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• **Supplementary File** •

Distribution of polarization squeezed light through a 20 km fiber channel

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Appendix A The theoretical analysis

Polarization squeezed light plays a key role in quantum memory for transferring quantum information between atoms and photons [1, 2], and has potential applications in quantum precision measurement [3–6]. Polarization squeezed light can be generated by coupling a quadrature squeezed state and an orthogonal polarization strong coherent beam on a polarization beam splitter (PBS) [7, 8]. The polarization quantum state of light can be characterized by four Stokes operators on a Poincaré sphere, where $\hat{S}_0$ represents the intensity of light field, $\hat{S}_1$, $\hat{S}_2$ and $\hat{S}_3$ are horizontal, linear at 45°, and right-circular polarizations, respectively. Four Stokes operators are expressed by [7]

$$\begin{aligned}\hat{S}_0 &= \hat{a}_H^\dagger \hat{a}_H + \hat{a}_V^\dagger \hat{a}_V, \qquad \hat{S}_1 = \hat{a}_H^\dagger \hat{a}_H - \hat{a}_V^\dagger \hat{a}_V,\\ \hat{S}_2 &= \hat{a}_H^\dagger \hat{a}_V e^{i\theta} + \hat{a}_V^\dagger \hat{a}_H e^{-i\theta}, \quad \hat{S}_3 = i\hat{a}_V^\dagger \hat{a}_H e^{-i\theta} - i\hat{a}_H^\dagger \hat{a}_V e^{i\theta},\end{aligned} \tag{A1}$$

where $\hat{a}^\dagger$ and $\hat{a}$ are the creation and annihilation operators respectively, the subscripts of H and V represent horizontal and vertical polarization modes respectively, θ is the relative phase between these modes. The commutation relations of the Stokes operators are expressed as $[\hat{S}_0,\ \hat{S}_{i,j,k}] = 0$ and $[\hat{S}_i,\ \hat{S}_j] = 2i\hat{S}_k$, in which $i, j, k = 1, 2, 3$ are circularly interchangeable. Since the Stokes operators $\hat{S}_1$, $\hat{S}_2$, and $\hat{S}_3$ are non-commutable, it is not possible to measure them accurately simultaneously.

In the continuous-variable quantum information processing, the information is encoded in the amplitude and phase quadratures of photonics harmonic oscillators, which are defined by $\hat{x} = \hat{a} + \hat{a}^\dagger$ and $\hat{p} = i(\hat{a}^\dagger - \hat{a})$, respectively. The annihilation operator can be expressed as a linear operator $\hat{a} = \alpha + \delta\hat{a}$, where α and $\delta\hat{a}$ are the amplitude and the quantum noise operator [7]. For a vacuum state, the variances of amplitude and phase quadratures are $V_0^x = V_0^p = 1$, which corresponds to the shot noise level (SNL). The variances of four Stokes operators are given by [7]

$$\begin{aligned}V_{\hat{S}_0} &= V_{\hat{S}_1} = \frac{\alpha_H^2 V_{x_H} + \alpha_V^2 V_{x_V}}{\alpha_H^2 + \alpha_V^2},\\ V_{\hat{S}_2} &= cos^2\theta \frac{\alpha_V^2 V_{x_H} + \alpha_H^2 V_{x_V}}{\alpha_H^2 + \alpha_V^2} + sin^2\theta \frac{\alpha_V^2 V_{p_H} + \alpha_H^2 V_{p_V}}{\alpha_H^2 + \alpha_V^2},\\ V_{\hat{S}_3} &= sin^2\theta \frac{\alpha_V^2 V_{x_H} + \alpha_H^2 V_{x_V}}{\alpha_H^2 + \alpha_V^2} + cos^2\theta \frac{\alpha_V^2 V_{p_H} + \alpha_H^2 V_{p_V}}{\alpha_H^2 + \alpha_V^2},\end{aligned} \tag{A2}$$

respectively, where $V_x = \langle(\delta\hat{x})^2\rangle - \langle\delta\hat{x}\rangle^2$ and $V_p = \langle(\delta\hat{p})^2\rangle - \langle\delta\hat{p}\rangle^2$ are the variances of amplitude and phase quadratures respectively, and $\frac{1}{\alpha_H^2+\alpha_V^2}$ is the normalization factor. If the variance of a certain Stokes operator is lower than that of a coherent beam with equal power, this Stokes operator is said to be squeezed.

In our experiment, we prepare a polarization squeezed light and distribute it through a fiber channel. The inevitable loss and noise in the fiber channel lead to decoherence of the polarization squeezed light. For the polarization squeezed light, quantum variances of Stokes operators after the transmission through a fiber channel with attenuation of α are given by

$$\begin{aligned}V'_{\hat{S}_1} &= \eta V_{\hat{S}_1} + (1-\eta)(V_0 + W),\\ V'_{\hat{S}_2} &= \eta V_{\hat{S}_2} + (1-\eta)(V_0 + W),\\ V'_{\hat{S}_3} &= \eta V_{\hat{S}_3} + (1-\eta)(V_0 + W),\end{aligned} \tag{A3}$$

respectively, where $\eta = \eta_1 \times \eta_2$ is the total efficiency, including the transmission efficiency of $\eta_1 = 10^{-\alpha L/10}$ in the fiber channel with L kilometer and the fiber coupling efficiency of $\eta_2 = 94\%$, $V_0 = 1$ is the variance of vacuum noise, and W is the variance of excess noise. By measuring the variance of Stokes operators of the output mode, the decoherence of the polarization squeezed light can be characterized.

