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Stability analysis and stabilization of semi-Markov jump linear systems with unavailable sojourn-time information

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Abstract This study addresses the stability and stabilization problems of discrete-time semi-Markov jump linear systems (S-MJLSs) with unavailable sojourn-time information. The sojourn-time probability mass functions (S-TPMFs) of discrete-time semi-Markov chains are no longer confined to the geometric distribution, and it is difficult to obtain accurate and comprehensive information for S-TPMFs in practice. This is because S-TPMFs are usually deduced from the statistical characteristics according to the sampled-data, while adequate samples are often costly and time consuming. In this study, when the S-TPMFs for semi-Markov chains are assumed to be unavailable, the σ-error mean square stability is investigated for discrete-time S-MJLSs with some widely used assumptions; for semi-Markov chains, only the transition probability matrix of the embedded chain is used. In addition, the existence conditions of the effective controller are provided for closed-loop systems without using the information of S-TPMFs. Numerical examples are presented to illustrate the validity of the obtained theoretical results.

Keywords semi-Markov chain, semi-Markov jump linear systems, σ-error mean square stability, unavailable sojourn-time information

1 Introduction

Stochastic switching commonly exists in many realistic systems, such as aircraft [1], electric circuit systems [2], vehicles [3], and communication networks [4], which are usually caused by external and internal disturbances, for example, configuration changes, unexpected events, and random faults [5–7]. Markov jump systems, as an important part of stochastic switching systems, have been widely investigated, for linear systems [5, 6, 8–12] and nonlinear systems [13–19]. Markov chains have memoryless property, which demands that the sojourn-time between consecutive mode switching should satisfy the geometrical distribution for discrete-time case, and exponential distribution for continuous-time case [20], respectively. This restricts the application scope of Markov jump systems.

The semi-Markov jump system is an extension of the Markov jump system, whose sojourn-time is unnecessarily confined to either geometric distribution or exponential distribution. Consequently, semi-Markov jump systems are capable of depicting a wider range of stochastic switching systems [21–25]. At the same time, the generality of the semi-Markov chain also brings more challenges to the stability analysis for semi-Markov jump systems. Numerous results have been reported on the stability and stabilization problems of such systems [26–38].

Recent years have shown that the semi-Markov kernel approach is effective in tackling the stability and stabilization problems for discrete-time S-MJLSs [27, 28, 39–43]. For example, the mth mean stability is presented for discrete-time and continuous-time positive S-MJLSs with state resets in [27]. The σ-error mean square stability (σ-MSS[1)]) has been widely used to study S-MJLSs, (e.g., [42, 43]), which was firstly defined for S-MJLSs in [39], assuming that each sojourn-time is truncated by the given upper

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1) For convenience, σ-MSS will denote σ-error mean square stability or σ-error mean square stable in the following.

bound. In [40], the stability and stabilization were investigated for discrete-time S-MJLSs with general and exponentially periodic distributions of sojourn-time, respectively.

The semi-Markov kernel is usually calculated using S-TPMFs and the transition probability matrix of the embedded chain. Thus, in order to tackle the stability problem for discrete-time S-MJLSs with the semi-Markov kernel approach, almost all existing results assume that specific S-TPMFs are available. However, in practice, it is difficult to obtain accurate and comprehensive information for the S-TPMFs, because they are usually deduced from statistical characteristics according to the sampled data. In fact, adequate samples for random sojourn-time and mode transitions of semi-Markov chains are costly and time-consuming [44]; inadequate samples may result in an incorrect presumption for S-TPMFs, which leads to the designed controllers failing to stabilize the S-MJLS. However, the stability criteria established for discrete-time S-MJLSs with the semi-Markov kernel approach, are usually dependent on the given upper bound of sojourn-time, which significantly affects the practical use of derived results [44]. This leads to the fact that the results in [39,42,43] are not applicable when the specific information of S-TPMFs is unavailable.

Motivated by the aforementioned discussions, this study addresses the stability problem for discrete-time S-MJLSs with unavailable S-TPMFs information. The major contributions of this study are as follows:

(1) The stability problem is investigated for discrete-time S-MJLSs without using any S-TPMFs information, where only the transition probability matrix of embedded chains is used for semi-Markov chain. The results in [27, 39, 40, 43] depend heavily on the S-TPMFs information, which cannot be applied to the situation considered in this study.

(2) A novel analysis technique is developed to study the σ-MSS for discrete-time S-MJLSs under some widely used assumptions, and the established stability criteria remove the dependence of the upper bound for sojourn-time in [39, 40, 42, 43, 45].

The remainder of this paper is organized as follows. The model is introduced in Section 2, along with some necessary definitions. Section 3 is divided into two parts. The stability criteria are established for unforced discrete-time S-MJLSs in the first part, and in the second part, the existence conditions for stabilizing the controller are presented for closed-loop S-MJLSs. Finally, two numerical examples are provided to demonstrate the validity of the obtained theoretical results.

Notations. Set $\mathbb{N}^+ = \{1, 2, \ldots\}$, $\mathbb{N} = \mathbb{N}^+ \cup \{0\}$, and $\Gamma = \{1, 2, \ldots, \vartheta\}$, where ϑ is a finite positive integer. The d-dimensional Euclidean space is represented by $\mathbb{R}^d$. $\mathbb{N}_{[n_1,n_2]}$ represents the set $\{n \in \mathbb{N} | n_1 \leqslant n \leqslant n_2\}$. $(\Omega, \mathcal{F}, \mathbb{P})$ is a complete probability space, and $\mathbb{E}$ is the mathematical expectation defined in this space. P' and P^{-1} represent the transposition and inverse of matrix P, respectively. $\mathrm{diag}\{\cdots\}$ is defined as a block-diagonal matrix, and the asterisk $*$ is employed as an ellipsis for terms introduced by symmetry. $\lambda_{\min}(P)$ ($\lambda_{\max}(P)$) describes the smallest (largest) eigenvalue of matrix P. $\mathrm{diag}_{(\vartheta)}\{P\}$ is defined as a $\vartheta \times \vartheta$ block-diagonal matrix, whose diagonal entries are P. Notation $P \succ 0$ ($P \prec 0$) denotes the matrix P is positive (negative) definite and real symmetric. The symbols $\underline{\mathrm{Ge}}(a_1)$, $\underline{\mathrm{P}}(a_2)$, and $\underline{\mathrm{b}}(a_3, a_4)$ denote the geometric distribution with parameter a_1, Poisson distribution with parameters a_2, and binomial distribution with parameters a_3 and a_4, respectively. The mathematical expectation of x conditional on y is defined as $\mathbb{E}x|_y$. The symbol $I(\cdot)$ denotes the indicator function.

2 Preliminaries

Consider the following discrete-time S-MJLS:

$$x(n+1) = A_{r(n)}x(n) + B_{r(n)}u(n),\ n \in \mathbb{N}, \tag{1}$$

where $x(n) \in \mathbb{R}^{d_x}$ is the system state, $\{r(n)\}_{n\in\mathbb{N}}$ is a semi-Markov chain taking values from the finite set Γ, and $u(n) \in \mathbb{R}^{d_u}$ is the control input. Both A_i and B_i are defined as real matrices with appropriate dimensions for any $i \in \Gamma$. For simplicity, let initial mode $r(0) = r_0$ and the initial state $x(0) = x_0$.

