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Global output feedback regulation of time-varying nonlinear systems via the dual-gain method

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Notably, owing to the limited information available, the problem of output feedback stabilization/regulation for nonlinear systems has been explored based on state observers. In addition, the sensors accounting for state measurement could fail to detect the system states accurately due to the limitations of manufacturing techniques and instruments. An example was given in [1], where a displacement sensor suffers from ±10% sensitivity error. Even worse, systems might suffer from a variety of nonlinear characteristics which undoubtedly challenge the existing output feedback design and stability analysis strategies. Hence, this study considers the following nonlinear system:

$$\begin{cases}\dot{x}_i(t)=x_{i+1}(t)+f_i(t,x(t),u(t)), i=1,\ldots,n-1,\\ \dot{x}_n(t)=u(t)+f_n(t,x(t),u(t)),\end{cases} \tag{1}$$

where $x(t)=[x_1(t),\ldots,x_n(t)]^{\mathrm{T}}\in\mathbb{R}^n$ is the system state, $f_i:\mathbb{R}^+\times\mathbb{R}^n\times\mathbb{R}\to\mathbb{R}$ is continuous in the first argument and locally Lipschitz in the rest of ones with $i=1,\ldots,n$. $y(t)\in\mathbb{R}$ is the measurement output defined as $y(t)=\theta(t)x_1(t)$ and $u(t)\in\mathbb{R}$ is the control input. $\theta(t):\mathbb{R}^+\to\mathbb{R}$ is an unknown bounded continuous function.

The target of this study is to construct an output feedback regulator for system (1) with any initial condition $x(t_0)=x_0\in\mathbb{R}^n, t_0\geqslant 0$ such that (i) all the closed-loop signals and the actual control are globally uniformly bounded on $[t_0,+\infty)$, (ii) $u(t)$ and $x(t)$ converge to the origin. The following assumptions are indispensable.

Assumption 1. There is a known parameter $\bar{\theta}>0$ satisfying inequality $|1-\theta(t)|\leqslant\bar{\theta}<1$, where $\bar{\theta}$ is an allowable sensitivity error.

Assumption 2. For $i=1,\ldots,n$, there exist a known constant $p>0$, an unknown constant $c>0$, and a continuous known function $\phi(t)>0$ defined on $[0,+\infty)$ satisfying

$$|f_i(t,x,u)|\leqslant c\phi(t)(1+|y|^p)(|x_1|+\cdots+|x_i|). \tag{2}$$

Assumption 1 explained/used in [2] is standard and provides a clear description of the allowable range of $\theta(t)$. The inequality (2) indicates that the nonlinearities of system (1) are bounded by a time-varying function multiplying the unmeasured states with an unknown constant and a polynomial form of the output function, which implies that the system contains more severe parameter unknowns, time variations, and nonlinearities. By driving $\phi(t)$ to approach $+\infty$ as time increases, the ever-growing property of $\phi(t)$ challenges some existing methods, such as the dual-domination approach [3], the improved dynamic gain method [4], and the dynamic-gain scaling approach [5]. This motivates us to find a new solution which outlines two innovations in this study. (i) The dual-domination approach [3] is creatively improved. Specifically, a constant gain and a dynamic gain, working as dual gains, are introduced so that ever-growing system nonlinearities with unknown growth rates and unknown sensor sensitivity can be dominated/compensated successfully. (ii) The proposed methodology might lead to the possibility of performing/unifying the output feedback design associated with a more general kind of nonlinear systems suffering from growth rates and unknown measurement/sensor sensitivity.

Main result. The notations given in Appendix A are adopted throughout the study.

