optimization is

$$\boldsymbol{B}_{\rm J}^{\rm PRO2} = \left[\begin{array}{cccccccc:ccccc} 4 & 0 & 1 & 1 & 5 & 1 & 1 & 2 & 0 & 1 & 0 & 0 & 0 \\ 1 & 3 & 1 & 1 & 3 & 3 & 3 & 2 & 0 & 0 & 1 & 0 & 0 \\ \hdashline 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 1 & 0 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 5 & 4 & 1 & 2 & 2 \end{array}\right]. \quad (5)$$

Simulation results. The bit error rate (BER) performance over the additive white Gaussian noise (AWGN) channel via joint belief propagation iterative decoding is evaluated. The source code and channel code are obtained through the 'copy and permute' operation via the PEG algorithm/ACE-PEG algorithm. The maximum frame length is 2400 bits for $p = 0.04$ and 3200 bits for $p = 0.01$. The maximum number of iterations is set to 50, and that of frames with errors is set to 100.

The BER simulation results of DP-LDPC codes from [6] and the proposed $\boldsymbol{B}_{\rm J}^{\rm PRO1}$ are provided in Figure 1(a). When the BER is at the level of 10^{-5}, $\boldsymbol{B}_{\rm J}^{\rm PRO1}$ brings a gain of 1.0 dB over that from [6]. And at the BER level of 10^{-6}, the proposed DP-LDPC codes achieve a gain of 0.7 dB over the one from [6], obtained also via the ACE-PEG algorithm.

Figure 1(b) depicts the compared results between the optimized code $\boldsymbol{B}_{\rm J}^{\rm PRO2}$ and that from [6] for $p = 0.01$. It can be found that the similar results can be achieved.

Conclusion. In this work, a novel design method of DP-LDPC codes for the JSCC system via the ACE-PEG algorithm has been proposed. The designed base matrix contains more higher-degree VNs than the conventional ones, and thus increases the amount of message between the cycles and/or stopping sets and the remaining part of the Tanner graph via the ACE-PEG algorithm. Simulation results have demonstrated that the proposed design approach of DP-LDPC codes obtains significant performance enhancement in both waterfall and error-floor regions compared to the selected benchmark schemes from the latest literature.

Acknowledgements This work was supported by National Natural Science Foundation of China (Grant No. 61871337).

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