To introduce the definition of a semi-Markov chain, some necessary notations and definitions are provided first. Let ϱ_n and ζ_n, respectively, be the time instant and the index of the subsystem mode at the nth jump of the stochastic process $\{r(n)\}_{n\in\mathbb{N}}$. The sojourn-time between the nth and $(n+1)$th jump is defined as η_n, where $\eta_n = \varrho_{n+1} - \varrho_n$ for $n \in \mathbb{N}$ and $\varrho_0 = 0$. The Markov renewal chain (MRC) is defined as follows.

Definition 1 ([40]). If $\{(\zeta_n, \varrho_n)\}_{n\in\mathbb{N}}$ is a discrete-time stochastic process, and for $\forall k \in \mathbb{N}^+$ and $i, j \in \Gamma$,

$$\begin{aligned}\mathbb{P}(\zeta_{n+1} = j, \eta_n = k|\zeta_n = i, \eta_{n-1}, \ldots, \zeta_1, \eta_0, \zeta_0) &= \mathbb{P}(\zeta_{n+1} = j, \eta_n = k|\zeta_n = i)\\ &= \mathbb{P}(\zeta_1 = j, \eta_0 = k|\zeta_0 = i),\end{aligned} \tag{2}$$

then $\{(\zeta_n, \varrho_n)\}_{n\in\mathbb{N}}$ is said to be a homogeneous MRC.

Stochastic process $\{\zeta_n\}_{n\in\mathbb{N}}$ is a Markov chain, which is usually called the embedded chain of MRC $\{(\zeta_n, \varrho_n)\}_{n\in\mathbb{N}}$. Let $\Theta = [\theta_{ij}]_{\vartheta\times\vartheta}$ be the transition probability matrix of $\{\zeta_n\}_{n\in\mathbb{N}}$ with $\theta_{ij} = \mathbb{P}(\zeta_{n+1} = j|\zeta_n = i)$ and $\theta_{ii} = 0$ for $\forall i, j$. In this study, the probability distribution of the sojourn-time is assumed to have the following property.

Assumption 1 ([15]). Let $\{(\zeta_n, \varrho_n)\}_{n\in\mathbb{N}}$ be an MRC and satisfy that for $\forall k \in \mathbb{N}^+$ and $i, j \in \Gamma$,

$$\mathbb{P}(\eta_n = k|\zeta_{n+1} = j, \zeta_n = i) = \mathbb{P}(\eta_n = k|\zeta_n = i) = \mathbb{P}(\eta_0 = k|\zeta_0 = i),$$

that is, the probability distribution of sojourn-time depends only on the current mode, and is independent of the next one.

Under Assumption 1, the semi-Markov kernel can be defined as

$$\begin{aligned}\pi_{ij}(k) &\triangleq \mathbb{P}(\zeta_{n+1} = j, \eta_n = k|\zeta_n = i) = \mathbb{P}(\zeta_{n+1} = j|\zeta_n = i)\mathbb{P}(\eta_n = k|\zeta_{n+1} = j, \zeta_n = i)\\ &= \theta_{ij}\omega_i(k),\end{aligned} \tag{3}$$

where $\omega_i(k) \triangleq \mathbb{P}(\eta_{n+1} = k|\zeta_n = i)$ and $k \in \mathbb{N}^+$.

Remark 1. In Assumption 1, the probability distribution of sojourn-time is assumed to be independent of the next system mode. It is worth noting that this assumption is widely adopted for considering the stability/stabilization of semi-Markov jump systems [22, 26, 33, 46, 47]. Obviously, under Assumption 1, if the distribution $\omega_i(k)$ of sojourn-time is limited to the geometric distribution, then the defined semi-Markov chain becomes a discrete-time Markov chain.

With the help of the definition for MRC, we can define a semi-Markov chain as follows.

Definition 2 ([48]). Let $\{(\zeta_n, \varrho_n)\}_{n\in\mathbb{N}}$ be an MRC, the stochastic process $\{r(n)\}_{n\in\mathbb{N}}$ is said to be a semi-Markov chain, if for $\forall k \in \mathbb{N}$, $r(k) = \zeta_{N(k)}$, where $N(k) \triangleq \max\{n \in \mathbb{N}|\varrho_n \leqslant k\}$ is a discrete-time counting process.

A stability concept for the unforced system (1) can be provided, that is, the control input $u(n) \equiv 0$ in system (1).

Definition 3 ([39]). Given the upper bound $\mathcal{T}_i$ of sojourn-time for the ith mode, the unforced system (1) is said to be σ-MSS, if for any initial value $x_0 \in \mathbb{R}$, the initial mode $r_0 \in \Gamma$, $i \in \Gamma$, and $\iota \in \mathbb{N}^+$,

$$\lim_{n\to\infty} \mathbb{E}[\|x(n)\|^2]|_{x_0, r_0, \eta_\iota I_{(\zeta_\iota = i)} \leqslant \mathcal{T}_i} = 0,$$

where $\sigma \triangleq \sum_{i\in\Gamma} |\ln F_i(\mathcal{T}_i)|$ denotes the approximation error component of the ith mode and $F_i(\mathcal{T}_i) = P(\eta_\iota \leqslant \mathcal{T}_i|\zeta_\iota = i)$.

Remark 2. In most existing literature on σ-MSS, the error σ should be calculated to characterize the approximate effect. In this study, it is unnecessary because the error σ can always be sufficiently small. As pointing out in Remark 3 of [39], if $\mathcal{T}_i \to \infty$ for each mode $i \in \Gamma$, then $\sigma \to 0$. In this study, the stability criteria presented do not depend on the given upper bound $\mathcal{T}_i$, we can always choose a sufficiently large $\mathcal{T}_i$ such that the corresponding error σ is sufficiently small.

Remark 3. Many important results have been established for discrete-time S-MJLSs in [39,40,42,43,45]. However, the stability criteria in [39, 40, 42, 43, 45] cannot be used for discrete-time S-MJLSs, when the S-TPMFs information is unavailable. However, as pointed out in Remark 3 of [44], sufficient conditions provided in [39,40,42,43,45] depend on the given upper bound of sojourn-time. In this study, by assuming that the probability distribution of sojourn-time depends only on the current mode, σ-MSS is presented for discrete-time S-MJLSs without using any S-TPMF information, and the stability criteria are not affected by the exactly truncated upper bounds of sojourn-time.

3 Main results

In this section, the criteria of σ-MSS are presented for the unforced system (1) first, and thereafter the existence conditions of state-feedback stabilizing controllers are provided such that the resulting closed-loop system (1) is σ-MSS, where the sojourn-time information is assumed to be unavailable.

