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Global optimization of double protograph LDPC codes for JSCC scheme

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A double low-density parity-check (D-LDPC) coding system was proposed in [1] as a typical joint source-channel coding (JSCC) scheme [1–5]. In this system, two LDPC coding matrices perform source and channel coding, respectively. An important concept in a D-LDPC coding system is the introduction of a linking matrix between the check nodes (CNs) of the source coding matrix and the variable nodes (VNs) of the channel LDPC coding matrix. The linking matrix sets up a fundamental connection and performs the exchange of information between source decoding and channel decoding.

Many studies have been conducted on the optimization of a D-LDPC coding system (a detailed review can be found in Appendix A). These studies focused on two aspects: partial optimization and global optimization. Several global optimization studies have been conducted, such as the joint component design, which is based on a multiobjective optimization differential evolution [3], and the optimal allocation of degree-2 VNs [4]. These global optimization studies are all based on the condition that the linking matrix is an identity matrix. Recently, a nonidentity linking matrix was introduced into the D-LDPC coding system to improve its performance in the waterfall region [5]. However, the study only focused on the optimization of the nonidentity linking matrix when the extending matrix is a zero matrix, which is a kind of partial optimization. In addition, it was only considered that the extending matrix is a zero matrix. In this study, we propose a global optimization algorithm for double-protograph LDPC (DP-LDPC) codes with a nonzero extending matrix and a nonidentity linking matrix. The main contributions of our study are summarized as follows. (i) General encoding and decoding methods for D-LDPC codes are proposed; (ii) the characteristics of the structure for the DP-LDPC codes are analyzed, and new design strategies and guidelines are presented; (iii) an iterative code searching method for optimizing several DP-LDPC codes is proposed.

System model. A protograph can be represented by a base matrix $\boldsymbol{B} = [b^{ij}]$, where $b^{ij} \in \mathbb{N}$ represents the edges connecting the i-th CN with the j-th VN. In a DP-LDPC coding system, an $(m_{\mathrm{S}} + m_{\mathrm{C}}) \times (n_{\mathrm{S}} + n_{\mathrm{C}})$ joint protograph $\boldsymbol{B}_{\mathrm{J}}$ matrix can be defined as

$$\boldsymbol{B}_{\mathrm{J}} = \left[\begin{array}{c:c} \boldsymbol{B}_{\mathrm{S}} & \boldsymbol{B}_{\mathrm{L}} \\ \hdashline \boldsymbol{B}_{\mathrm{E}} & \boldsymbol{B}_{\mathrm{C}} \end{array}\right], \tag{1}$$

where $\boldsymbol{B}_{\mathrm{S}}$ is the source component, $\boldsymbol{B}_{\mathrm{C}}$ is the channel component, $\boldsymbol{B}_{\mathrm{L}} = [\boldsymbol{0}\ \boldsymbol{B}_{\mathrm{L}}^*]$ is the linking component, and $\boldsymbol{B}_{\mathrm{E}}$ is the extending component. The corresponding $(M_{\mathrm{S}} + M_{\mathrm{C}}) \times (N_{\mathrm{S}} + N_{\mathrm{C}})$ joint parity-check matrix $\boldsymbol{H}_{\mathrm{J}}$ (including $\boldsymbol{H}_{\mathrm{S}}$, $\boldsymbol{H}_{\mathrm{C}}$, $\boldsymbol{H}_{\mathrm{L}}$, and $\boldsymbol{H}_{\mathrm{E}}$) can be obtained by performing "copy and permute" operations. The detailed system model can be found in Appendix B.

General DP-LDPC encoding and decoding systems. Let a $1 \times N_s$ source sequence $\boldsymbol{s}$ that obtains a value from a binary independent and identically distributed (i.i.d) Bernoulli source ($\xi_1 < 0.5$). For encoding, a generated matrix in a systematic form $\boldsymbol{G}_{\mathrm{S}} = [\boldsymbol{I}_{\mathrm{S}}\ (\boldsymbol{P}_{\mathrm{S}})^{\mathrm{T}}]$ is initially obtained from $[\boldsymbol{H}_{\mathrm{S}}\ \boldsymbol{H}_{\mathrm{L}}^*]$ by performing Gaussian elimination, where the $N_s \times N_s$ $\boldsymbol{I}_{\mathrm{S}}$ matrix is an identity matrix, the size of $\boldsymbol{P}_{\mathrm{S}}$ is $N_s \times M_s$, and $(\cdot)^{\mathrm{T}}$ is the matrix transpose operation. Thus, the compressed bits $\boldsymbol{b}$ can be obtained using

$$[\boldsymbol{s}\ \boldsymbol{b}] = \boldsymbol{s} \cdot \boldsymbol{G}_{\mathrm{S}} = \boldsymbol{s} \cdot [\boldsymbol{I}_{\mathrm{S}}\ (\boldsymbol{P}_{\mathrm{S}})^{\mathrm{T}}]. \tag{2}$$