For system (1), construct the observer as follows:

$$\begin{cases}\dot{\hat{x}}_i(t)=\hat{x}_{i+1}(t)-r^i(t)a_i\hat{x}_1(t),\ i=1,\ldots,n-1,\\ \dot{\hat{x}}_n(t)=u(t)-r^n(t)a_n\hat{x}_1(t),\end{cases} \tag{3}$$

where $\hat{x}(t)=[\hat{x}_1(t),\ldots,\hat{x}_n(t)]^{\mathrm{T}}$ is the estimate of the state $x(t)$, $r(t)=L_1(t)(L_2^2(t)+b)$ is continuously differentiable, $L_1(t)$ satisfies the following differentiable equation:

$$\dot{L}_1(t)=-\rho_1L_1^2(t)+\rho_2(1+|y(t)|^p)^2L_1(t),\ L_1(t_0)=1,\ t_0\geqslant 0, \tag{4}$$

with b,ρ_1, and ρ_2 being positive constants determined subsequently, and a time-varying function $L_2(t)$ with $L_2(t_0)\geqslant 1$ is increasing, continuously differentiable and satisfies

$$\begin{cases}\text{(i)} & \lim\limits_{t\to+\infty}L_2(t)=+\infty,\\ \text{(ii)} & \lim\limits_{t\to+\infty}\frac{\dot{L}_2(t)}{L_2^2(t)}=0,\\ \text{(iii)} & \lim\limits_{t\to+\infty}\frac{\phi^2(t)}{L_2(t)}=0.\end{cases} \tag{5}$$

Positive constants $a_1,\ldots,a_n,b_1,\ldots,b_n$ are selected to

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guarantee that Λ_1 and Λ_3 are Hurwitz, respectively, where

$$\Lambda_1=\begin{bmatrix} -a_1 & 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -a_{n-1} & 0 & 0 & \cdots & 1 \\ -a_n & 0 & 0 & \cdots & 0 \end{bmatrix}, \Lambda_3=\begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ -b_1 & -b_2 & \cdots & -b_n \end{bmatrix}, \quad (6)$$

and positive definite symmetric matrices P and Q satisfy

$$\begin{cases} \Lambda_1^{\mathrm{T}}P+P\Lambda_1 \leqslant -2I,\ d_1 I \leqslant BP+PB \leqslant d_2 I, \\ \Lambda_3^{\mathrm{T}}Q+Q\Lambda_3 \leqslant -I,\ d_3 I \leqslant QB+BQ \leqslant d_4 I, \end{cases} \quad (7)$$

where d_1, d_2, d_3, and d_4 are positive constants which are assigned in advance, $B = \mathrm{diag}\{v, v+1, \ldots, v+n-1\}$ is a diagonal matrix with the constant v satisfying $0 < v \leqslant \frac{1}{4p}$.

Now, we give the main result of this study.

Theorem 1. If Assumptions 1 and 2 hold for system (1), then there exists a regulator that has the following properties:

(i) all the signals, denoted by $x(t), \hat{x}(t), L_1(t)$, and $u(t)$, of the closed-loop system are bounded on $[t_0, +\infty)$.

(ii) $\lim_{t\to+\infty} u(t) = \lim_{t\to+\infty} x(t) = \lim_{t\to+\infty} \hat{x}(t) = 0$.

Proof. The proof is divided into four parts.

Part I. Analysis of the error dynamics. Define the estimation error as follows:

$$\varepsilon_i = \frac{x_i - \hat{x}_i}{r^{v+i-1}},\ i = 1, \ldots, n. \quad (8)$$