3.1 MSS for unforced systems

In this subsection, stability criteria were established for the unforced S-MJLS (1). Before proceeding further, a lemma is presented as follows:

Lemma 1. For unforced S-MJLS (1), if there exists a set of positive definite matrices P_i such that for $\forall i \in \Gamma$,

(H_1) $A_i'\mathcal{P}^i A_i - P_i \prec 0$,

(H_2) $A_i'\mathcal{P}^i A_i - \mathcal{P}^i \prec 0$,

where $\mathcal{P}^i = \sum_{j\in\Gamma}\theta_{ij}P_j$, then

$$\lim_{n\to\infty}\mathbb{E}\|x(\varrho_n)\|^2 = 0. \tag{4}$$

Proof. Let the Lyapunov function

$$V_i(x(n)) = x'(n)P_i x(n),$$

where $i \in \Gamma$ and $n \in \mathbb{N}$. For $i, j \in \Gamma$ and $k_n \in \mathbb{N}$, using (3), one has

$$\mathbb{P}(r(\varrho_{n+1}) = j, \eta_n = k_n | r(\varrho_n) = i) = \theta_{ij}\mathbb{P}(\eta_n = k_n | r(\varrho_n) = i). \tag{5}$$

From (5), it is not difficult to see that

$$\begin{aligned}
&\mathbb{E}[V_{r(\varrho_{n+1})}(x(\varrho_{n+1}))|x(\varrho_n), r(\varrho_n) = i]\\
&= \sum_{k_n=1}^{+\infty}\sum_{j\in\Gamma} x'(\varrho_{n+1})P_j x(\varrho_{n+1}) I_{(\eta_n=k_n)}\mathbb{P}(r(\varrho_{n+1}) = j, \eta_n = k_n | r(\varrho_n) = i)\\
&= \sum_{k_n=1}^{+\infty}\sum_{j\in\Gamma} x'(\varrho_{n+1})P_j x(\varrho_{n+1}) I_{(\eta_n=k_n)}\mathbb{P}(\eta_n = k_n | r(\varrho_n) = i)\theta_{ij}.
\end{aligned}$$

It follows that

$$\mathbb{E}[V_{r(\varrho_{n+1})}(x(\varrho_{n+1}))|x(\varrho_n), r(\varrho_n) = i] = \sum_{k_n=1}^{+\infty}\mathbb{P}(\eta_n = k_n | r(\varrho_n) = i) I_{(\eta_n=k_n)} x'(\varrho_{n+1})\mathcal{P}^i x(\varrho_{n+1}),$$

where $\mathcal{P}^i = \sum_{j\in\Gamma}\theta_{ij}P_j$. Noticing the form of unforced S-MJLS (1), we have

$$\mathbb{E}[V_{r(\varrho_{n+1})}(x(\varrho_{n+1}))|x(\varrho_n), r(\varrho_n) = i] = \sum_{k_n=1}^{+\infty} P(\eta_n = k_n | r(\varrho_n) = i) x'(\varrho_n)(A_i')^{k_n}\mathcal{P}^i A_i^{k_n} x(\varrho_n). \tag{6}$$

In view of (H_2), we derive that for any integral $\iota \geqslant 2$,

$$(A_i')^{\iota}\mathcal{P}^i A_i^{\iota} - (A_i')^{\iota-1}\mathcal{P}^i A_i^{\iota-1} \prec 0,$$

which yields that

$$\begin{aligned}
&(A_i')^{k_n}\mathcal{P}^i A_i^{k_n} - A_i'\mathcal{P}^i A_i\\
&\quad= \Big((A_i')^{k_n}\mathcal{P}^i A_i^{k_n} - (A_i')^{k_n-1}\mathcal{P}^i A_i^{k_n-1}\Big) + \Big((A_i')^{k_n-2}\mathcal{P}^i A_i^{k_n-2} - (A_i')^{k_n-3}\mathcal{P}^i A_i^{k_n-3}\Big)\\
&\qquad + \cdots + \Big((A_i')^2\mathcal{P}^i A_i^2 - (A_i')\mathcal{P}^i A_i\Big)\\
&\quad= \sum_{\iota=2}^{k_n}[(A_i')^{\iota}\mathcal{P}^i A_i^{\iota} - (A_i')^{\iota-1}\mathcal{P}^i A_i^{\iota-1}] \prec 0.
\end{aligned} \tag{7}$$

Substituting (7) into (6) yields that

$$\mathbb{E}[V_{r(\varrho_{n+1})}(x(\varrho_{n+1}))|x(\varrho_n), r(\varrho_n) = i] \leqslant \sum_{k_n=1}^{+\infty}\mathbb{P}(\eta_n = k_n | r(\varrho_n) = i) x'(\varrho_n) A_i'\mathcal{P}^i A_i x(\varrho_n)$$

$$=x'(\varrho_n)A_i'\mathcal{P}^iA_ix(\varrho_n), \tag{8}$$

where $\sum_{k_n=1}^{+\infty}\mathbb{P}(\eta_n=k_n|r(\varrho_n)=i)=1$. It follows from ($\text{H}_1$) that $\beta\triangleq\min_{i\in\Gamma}\{\lambda_{\min}(-A_i'\mathcal{P}^iA_i+P_i)\}>0$. Noticing ($\text{H}_1$) and (8), for any $r(\varrho_n)=i\in\Gamma$, we have

$$\begin{aligned}\mathbb{E}[V_{r(\varrho_{n+1})}(x(\varrho_{n+1}))|x(\varrho_n),r(\varrho_n)=i]-V_i(x(\varrho_n))\leqslant&x'(\varrho_n)A_i'\mathcal{P}^iA_ix(\varrho_n)-x'(\varrho_n)P_ix(\varrho_n)\\=&x'(\varrho_n)(A_i'\mathcal{P}^iA_i-P_i)x(\varrho_n)\\\leqslant&-\beta\|x(\varrho_n)\|^2.\end{aligned}$$

Consequently, for any $r(\varrho_n)\in\Gamma$ and $n\in\mathbb{N}$,

$$\beta\|x(\varrho_n)\|^2\leqslant\Big[V_{r(\varrho_n)}(x(\varrho_n))-\mathbb{E}[V_{r(\varrho_{n+1})}(x(\varrho_{n+1}))|x(\varrho_n),r(\varrho_n)]\Big]. \tag{9}$$

By taking the expectation on the both sides of (9), one has

$$\mathbb{E}\|x(\varrho_n)\|^2\leqslant\frac{1}{\beta}\Big[\mathbb{E}V_{r(\varrho_n)}(x(\varrho_n))-\mathbb{E}V_{r(\varrho_{n+1})}(x(\varrho_{n+1}))\Big]. \tag{10}$$

It follows from (10) that for $\forall n\in\mathbb{N}$,

$$\sum_{\iota=0}^{n}\mathbb{E}\|x(\varrho_\iota)\|^2\leqslant\frac{1}{\beta}\Big[\mathbb{E}V_{r(0)}(x(0))-\mathbb{E}V_{r(\varrho_{n+1})}(x(\varrho_{n+1}))\Big]\leqslant\frac{1}{\beta}\mathbb{E}V_{r(0)}(x(0))<+\infty, \tag{11}$$

which implies that

$$\lim_{n\to\infty}\mathbb{E}\|x(\varrho_n)\|^2=0. \tag{12}$$

Based on Lemma 1, sufficient conditions will be given to ensure that the unforced S-MJLS (1) is σ-MSS.