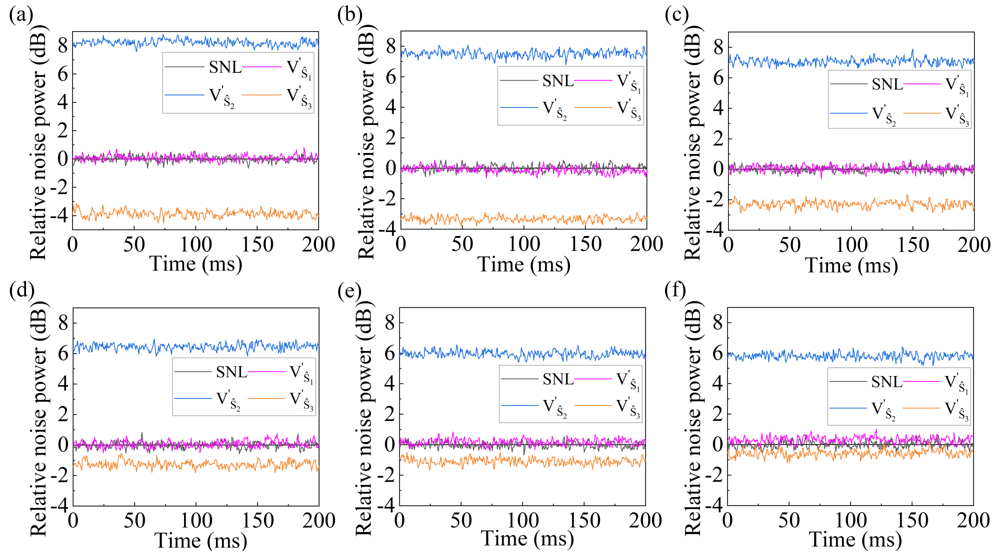

Figure B1 Relative noise powers of the output of polarization squeezed light at different transmission distances. (a-f) The measured noise of Stokes operators at transmission distances of 0.002, 1, 5, 10, 15, and 20 km, respectively. The measurement frequency is 6 MHz. The resolution bandwidth and video bandwidth of the spectrum analyzer are 100 kHz and 300 Hz, respectively.

Table B1 The measured quantum noise of Stokes parameters at different transmission distances.

Transmission distance (km)	Quantum noise of $\hat{S}_1$ (dB)	Quantum noise of $\hat{S}_2$ (dB)	Quantum noise of $\hat{S}_3$ (dB)
0.002	+0.08 $\pm$ 0.28	+8.21 $\pm$ 0.28	-3.88 $\pm$ 0.30
1	-0.09 $\pm$ 0.28	+7.50 $\pm$ 0.31	-3.34 $\pm$ 0.29
5	+0.03 $\pm$ 0.29	+7.06 $\pm$ 0.30	-2.30 $\pm$ 0.29
10	+0.07 $\pm$ 0.28	+6.44 $\pm$ 0.29	-1.26 $\pm$ 0.28
15	+0.17 $\pm$ 0.29	+5.96 $\pm$ 0.28	-1.09 $\pm$ 0.30
20	+0.38 $\pm$ 0.28	+5.79 $\pm$ 0.30	-0.57 $\pm$ 0.28

Appendix B The details of experiment

As shown in Fig. 1(b) in the main text, a continuous-wave single-mode fiber laser at telecommunication wavelength of 1550 nm is divided into four parts, which serve as the seed beam of the OPA, the local oscillator, the bright coherent beam for preparing a polarization squeezed light, and the input beam of the SHG, respectively.

In the measurement of variance of Stokes operator of $\hat{S}_1$, the output beams of the PBS3 are measured by a homodyne detector directly. In the measurement of variance of Stokes operator $\hat{S}_2$, the HWP is set to 22.5°. In the measurement of variance of Stokes operator $\hat{S}_3$, the HWP and QWP are set to 22.5° and 45° respectively. In the measurement of the excess noise in the fiber channel, we replace the amplitude-squeezed state with a weak coherent beam with the same power. In this case, the variances of three Stokes operators are $V^c_{\hat{S}_1} = V^c_{\hat{S}_2} = V^c_{\hat{S}_3} = 1$, where the superscript represents the coherent state. According to Eq. (3), the excess noise W can be determined by measuring the variance of any Stokes operators after the transmission. We measure the variance of $\hat{S}_2$ at different transmission distances to estimate the transmission efficiencies and excess noises.

The measured Stokes parameters of the polarization squeezed light after the transmission are shown in Fig. B1, and the detailed quantum noise of each Stokes parameter at different transmission distances are shown in Table B1.

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