Then, from (1), (3), and (8), there holds

$$\dot{\varepsilon} = r\Lambda_1\varepsilon - \frac{\dot{r}}{r}B\varepsilon + \frac{r}{r^v}\Lambda_2 x_1 + F, \quad (9)$$

where $\varepsilon = [\varepsilon_1, \ldots, \varepsilon_n]^{\mathrm{T}}$, $F = [\frac{f_1}{r^v}, \frac{f_2}{r^{v+1}}, \ldots, \frac{f_n}{r^{v+n-1}}]^{\mathrm{T}}$, $\Lambda_2 = [a_1, a_2, \ldots, a_n]^{\mathrm{T}}$, the definition of B can be found below (7), and Λ_1 is given in (6). Choose $V_1(\varepsilon) = \beta\varepsilon^{\mathrm{T}}P\varepsilon$, where the constant $\beta > 0$. Through complicated calculations in Appendix B, there holds

$$\begin{aligned} \dot{V}_1 \leqslant & -\beta(d_1\rho_2(1+|y|^p)^2+r-d_2\rho_1 L_1)\|\varepsilon\|^2+\beta r\frac{x_1^2}{r^{2v}}\|P\Lambda_2\|^2 \\ & +2nc\|P\|\beta\phi\|\varepsilon\|(1+|y|^p)\sum_{i=1}^{n}\frac{|x_i|}{r^{v+i-1}}. \end{aligned} \quad (10)$$

Part II. Construction of a regulator. Introduce the coordinate transformations as follows:

$$\zeta_1=\frac{x_1}{r^v}, \zeta_i=\frac{\hat{x}_i}{r^{v+i-1}M^{i-1}}, \bar{v}=\frac{u}{r^{v+n}M^n}, i=2, \ldots, n, \quad (11)$$

where the constant gain $M \geqslant 1$ is determined later. Then, using (1), (3), (8), and (11), there holds

$$\begin{cases} \dot{\zeta}_1 = Mr\zeta_2 - v\frac{\dot{r}}{r}\zeta_1 + r\varepsilon_2 + \frac{f_1}{r^v}, \\ \dot{\zeta}_i = Mr\zeta_{i+1}+\frac{ra_i}{M^{i-1}}\varepsilon_1-\frac{ra_i}{M^{i-1}}\zeta_1-(v+i-1)\frac{\dot{r}}{r}\zeta_i, \\ \quad i = 2, 3, \ldots, n-1, \\ \dot{\zeta}_n = Mr\bar{v}+\frac{ra_n}{M^{n-1}}\varepsilon_1-\frac{ra_n}{M^{n-1}}\zeta_1-(v+n-1)\frac{\dot{r}}{r}\zeta_n. \end{cases} \quad (12)$$

Assign $\bar{v} = -\frac{b_1 y}{r^v} - b_2\zeta_2 - \cdots - b_n\zeta_n$, where $b_1, \ldots, b_n$ are determined by (6). This and Eq. (11) lead to the actual regulator/controller as

$$u(t)=-r^n(t)M^n b_1 y(t)-\sum_{i=2}^{n} r^{n-i+1}(t)M^{n-i+1}b_i\hat{x}_i(t). \quad (13)$$

Substituting (13) into (12) yields

$$\begin{aligned} \dot{\zeta} = & \ rM\Lambda_3\zeta - \frac{\dot{r}}{r}B\zeta + r\Lambda_4(\varepsilon_1 - \zeta_1) + rJ\varepsilon_2 \\ & +rMZb_1(1-\theta)\zeta_1 + K, \end{aligned} \quad (14)$$

where $J = [1, 0, \ldots, 0]^{\mathrm{T}}$, $\Lambda_4 = [0, \frac{a_2}{M}, \ldots, \frac{a_n}{M^{n-1}}]^{\mathrm{T}}$, $\zeta = [\zeta_1, \ldots, \zeta_n]^{\mathrm{T}}$, $Z = [0, \ldots, 0, 1]^{\mathrm{T}}$, $K = [\frac{f_1}{r^v}, 0, \ldots, 0]^{\mathrm{T}}$, the definitions of B and Λ_3 are given below (7) and in (6), respectively. Choose $V_2(\zeta) = \zeta^{\mathrm{T}}Q\zeta$. Through complicated calculations in Appendix C, there holds