Theorem 1. If there exist positive constants κ_i, $\mathcal{T}_i$, and a set of positive definite matrices P_i such that for $\forall i\in\Gamma$, (H_1) and (H_2) hold, and

(H_3) $A_i'P_iA_i-\kappa_iP_i\prec 0$,

then unforced S-MJLS (1) is σ-MSS.

Proof. For any $i\in\Gamma$ and $n\in\mathbb{N}$, let the Lyapunov function

$$V_i(x(n))=x'(n)P_ix(n).$$

If $r(\varrho_n)=i\in\Gamma$, and $\mathcal{T}_i$ is the given upper bound of the sojourn-time for the ith mode, then for $\forall\bar{n}\in\mathbb{N}_{[\varrho_n+1,\varrho_{n+1}]}$, we have $\bar{n}-\varrho_n\leqslant\mathcal{T}_i$. Using ($\text{H}_3$), we have

$$\begin{aligned}V_{r(\varrho_n)}(x(\bar{n}))=x'(\bar{n})P_ix(\bar{n})=x'(\varrho_n)(A_i')^{\bar{n}-\varrho_n}P_iA_i^{\bar{n}-\varrho_n}x(\varrho_n)\leqslant\kappa_i^{\bar{n}-\varrho_n}V_{r(\varrho_n)}(x(\varrho_n))\\\leqslant\lambda_{\max}(P_i)(\bar{\kappa}_i)^{\mathcal{T}_i}\|x(\varrho_n)\|^2,\end{aligned} \tag{13}$$

where $\bar{\kappa}_i=\max\{\kappa_i,1\}$ and $r(\varrho_n)=i$. It follows that for $\bar{n}\in\mathbb{N}_{[\varrho_n+1,\varrho_{n+1}]}$,

$$V_{r(\varrho_n)}(x(\bar{n}))\leqslant c_1\|x(\varrho_n)\|^2,$$

where $c_1=\max_{i\in\Gamma}\{\lambda_{\max}(P_i)(\bar{\kappa}_i)^{\mathcal{T}_i}\}$. Consequently,

$$\mathbb{E}V_{r(\varrho_n)}(x(\bar{n}))\leqslant c_1\mathbb{E}\|x(\varrho_n)\|^2. \tag{14}$$

In addition, it can be verified that

$$c_2\mathbb{E}\|x(\bar{n})\|^2\leqslant\mathbb{E}V_{r(\varrho_n)}(x(\bar{n})),$$

where $c_2=\min_{i\in\Gamma}\{\lambda_{\min}(P_i)\}$. Thus, for $\forall\bar{n}\in\mathbb{N}$, there is a corresponding n such that $\bar{n}\in\mathbb{N}_{[\varrho_n+1,\varrho_{n+1}]}$ and

$$\mathbb{E}\|x(\bar{n})\|^2\leqslant\frac{c_1}{c_2}\mathbb{E}\|x(\varrho_n)\|^2. \tag{15}$$

Combining Lemma 1 with (15) yields that

$$\lim_{\bar{n}\to\infty}\mathbb{E}\|x(\bar{n})\|^2|_{x_0,r_0,\eta_nI_{(\zeta_n=i)}\leqslant\mathcal{T}_i}=0.$$

It follows that unforced S-MJLS (1) is σ-MSS.

Remark 4. Conditions (H_1)–(H_3) in Theorem 1 are commonly used to study the stability of discrete-time Markov/semi-Markov jump linear systems. Condition (H_1) is a sufficient and necessary condition to obtain the mean square stability (MSS) for unforced system (1) by assuming $\{r(n)\}_{n\in\mathbb{N}}$ is a Markov chain [5]. Condition (H_2) is similar to some assumptions, which are necessary to handle the stability for discrete-time S-MJLS with a semi-Markov kernel approach, for example, assumption (16) in [40] and assumption (31) in [43]. Condition (H_3) establishes the relationship between $V_{r(\varrho_n)}(x(\bar{n}))$ and $V_{r(\varrho_n)}(x(\varrho_n))$ for any $\bar{n} \in \mathbb{N}_{[\varrho_n,\varrho_{n+1}]}$, which was also utilized in [39, 40, 43].

Our next result provides a different set of conditions to ensure that the unforced S-MJLS (1) is σ-MSS.

Theorem 2. If there exist constants $\kappa_i > 0, \mathcal{T}_i > 0$, $\mu_i > -1$, and a set of positive definite matrices P_i such that for $\forall i \in \Gamma$, (H_2) and (H_3) in Theorem 1 hold, and

(H_4) $A_i'\mathcal{P}^i A_i - (1+\mu_i)P_i \prec 0$,

(H_5) $\max_{i\in\Gamma}\sum_{j\in\Gamma}(1+\mu_i)\theta_{ij} < 1$,

where $\mathcal{P}^i = \sum_{j\in\Gamma}\theta_{ij}P_j$, then unforced S-MJLS (1) is σ-MSS.

Proof. For $\forall i \in \Gamma$ and $n \in \mathbb{N}$, let the Lyapunov function

$$V_i(x(n)) = x'(n)P_i x(n).$$

Set $\mathcal{F}_n = \boldsymbol{\sigma}(\{x(\varrho_k)\}_{k\in\mathbb{N}_{[0,n]}}, \{r(\varrho_k)\}_{k\in\mathbb{N}_{[0,n]}})$ with $n \in \mathbb{N}$, which is a $\boldsymbol{\sigma}$-algebra generated by $\{x(\varrho_k)\}_{k\in\mathbb{N}_{[0,n]}}$ and $\{r(\varrho_k)\}_{k\in\mathbb{N}_{[0,n]}}$. It can be checked that

$$\mathbb{E}[V_{r(\varrho_{n+1})}(x(\varrho_{n+1}))|\mathcal{F}_n] = \mathbb{E}[V_{r(\varrho_{n+1})}(x(\varrho_{n+1}))|x(\varrho_n), r(\varrho_n)]. \tag{16}$$

If $r(\varrho_n) = i$, then using (8) in the proof of Lemma 1 and (H_4), we have

$$\mathbb{E}[V_{r(\varrho_{n+1})}(x(\varrho_{n+1}))|x(\varrho_n), r(\varrho_n) = i] \leqslant x'(\varrho_n)A_i'\mathcal{P}^i A_i x(\varrho_n) \leqslant (1+\mu_i)V_i(x(\varrho_n)),\ n \in \mathbb{N}.$$

It follows that for $\forall n \in \mathbb{N}$,

$$\mathbb{E}[V_{r(\varrho_{n+1})}(x(\varrho_{n+1}))|x(\varrho_n), r(\varrho_n)] \leqslant (1+\mu_{r(\varrho_n)})V_{r(\varrho_n)}(x(\varrho_n)). \tag{17}$$

In view of (16) and (17), for $\forall n \in \mathbb{N}$, one has

$$\mathbb{E}[V_{r(\varrho_{n+1})}(x(\varrho_{n+1}))|\mathcal{F}_n] \leqslant (1+\mu_{r(\varrho_n)})V_{r(\varrho_n)}(x(\varrho_n)). \tag{18}$$