If $\boldsymbol{H}_{\mathrm{L}}^*$ is an identity matrix, $\boldsymbol{P}_{\mathrm{S}} = \boldsymbol{H}_{\mathrm{S}}$. Then, the source sequence, which is connected using $\boldsymbol{H}_{\mathrm{E}}^*$ (denoted by $\boldsymbol{s}_p$), and the compressed bits $\boldsymbol{b}$ are combined into a new sequence $[\boldsymbol{s}_p\ \boldsymbol{b}]$, and a generated matrix $\boldsymbol{G}_{\mathrm{C}} = [\boldsymbol{I}_{\mathrm{C}}\ (\boldsymbol{P}_{\mathrm{C}})^{\mathrm{T}}]$ is obtained from $[\boldsymbol{H}_{\mathrm{E}}^*\ \boldsymbol{H}_{\mathrm{C}}]$, where $\boldsymbol{I}_{\mathrm{C}}$ is an $(M_{\mathrm{S}}+N_{\mathrm{E}}) \times (M_{\mathrm{S}}+N_{\mathrm{E}})$ identity matrix. $[\boldsymbol{s}_p\ \boldsymbol{b}]$ is encoded using $\boldsymbol{G}_{\mathrm{C}}$ as follows:

$$\boldsymbol{c} = [\boldsymbol{s}_p\ \boldsymbol{b}] \cdot \boldsymbol{G}_{\mathrm{C}} = [\boldsymbol{s}_p\ \boldsymbol{b}] \cdot [\boldsymbol{I}_{\mathrm{C}}\ (\boldsymbol{P}_{\mathrm{C}})^{\mathrm{T}}], \tag{3}$$

where $\boldsymbol{c} = [\boldsymbol{s}_p\ \boldsymbol{b}\ \boldsymbol{p}]$ and the $1 \times M_{\mathrm{C}}$ $\boldsymbol{p}$ is the parity bit sequence. Then, sequence $[\boldsymbol{b}\ \boldsymbol{p}]$ is sent to an additive white Gaussian noise channel.

We consider that the whole codeword $\boldsymbol{u} = [\boldsymbol{s}\ \boldsymbol{b}\ \boldsymbol{p}]$ satisfies $\boldsymbol{u}(\boldsymbol{H}_{\mathrm{J}})^{\mathrm{T}} = [\boldsymbol{0}]$. Thus, the corrupted sequence $\boldsymbol{y}$ at the receiver can be decoded using the joint belief propagation (JBP) algorithm, considering the joint Tanner graph as a whole. Details of the general encoding and decoding processes can be found in Appendix C.

Design strategies and guidelines. By combining different constructions of $\boldsymbol{B}_{\mathrm{S}}$, $\boldsymbol{B}_{\mathrm{C}}$, $\boldsymbol{B}_{\mathrm{E}}$, and $\boldsymbol{B}_{\mathrm{L}}$ and analyzing their decoding thresholds, the following design strategies are pro-

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posed. (i) The base matrices $\boldsymbol{B}_{\mathrm{S}}$ and $\boldsymbol{B}_{\mathrm{E}}$ are considered a whole, which is denoted by a source-extending coding base matrix $\boldsymbol{B}_{\mathrm{S\&E}}$ because it determines the source decoding threshold $(\xi_1)_{\mathrm{th}}$. (ii) The $\boldsymbol{B}_{\mathrm{C}}$ and $\boldsymbol{B}_{\mathrm{L}}$ matrices are considered a whole, which is denoted by a linking channel coding base matrix $\boldsymbol{B}_{\mathrm{L\&C}}$ because the mismatch between them generates a "bad" code. (iii) Now, the joint base matrix becomes

$$\boldsymbol{B}_{\mathrm{J}} = \left[\begin{array}{c:c} \boldsymbol{B}_{\mathrm{S}} & \boldsymbol{B}_{\mathrm{L}} \\ \boldsymbol{B}_{\mathrm{E}} & \boldsymbol{B}_{\mathrm{C}} \end{array}\right] = \left[\begin{array}{c:c} \boldsymbol{B}_{\mathrm{S\&E}} & \boldsymbol{B}_{\mathrm{L\&C}} \end{array}\right]. \tag{4}$$