$$\begin{aligned} \dot{V}_2 \leqslant & \ d_4\rho_1 L_1\|\zeta\|^2+(\bar{m}_1(t)+2r\|Q\Lambda_2\|+r\|Q\|^2+r)\|\zeta\|^2 \\ & -rM(1-2b_1|1-\theta|\cdot\|Q\|)\|\zeta\|^2+(1+|y|^p)^2\|\zeta\|^2 \\ & +r(1+\|Q\Lambda_2\|^2)\|\varepsilon\|^2-d_3\rho_2(1+|y|^p)^2\|\zeta\|^2, \end{aligned} \quad (15)$$

where $\bar{m}_1(t) = c^2\|Q\|^2\phi^2(t)$ is continuous and satisfies $\lim_{t\to+\infty}\frac{\bar{m}_1(t)}{L_2(t)} = 0$.

Part III. Determination of parameters. Choose a continuously differentiable function $V_e(\varepsilon, \zeta) = V_1(\varepsilon) + V_2(\zeta)$. Then, the calculations in Appendix D show that

$$\begin{aligned} \dot{V}_e \leqslant & -(\beta r - rk_1 - \bar{m}(t) - \beta d_2\rho_1 L_1)\|\varepsilon\|^2 - (rM\sigma - rk_3 \\ & -\bar{m}(t) - d_4\rho_1 L_1)\|\zeta\|^2 - (\beta d_1\rho_2 - 1)(1+|y|^p)^2\|\varepsilon\|^2 \\ & -(d_3\rho_2 - 2)(1+|y|^p)^2\|\zeta\|^2, \end{aligned} \quad (16)$$

where $\beta = 1 + k_1 = 2 + \|Q\Lambda_2\|^2, k_3 = k_2 + \beta\|P\Lambda_2\|^2 = 1+2\|Q\Lambda_2\|+\|Q\|^2+\beta\|P\Lambda_2\|^2, \sigma = 1 - 2b_1\bar{\theta}\|Q\|$, and $\bar{m}(t) = \bar{m}_1(t) + 9\beta^2c^2n^3M^{2n-2}\|P\|^2\phi^2(t)$ satisfies $\lim_{t\to+\infty}\frac{\bar{m}(t)}{L_2(t)} = 0$. If the designed parameters ρ_1, ρ_2, b, and M are chosen to satisfy the conditions in the following order:

$$\begin{aligned} & \text{①}\rho_1 > 0,\ \text{②}\rho_2 \geqslant \max\left\{\rho_1, \frac{2}{d_3}, \frac{1}{\beta d_1}\right\}, \\ & \text{③}b \geqslant \max\{\beta d_2\rho_1, d_4\rho_1\}, \text{④}M \geqslant \max\left\{1, \frac{1+k_3}{\sigma}\right\}, \end{aligned} \quad (17)$$

then Eq. (16) can be written as

$$\dot{V}_e \leqslant -(L_2^2 L_1 - \bar{m})(\|\varepsilon\|^2 + \|\zeta\|^2). \quad (18)$$

Part IV. Stability analysis. By the existence and continuation properties of solutions, the solution of the closed-loop system can be defined on $[t_0, T_f)$, where T_f may be finite or $+\infty$. Introducing $W(t) \triangleq [\varepsilon(t), \zeta(t)]^{\mathrm{T}}$, we have the following claims. Claim 1. $W(t)$ is bounded on $[t_0, T_f)$ with $T_f<+\infty$; Claim 2. $T_f = +\infty$; Claim 3. $\lim_{t\to+\infty} W(t) = 0$; Claim 4. $\lim_{t\to+\infty} x(t) = \lim_{t\to+\infty}\hat{x}(t) = 0$; Claim 5. $\lim_{t\to+\infty} u(t) = 0$; Claim 6. $L_1(t)$ is bounded on $[t_0, +\infty)$. The detailed proof of Claims 1–6 is given in Appendix E.

Simulation examples. The simulation examples are included in Appendix F.

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Supporting information Appendixes A–F. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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