By utilizing the definition of $\mathcal{F}_n$ and (18), we derive that

$$\begin{aligned}\mathbb{E}[V_{r(\varrho_{n+1})}(x(\varrho_{n+1}))|\mathcal{F}_{n-1}] &= \mathbb{E}[\mathbb{E}[V_{r(\varrho_{n+1})}(x(\varrho_{n+1}))|\mathcal{F}_n]|\mathcal{F}_{n-1}] \\ &\leqslant (1+\mu_{r(\varrho_n)})\mathbb{E}[V_{r(\varrho_n)}(x(\varrho_n))|\mathcal{F}_{n-1}] \\ &\leqslant \prod_{k=n-1}^{n}(1+\mu_{r(\varrho_k)})V_{r(\varrho_{n-1})}(x(\varrho_{n-1})).\end{aligned}$$

Repeating this procedure yields that

$$\mathbb{E}[V_{r(\varrho_{n+1})}(x(\varrho_{n+1}))|\mathcal{F}_0] \leqslant V_{r_0}(x_0)\prod_{k=0}^{n}(1+\mu_{r(\varrho_k)}).$$

Thus, we obtain that

$$\mathbb{E}[V_{r(\varrho_{n+1})}(x(\varrho_{n+1}))] \leqslant (1+\mu_{r_0})V_{r_0}(x_0)\mathbb{E}\left[\prod_{k=1}^{n}(1+\mu_{r(\varrho_k)})\right]. \tag{19}$$

It can be verified that

$$\begin{aligned}\mathbb{E}\left[\prod_{k=1}^{n}(1+\mu_{r(\varrho_k)})\right] &= \sum_{i_1,\ldots,i_n}\prod_{k=1}^{n}(1+\mu_{i_k})\mathbb{P}(r(\varrho_1) = i_1, \ldots, r(\varrho_n) = i_n) \\ &= \sum_{i_1,\ldots,i_n}\prod_{k=1}^{n}(1+\mu_{i_k})\theta_{i_{k-1}i_k} = \prod_{k=1}^{n}\sum_{i_k}(1+\mu_{i_k})\theta_{i_{k-1}i_k}\end{aligned}$$

$$\leqslant \varpi^n, \tag{20}$$

where $\varpi = \max_{i\in\Gamma}\sum_{j\in\Gamma}(1+\mu_i)\theta_{ij}$ and $i_0 = r_0$. Combining (19) and (20) with (H_5) yields that

$$\lim_{n\to\infty} \mathbb{E}[V_{r(\varrho_n)}(x(\varrho_n))] = 0. \tag{21}$$

Using (H_3), we can obtain (15) as in the proof of Theorem 1. Thereafter, for the given $\mathcal{T}_i$, in view of (21) and (15), one has

$$\lim_{n\to\infty} \mathbb{E}\|x(n)\|^2|_{x_0,r_0,\eta_n I_{(\zeta_n=i)}\leqslant\mathcal{T}_i} = 0.$$

It follows that unforced S-MJLS (1) is σ-MSS.

Remark 5. It can be verified that (H_4) in Theorem 2 is an extension of (H_1) in Theorem 1, where (H_4) becomes (H_1) by letting $\mu_i \equiv 0$. The σ-MSS was presented in [43] for S-MJLS using the semi-Markov kernel approach. Compared with Theorem 3.3 in [43], a novel analysis method is introduced in Theorem 2 to study the σ-MSS for S-MJLSs, and there are no specific requirements for the information of S-TPMFs and the given upper bound of sojourn-time.

Remark 6. It is known that (H_1) is a sufficient and necessary condition to obtain the MSS for discrete-time Markov jump linear systems (see [5]), where the geometric distribution information is not assumed. In this paper, σ-MSS is investigated for S-MJLSs, where the sojourn-time information is also not required. Compared with the classical result in [5], this paper has the following two contributions:

(1) Markov chain is extended to semi-Markov chain. In addition, (H_1) is further generalized to (H_4) in Theorem 2, where σ-MSS is studied for S-MJLSs under the general condition (H_4);

(2) A novel analysis technique is developed in this study, which combines the Lyapunov function analysis method with analysis technique of Markov chains.

Remark 7. Numerous important results have been reported for the stability analysis and control problem for S-MJLS in [27, 28, 39–43] by using the semi-Markov kernel approach, and the stability criteria in [39, 40, 42, 43, 45] depend on the given upper bound of sojourn-time. However, the S-TPMFs information is difficult to obtain in practice. This is because S-TPMFs are usually deduced from the statistical characteristics according to the sampled-data, while adequate samples are often costly and time consuming. In addition, the stability results depend on the upper bound of sojourn-time, which seriously affects their application [44]. Thus, in this study, σ-MSS was considered without using the S-TPMFs information. The main contributions of Theorems 1 and 2 lie in the following two parts:

(1) Under some widely used assumptions, the stability criteria are provided without using any S-TPMFs information, which means that the results in Theorems 1 and 2, are independent of the specific form for S-TPMFs.

(2) The exactly truncated upper bound of the sojourn-time is not embodied in the stability criteria.

3.2 Controller design for closed-loop systems

Based on Theorems 1 and 2, the sufficient conditions of stabilizing controller are presented for S-MJLS (1) in this subsection. In this study, the mode-dependent controller is assumed to have the following form:

$$u(n) = K_{r(n)}x(n), \tag{22}$$

where $r(n) = i \in \Gamma$, $n \in \mathbb{N}$ and K_i is the control gain to be determined. Therefore, we obtain the following closed-loop S-MJLS:

$$x(n+1) = \bar{A}_{r(n)}x(n), \tag{23}$$

where $\bar{A}_{r(n)} = A_{r(n)} + B_{r(n)}K_{r(n)}$ and $n \in \mathbb{N}$. In the following, effective state-feedback gains are designed for the closed-loop system (23) with the help of Theorems 1 and 2, respectively.

Theorem 3. Assuming that there exist constants $\kappa_i \in \mathbb{R}^+$ and matrices L_i, Q_i, U_i for $i \in \Gamma$ such that the following inequalities hold:

$$\begin{bmatrix} L_i - Q_i' - Q_i & A_iQ_i + B_iU_i \\ * & -\kappa_i L_i \end{bmatrix} \prec 0, \tag{24}$$

$$\begin{bmatrix} \Upsilon_i & (\mathcal{A}_i\mathcal{Q}_i + \mathcal{B}_i\mathcal{U}_i)R_i \\ * & L_i \end{bmatrix} \prec 0, \tag{25}$$

$$\begin{bmatrix} \Upsilon_i & (\mathcal{A}_i\mathcal{Q}_i + \mathcal{B}_i\mathcal{U}_i)R_i \\ * & \bar{L}_i \end{bmatrix} \prec 0, \tag{26}$$

where

$$\Upsilon_i = \mathcal{L} - \mathcal{Q}_i - \mathcal{Q}_i',\ \mathcal{Q}_i = \mathrm{diag}_{(\vartheta)}\{Q_i\},\ \mathcal{L} = \mathrm{diag}\{L_1, L_2, \ldots, L_\vartheta\},$$
$$\bar{L}_i = \sum_{j\in\Gamma}\theta_{ij}L_j,\ R_i = (\sqrt{\theta_{i1}}I, \sqrt{\theta_{i2}}I, \ldots, \sqrt{\theta_{i\vartheta}}I)',\ \mathcal{A}_i = \mathrm{diag}_{(\vartheta)}\{A_i\},$$

$\mathcal{B}_i = \mathrm{diag}_{(\vartheta)}\{B_i\}$, and $\mathcal{U}_i = \mathrm{diag}_{(\vartheta)}\{U_i\}$. Then, the closed-loop system (23) is σ-MSS with state-feedback gain $K_i = U_iQ_i^{-1}$ for $\forall i \in \Gamma$.