(iv) The design of $\boldsymbol{B}_{\mathrm{J}}$ is converted into the design of $\boldsymbol{B}_{\mathrm{S\&E}}$ and $\boldsymbol{B}_{\mathrm{L\&C}}$, as well as the optimal match between them.

The detailed design principles are summarized as follows: (a) The number of degree-2 VNs in $\boldsymbol{B}_{\mathrm{J}}$ is at most $m_{\mathrm{S}} + m_{\mathrm{C}} - n_{\mathrm{punc}} - 1$. (b) The VN with the high degree exists in $\boldsymbol{B}_{\mathrm{L\&C}}$. (c) The degree-1 VNs exist in $\boldsymbol{B}_{\mathrm{L\&C}}$. (d) The left VNs except the pre-coder structure and degree-2 VNs in base matrix $\boldsymbol{B}_{\mathrm{J}}$ have the degree larger than 2. (e) The nonzero columns of $\boldsymbol{B}_{\mathrm{E}}$ in $\boldsymbol{B}_{\mathrm{S\&E}}$ must be set appropriately. Details of the design strategies and guidelines can be found in Appendix D.1.

Optimization method. We propose an iterative code searching algorithm between $\boldsymbol{B}_{\mathrm{S\&E}}$ and $\boldsymbol{B}_{\mathrm{L\&C}}$ based on the differential evolution (DE) algorithm. Compared with conventional code searching algorithms [3], which search the whole joint protograph based on the DE algorithm, the proposed iterative code searching algorithm exploits the characteristics of $\boldsymbol{B}_{\mathrm{S\&E}}$ and $\boldsymbol{B}_{\mathrm{L\&C}}$. As a result, the complexity of the proposed algorithm decreases significantly.

Consider the optimized cases in [3, 4], where the source statistic $\xi_1^{\mathrm{obj}} = 0.04$, $m_{\mathrm{S}} = 4$, and $n_{\mathrm{S}} = 2$ for the source base matrix, and $m_{\mathrm{C}} = 3$ and $n_{\mathrm{C}} = 5$ for the channel protograph. In addition, there are one degree-1 VN and one high-degree punctured VN in the channel component $\boldsymbol{B}_{\mathrm{L\&C}}$. The maximum number of degree-2 VNs can be calculated as 3. According to the allocation principles, these VNs should be assigned to the channel component $\boldsymbol{B}_{\mathrm{L\&C}}$. For the source component, there are no degree-2 VNs and one nonzero VN. Thus, they can be initialized as

$$\boldsymbol{B}_{\mathrm{L\&C}}^{\mathrm{ini}} = \begin{bmatrix} 0 & 0 & 0 & 1 & b_{\mathrm{lc}}^{15} \\ 0 & 0 & 0 & 0 & b_{\mathrm{lc}}^{25} \\ 1 & 0 & 0 & 0 & b_{\mathrm{lc}}^{35} \\ 0 & 1 & 1 & 0 & b_{\mathrm{lc}}^{45} \\ 0 & 1 & 1 & 1 & b_{\mathrm{lc}}^{55} \end{bmatrix} \text{ and } \boldsymbol{B}_{\mathrm{S\&E}}^{\mathrm{ini}} = \begin{bmatrix} b_{\mathrm{se}}^{11} & b_{\mathrm{se}}^{12} & b_{\mathrm{se}}^{13} & b_{\mathrm{se}}^{14} \\ b_{\mathrm{se}}^{21} & b_{\mathrm{se}}^{22} & b_{\mathrm{se}}^{23} & b_{\mathrm{se}}^{24} \\ b_{\mathrm{se}}^{31} & 0 & 0 & 0 \\ b_{\mathrm{se}}^{41} & 0 & 0 & 0 \\ b_{\mathrm{se}}^{51} & 0 & 0 & 0 \end{bmatrix}, \tag{5}$$