Proof. In this proof, it will be shown that conditions (24)–(26) can imply (H_1)–(H_3) in Theorem 1 for the closed-loop system (23). If we set

$$Y_i = Q_i^{-1},\ P_i = (Q_i')^{-1}L_iQ_i^{-1},\ K_i = U_iQ_i^{-1},$$

then it can be deduced from (24) that for $\forall i \in \Gamma$,

$$\begin{bmatrix} (Y_i')^{-1}P_iY_i^{-1} - (Y_i')^{-1} - Y_i^{-1} & A_iY_i^{-1} + B_iK_iY_i^{-1} \\ * & -\kappa_i(Y_i')^{-1}P_iY_i^{-1} \end{bmatrix} \prec 0. \tag{27}$$

Taking a congruence transformation to (27) with $\mathrm{diag}\{Y_i, Y_i\}$, it follows that

$$\begin{bmatrix} P_i - Y_i - Y_i' & Y_i'(A_i + B_iK_i) \\ * & -\kappa_iP_i \end{bmatrix} \prec 0. \tag{28}$$

In view of $(P_i - Y_i)'P_i^{-1}(P_i - Y_i) \succ 0$, we have $-Y_i'P_i^{-1}Y_i \prec P_i - Y_i - Y_i'$. Thereafter, it can be derived from (28) that

$$\begin{bmatrix} -Y_i'P_i^{-1}Y_i & Y_i'(A_i + B_iK_i) \\ * & -\kappa_iP_i \end{bmatrix} \prec 0. \tag{29}$$

By taking a congruence transformation to (29) with $\mathrm{diag}\{Y_i^{-1}P_i, I\}$, we obtain

$$\begin{bmatrix} -P_i & P_i(A_i + B_iK_i) \\ * & -\kappa_iP_i \end{bmatrix} \prec 0. \tag{30}$$

Combining (30) with Schur complement follows (H_3).

Let

$$\mathcal{Y} = \mathrm{diag}\{Y_1, Y_2, \ldots, Y_\vartheta\},\ \Xi = \mathrm{diag}\{P_1, P_2, \ldots, P_\vartheta\},\ \mathcal{K}_i = \mathcal{U}_i\mathcal{Q}^{-1}.$$

It follows that $\mathcal{Y} = \mathcal{Q}^{-1}$ and $\Xi = (\mathcal{Q}')^{-1}\mathcal{L}\mathcal{Q}^{-1}$. Thereafter, it can be verified from (25) that

$$\begin{bmatrix} \Psi & (\mathcal{A}_i + \mathcal{B}_i\mathcal{K}_i)\mathcal{Q}_iR_i \\ * & -L_i \end{bmatrix} \prec 0, \tag{31}$$

where $\Psi = (\mathcal{Y}')^{-1}\Xi\mathcal{Y}^{-1} - (\mathcal{Y}')^{-1} - \mathcal{Y}^{-1}$. Applying the congruence transformation to (31) with $\mathrm{diag}\{\mathcal{Y}, Y_i\}$, we obtain

$$\begin{bmatrix} \Xi - \mathcal{Y} - \mathcal{Y}' & \mathcal{Y}'(\mathcal{A}_i + \mathcal{B}_i\mathcal{K}_i)R_i \\ * & -Y_i'L_iY_i \end{bmatrix} \prec 0.$$

Since $(\Xi-\mathcal{Y})'\Xi^{-1}(\Xi-\mathcal{Y})\succ 0$, it follows that $-\mathcal{Y}\Xi_i^{-1}\Xi-\mathcal{Y}\prec\Xi-\mathcal{Y}-\mathcal{Y}'$. Thereafter,

$$\begin{bmatrix}-\mathcal{Y}'\Xi^{-1}\mathcal{Y} & \mathcal{Y}'(\mathcal{A}_i+\mathcal{B}_i\mathcal{K}_i)R_i\\ * & -Y_i'L_iY_i\end{bmatrix}\prec 0. \tag{32}$$

Utilizing the congruence transformation to (32) with $\mathrm{diag}\{\mathcal{Y}^{-1}\Xi, I\}$, we obtain

$$\begin{bmatrix}-\Xi & \Xi(\mathcal{A}_i+\mathcal{B}_i\mathcal{K}_i)R_i\\ * & -(Q_i')^{-1}L_iQ_i\end{bmatrix}\prec 0. \tag{33}$$

Noticing that Schur complement and (33), we obtain

$$R_i'(A_i+B_iK_i)'\Xi(A_i+B_iK_i)R_i-P_i\prec 0, \tag{34}$$

where $P_i=(Q_i')^{-1}L_iQ_i$. It can be verified that Eq. (34) is equivalent to (H_1). Similarly, it can be verified that Eq. (26) yields (H_2). Thus, in view of Theorem 1, the closed-loop system (23) is σ-MSS with a designed state-feedback gain $K_i=U_iQ_i^{-1}$ for $\forall i\in\Gamma$.

The last result in this study deals with the stabilization of the closed system (23) by utilizing Theorem 2.

Theorem 4. Assuming that (H_5) in Theorem 2 holds, and there exist constants $\kappa_i\in\mathbb{R}^+$, $\mu_i>-1$, and matrices L_i,Q_i,U_i for $i\in\Gamma$ such that the following inequalities hold:

$$\begin{bmatrix}L_i-Q_i'-Q_i & A_iQ_i+B_iU_i\\ * & -\kappa_iL_i\end{bmatrix}\prec 0, \tag{35}$$

$$\begin{bmatrix}\Upsilon_i & (\mathcal{A}_i\mathcal{Q}_i+\mathcal{B}_i\mathcal{U}_i)R_i\\ * & (1+\mu_i)L_i\end{bmatrix}\prec 0, \tag{36}$$

$$\begin{bmatrix}\Upsilon_i & (\mathcal{A}_i\mathcal{Q}_i+\mathcal{B}_i\mathcal{U}_i)R_i\\ * & \bar{L}_i\end{bmatrix}\prec 0, \tag{37}$$

where

$$\Upsilon_i=\mathcal{L}-\mathcal{Q}_i-\mathcal{Q}_i',\ \mathcal{Q}_i=\mathrm{diag}_{(\vartheta)}\{Q_i\},\ \mathcal{L}=\mathrm{diag}\{L_1,L_2,\ldots,L_\vartheta\},$$
$$\bar{L}_i=\sum_{j\in\Gamma}\theta_{ij}L_j,\ R_i=(\sqrt{\theta_{i1}}I,\sqrt{\theta_{i2}}I,\ldots,\sqrt{\theta_{i\vartheta}}I)',\ \mathcal{A}_i=\mathrm{diag}_{(\vartheta)}\{A_i\},$$

$\mathcal{B}_i=\mathrm{diag}_{(\vartheta)}\{B_i\}$, and $\mathcal{U}_i=\mathrm{diag}_{(\vartheta)}\{U_i\}$. Then, the closed-loop system (23) is σ-MSS with state-feedback gain $K_i=U_iQ_i^{-1}$ for $\forall i\in\Gamma$.

Proof. This analysis can be carried out along the same line as in the proof of Theorem 3.

4 Examples

Two numerical examples are presented in this section to validate the effectiveness of the obtained theoretical results.

Example 1. Assume that S-MJLS (1) has three subsystems, that is, $\Gamma=\{1,2,3\}$. The matrices in system (1) are defined as in [40] with

$$A_1=\begin{bmatrix}1.55 & -0.85\\ 0.50 & 0.15\end{bmatrix},\ A_2=\begin{bmatrix}5.41 & -4.73\\ 0.1 & 0.1\end{bmatrix},\ A_3=\begin{bmatrix}0.27 & 0.50\\ -1.31 & 2.03\end{bmatrix},$$
$$B_1=\begin{bmatrix}-0.1\\ 0.1\end{bmatrix},\ B_2=\begin{bmatrix}0.1\\ 1.1\end{bmatrix},\ B_3=\begin{bmatrix}0.1\\ -0.1\end{bmatrix}.$$

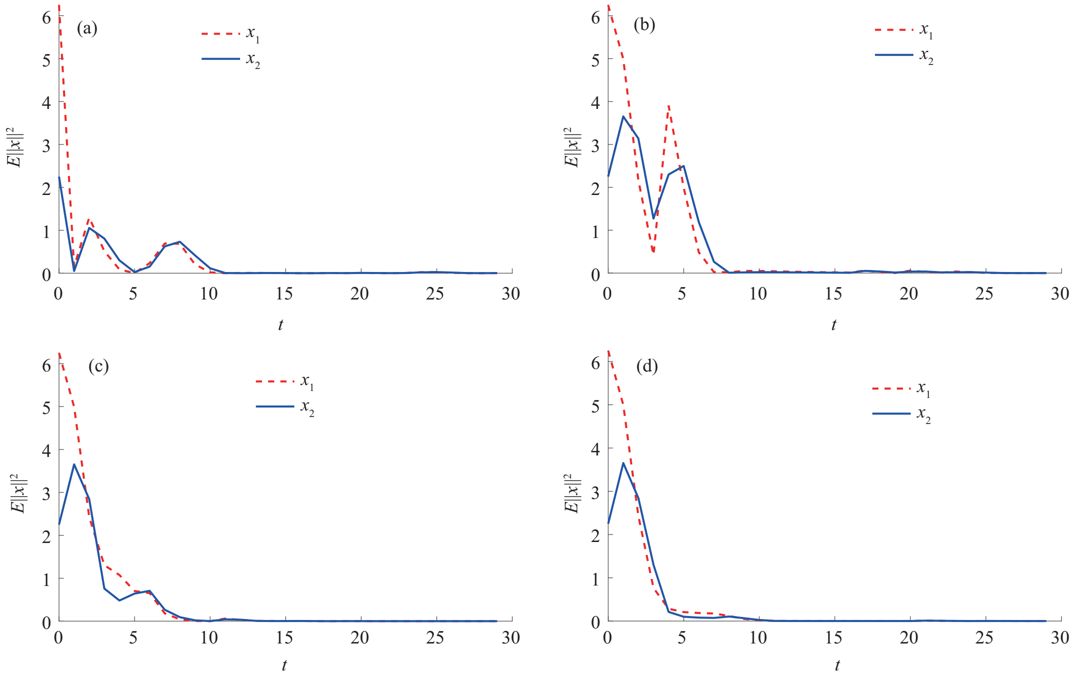

Figure 1 (Color online) (a) Average of 100 simulation results of state trajectories $x(t)$ for system (1) in Example 1, where the S-TPMF satisfying Case 1 and the controller is designed using Theorem 3; (b) state trajectories $x(t)$ for system (1) in Example 1, where the S-TPMF satisfying Case 2 and the controller is designed using Theorem 3; (c) state trajectories $x(t)$ for system (1) in Example 1, where the S-TPMF satisfying Case 1 and the controller is designed using Theorem 4; (d) state trajectories $x(t)$ for system (1) in Example 1, where the S-TPMF satisfying Case 2 and the controller is designed using Theorem 4.

The switching among these three modes is governed by a semi-Markov chain $\{r(n)\}_{n\in\mathbb{N}}$. Assume that this semi-Markov chain has a probability transition matrix

$$\Theta = \begin{bmatrix} 0 & 0.2262 & 0.7738 \\ 0.0912 & 0 & 0.9088 \\ 0.2463 & 0.7537 & 0 \end{bmatrix},$$

and S-TPMFs were unavailable. By solving the linear matrix inequality in Theorem 3, we can obtain the desired controller gains $K_1 = [2.1541, -2.3942]$, $K_2 = [4.5303, -3.9728]$, and $K_3 = [-3.5767, 2.2590]$.

To demonstrate the validity of the results obtained in this study through simulation diagrams. Two different S-TPMFs cases are listed as follows:

Case 1. $F_1 \sim \underline{\mathrm{Ge}}(0.4)$, $F_2 \sim \underline{\mathrm{P}}(2)$, and $F_3 \sim \underline{\mathrm{b}}(3, 0.2)$.

Case 2. $F_1 \sim \underline{\mathrm{P}}(3)$, $F_2 \sim \underline{\mathrm{Ge}}(0.4)$, and $F_3 \sim \underline{\mathrm{b}}(5, 0.4)$.

Thus, Assumption 1 is true for both Cases 1 and 2.

It is indicated in Figures 1(a) and (b) that S-MJLS (1) can be stabilized with the controller designed in Theorem 3, where the S-TPMFs are assumed to satisfy Cases 1 and 2, respectively. Thus, we find that the stability results in this study are independent of the specific form for S-TPMF.

Additionally, let $\mu_1 = -0.9$, $\mu_2 = -0.2$, and $\mu_3 = 0$ in Theorem 4. Using Theorem 4, we obtain the desired controller gains $K_1 = [2.2230, -3.0042]$, $K_2 = [4.5303, -3.9728]$, and $K_3 = [-3.3970, 2.4982]$. Figures 1(c) and (d) show that S-MJLS (1) can be stabilized with the controller designed in Theorem 4, where the S-TPMFs are assumed to satisfy Cases 1 and 2, respectively.