where $b_{\mathrm{lc}}^{15} + b_{\mathrm{lc}}^{25} + b_{\mathrm{lc}}^{35} + b_{\mathrm{lc}}^{45} + b_{\mathrm{lc}}^{55} > 2$, $b_{\mathrm{se}}^{11} + b_{\mathrm{se}}^{21} + b_{\mathrm{se}}^{31} + b_{\mathrm{se}}^{41} + b_{\mathrm{se}}^{51} > 2$ and $b_{\mathrm{se}}^{1j} + b_{\mathrm{se}}^{2j} > 2$ $(j = 2, 3, 4)$. To control the complexity, the maximum value of the elements is $b_{\max} = 3$. Then, Algorithm D1 in Appendix D is applied, and the optimized result can be obtained as follows:

$$\boldsymbol{B}_{\mathrm{opt}}^{0.04} = \left[\begin{array}{cccc:ccccc} 2 & 1 & 1 & 2 & 0 & 0 & 0 & 1 & 2 \\ 1 & 2 & 2 & 1 & 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 2 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{array}\right], \tag{6}$$

which has a source decoding threshold $(\xi_1)_{\mathrm{th}} = 0.135$ and a channel decoding $(E_b/N_0)_{\mathrm{th}} = -2.91$ dB for $\xi_1 = 0.04$. The proposed algorithm is also suitable for the case $\boldsymbol{B}_{\mathrm{E}} = 0$. The optimized result has better performance than the case of $\xi_{\mathrm{obj}} = 0.01$ in [5]. Design details can be found in Appendix D.

Simulation results and comparison. We compared $\boldsymbol{B}_{\mathrm{conv1}}^{0.04}$ [3] and $\boldsymbol{B}_{\mathrm{conv2}}^{0.04}$ [4]. The codelength of these DP-LDPC codes and the maximum number of iterations in the JBP decoding algorithm were set to 6400 and 100, respectively. The source model is a binary i.i.d Bernoulli source.

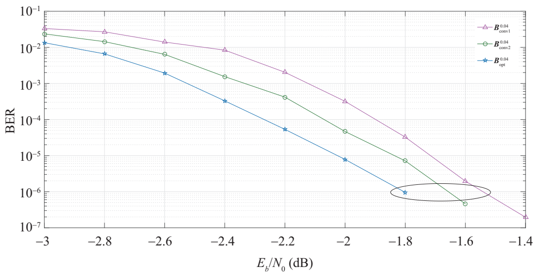

Figure 1 (Color online) BER performance of $\boldsymbol{B}_{\mathrm{conv1}}^{0.04}$, $\boldsymbol{B}_{\mathrm{conv2}}^{0.04}$, and $\boldsymbol{B}_{\mathrm{opt}}^{0.04}$ at $\xi_1 = 0.04$.

The bit error rate (BER) performance of $\boldsymbol{B}_{\mathrm{conv1}}^{0.04}$, $\boldsymbol{B}_{\mathrm{conv2}}^{0.04}$, and $\boldsymbol{B}_{\mathrm{opt}}^{0.04}$ is plotted in Figure 1. For BER $= 1 \times 10^{-6}$, $\boldsymbol{B}_{\mathrm{opt}}^{0.04}$ outperforms $\boldsymbol{B}_{\mathrm{conv1}}^{0.04}$ and $\boldsymbol{B}_{\mathrm{conv2}}^{0.04}$ by 0.25 and 0.15 dB, respectively, which is in line with the channel decoding threshold analysis. For the JBP decoding algorithm, the decoding complexity is dominated by the update calculation of the information from the CNs to VNs, which can be measured by the average degree $\overline{d_c}$ of a protograph. The decoding complexity of $\boldsymbol{B}_{\mathrm{opt}}^{0.04}$ decreases by 11.7% compared with that of $\boldsymbol{B}_{\mathrm{conv1}}^{0.04}$ and $\boldsymbol{B}_{\mathrm{conv2}}^{0.04}$. The simulation parameters and more simulation results can be found in Appendix E.

Therefore, it can be concluded that the optimized DP-LDPC codes have better performance and lower decoding complexity, compared with the conventional ones, which verifies the superiority of the proposed global optimization algorithm.

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Supporting information Appendixes A–E. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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