In Example 1 of [39], S-MJLS (1) with a given S-TPMF for sojourn-time is presented as the σ-MSS. The results in [39] are dependent on the sojourn-time S-TPMFs, which are rarely used to design a controller for S-MJLS (1) without the specific S-TPMFs information. In this study, the controller is designed for S-MJLS (1) in [39], where only the probability transition matrix is assumed for semi-Markov chains. Figure 1 shows that the designed controller is effective for different S-TPMFs.

Example 2. In this example, a discrete-time electronic throttle control system is investigated as in [45], which can be expressed in the form of system (1). Let $\chi(n)$, $\psi(n)$, and $\omega(n)$ denote the angular position of the valve, angular velocity of the valve, and electrical current consumed by the internal motor of the

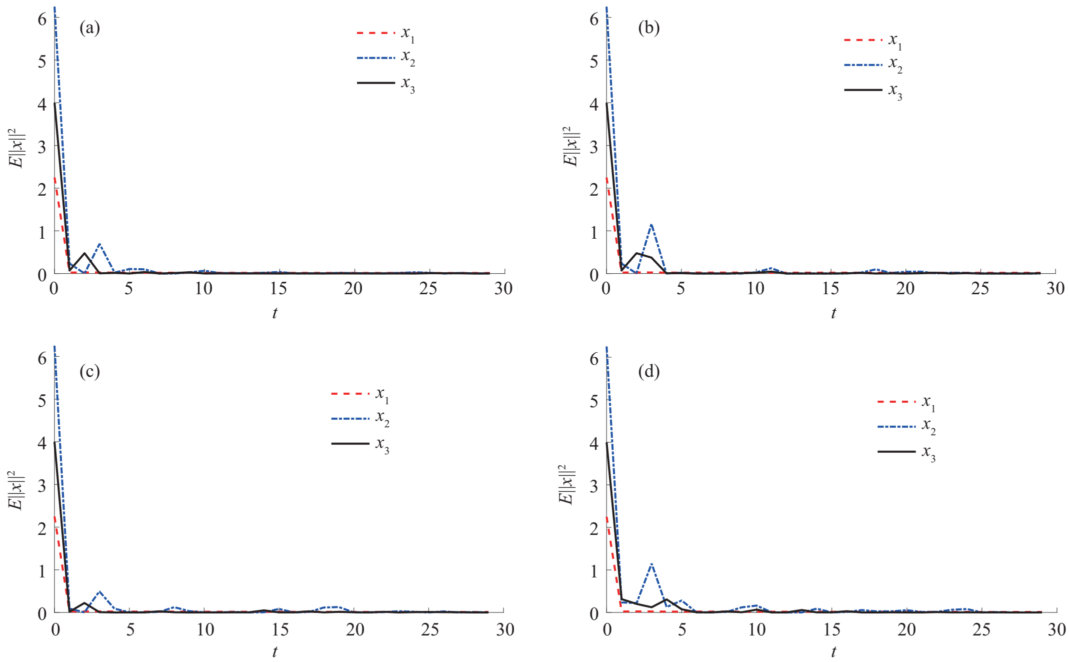

Figure 2 (Color online) (a) Average of 100 simulation results of state trajectories $x(t)$ for system (1) in Example 2, where the S-TPMF satisfying Case 1 and the controller is designed using Theorem 3; (b) state trajectories $x(t)$ for system (1) in Example 2, where the S-TPMF satisfying Case 2 and the controller is designed using Theorem 3; (c) state trajectories $x(t)$ for system (1) in Example 2, where the S-TPMF satisfying Case 1 and the controller is designed using Theorem 4; (d) state trajectories $x(t)$ for system (1) in Example 2, where the S-TPMF satisfying Case 2 and the controller is designed using Theorem 4.

throttle [2], respectively. Thereafter, the system state vector is defined as $x(n) = [\chi(n), \psi(n), \omega(n)]'$. Assuming that the power amplifier has three operation modes: normal, soft failure, and hard failure, which correspond to $r(n) = 1, 2$, and 3, respectively. For these three operation modes, the system parameters were defined as follows:

$$A_1 = \begin{bmatrix} 1 & 0.0109 & 0 \\ -0.1165 & 0.8072 & -1.5061 \\ 0 & -0.2285 & 0.7967 \end{bmatrix}, \; A_2 = \begin{bmatrix} 1 & 0.0152 & 0 \\ -0.0298 & 0.2737 & -1.1921 \\ 0 & -0.3584 & 0.3835 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} 1 & 0.0111 & 0 \\ -0.0229 & 0.7779 & -0.1899 \\ 0 & -0.6315 & 0.4178 \end{bmatrix}, \; B_1 = \begin{bmatrix} 0 \\ 0 \\ -0.0948 \end{bmatrix}, \; B_2 = \begin{bmatrix} 0 \\ 0 \\ -0.0735 \end{bmatrix}, \; B_3 = \begin{bmatrix} 0 \\ 0 \\ -0.2441 \end{bmatrix}.$$

Assuming that the switching among the three modes satisfies a semi-Markov chain $\{r(n)\}_{n\in\mathbb{N}}$, and it has a probability transition matrix:

$$\Theta = \begin{bmatrix} 0 & 0.6 & 0.4 \\ 0.3 & 0 & 0.7 \\ 0.4 & 0.6 & 0 \end{bmatrix},$$

and S-TPMFs were unavailable. Thereafter, the following desired controller gains are derived by utilizing Theorem 3:

$$K_1 = [0.1679, -8.3648, 17.4839], \; K_2 = [-4.2535, -3.8456, 7.5934], \; K_3 = [-3.1437, -4.8057, 1.5019].$$

In Figures 2(a) and (b), the electronic throttle is stable with the designed controller, where the S-TPMFs are assumed to satisfy Cases 1 and 2 in Example 1, respectively.

Additionally, let $\mu_1 = -0.9$, $\mu_2 = -0.2$, and $\mu_3 = 0$. Using Theorem 4, the desired controller gains are derived as follows:

$$K_1 = [0.1679, -8.3648, 17.4839], \; K_2 = [-4.0598, -5.4513, 10.2426], \; K_3 = [-6.8307, -4.6026, 5.0295].$$

Figures 2(c) and (d) show that the electronic throttle is stable with the designed controller, where the S-TPMFs are assumed to satisfy Cases 1 and 2 in Example 1, respectively. In addition, according to the values of control gains designed in [49] for the actual experimental platform, the corresponding values in Example 2 are within a reasonable range.

5 Conclusion

In this study, the stability problem has been investigated for discrete-time S-MJLSs with unavailable sojourn-time information. A novel analysis method has been developed to study the σ-error mean square stability for discrete-time S-MJLSs, and a set of mode-dependent controllers has been also designed for closed-loop systems, where only the probability transition matrix is used for the semi-Markov chain. Two numerical examples are provided to illustrate the validity of the obtained theoretical results.

In this paper, the stability and stabilization problems have been studied for discrete-time S-MJLSs with unavailable sojourn-time information. In the future, this idea can be expanded in other researches, such as the stability and stabilization of discrete-time (or continuous-time) semi-Markov jump nonlinear systems, and the stability and stabilization of discrete-time hidden semi-Markov jump nonlinear systems.

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