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Finite-time bearing-only formation of first-order multi-agent systems under pinning control

Chenjun LIU, Wei ZHU* & Fenglan SUN

Key Laboratory of Intelligent Analysis and Decision on Complex Systems, Chongqing University of Posts and Telecommunications, Chongqing 400065*, China*

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Formation control of multi-agent systems (MASs) has been applied in various fields, such as autonomous navigation and target encirclement, due to its ability to facilitate coordination and achieve the desired formation or space configuration. Bearing-based formation control offers greater practicality than position-based or distance-based approaches since bearing information is inexpensive and simple to acquire, such as utilizing optical cameras.

The finite-time control has been proposed for better anti-interference and faster convergence, such as [1]. However, the initial states affect the estimation of the stability time's upper bound for a symbolic or power function controller. Pinning control frequently develops the controllers for nodes in a large-scale network, where only a portion of agents can derive the reference state [2], such as the leader's position [3]. From a practical perspective, it is required to investigate the GPS-denial environment in which the agent may measure neighbors' bearing information. Furthermore, Appendix A provides an extensive review.

Based on the above discussion, we develop a finite-time tracking bearing-only formation of first-order MASs via pinning control. The main contributions are of three aspects. (i) In contrast to the results of finite-time bearing-based control [1, 4], the moving formation with the constant velocity can be tracked and the formation stabilization time can be directly set by users. (ii) A pinning strategy is presented to accomplish the desired formation using the neighbors' bearing information. (iii) To prevent collisions, we provide a less conservative sufficient condition that is independent of the scale of MASs.

Preliminaries and problem statements. Consider first-order MASs with n_l leaders and n_f followers ($n = n_\mathrm{l} + n_\mathrm{f}$, $n \geqslant 2$), which have the following dynamics:

$$\dot{x}_i(t) = u_i(t), \quad i = 1, \ldots, n, \tag{1}$$

where $x_i(t)$, $u_i(t) \in \mathbb{R}^d$ are the position and control input of each agent, respectively.

The network topology among agents can be described by a graph $G = (V, E)$, where $V = \{1, \ldots, n\}$ and $E \subseteq V \times V$ are the sets of nodes and edges, respectively. Let $V_\mathrm{l} = \{1, \ldots, n_\mathrm{l}\}$ and $V_\mathrm{f} = \{n_\mathrm{l} + 1, \ldots, n\}$ denote the sets of leaders and followers, respectively. Assuming that one orientation (i, j) of the s-th undirected edge is assigned to itself, we define $e_s := e_{ij} = x_j - x_i$ and $g_s := g_{ij} = e_s/\|e_s\|$ for $s \in \{1, \ldots, m\}$, where m denotes the number of undirected edges. Moreover, the edge set $\bar{E}$ is composed of each e_s. The orthogonal projection of g_s is $P_{g_s} := I_d - g_s g_s^\mathrm{T}$. Let $x = [x_1^\mathrm{T}, \ldots, x_n^\mathrm{T}]^\mathrm{T}$. Then, we have the bearing vector g and edge vector e in the similar compact form. The incidence matrix of the graph G is $H \in \mathbb{R}^{m\times n}$ satisfying $e = \bar{H}x$, where $\bar{H} = H \otimes I_d$. Let $x^* = [(x_1^*)^\mathrm{T}, \ldots, (x_n^*)^\mathrm{T}]^\mathrm{T}$ be the position for the target formation (G, x^*); then we have e^* and g^* analogously. In the leader-follower case, the bearing Laplacian matrix $L = \bar{H}^\mathrm{T}\mathrm{diag}(P_{g_1}, \ldots, P_{g_s})\bar{H} \in \mathbb{R}^{dn\times dn}$ can be divided into four matrix blocks: L_ll, L_ff, L_lf, and L_fl, to describe the relationship between leaders, between followers and between leaders and followers, respectively (see Appendix C.1 for detail definitions). Then, a pinning matrix $D \in \mathbb{R}^{m\times n}$ of the graph G is defined as

$$d_{si} = \begin{cases} a_{si}, & (i, j) \in E_1 \text{ or } (i, j) \in E_2, \\ b_{si}, & (j, i) \in E_1 \text{ or } (j, i) \in E_2, \\ 0, & \text{others}, \end{cases}$$

where $a_{si} < 1$, $b_{si} > -1$, $E_1 = \{(i, j) \in \bar{E}| i \in V_\mathrm{l}, j \in V_\mathrm{f}\}$ and $E_2 = \{(i, j) \in \bar{E}| i \in V_\mathrm{f}, j \in V_\mathrm{l}\}$. The detailed pinning control strategy is shown in Appendix C.2, ensuring that a part of agents acquire the leaders' relative bearing.

We want to design the control input $u_i(t)$ for each follower, using the neighbors' bearing information $\{g_{ij}(t)\}_{j\in\mathcal{N}_i}$, such that $x(t) \to x^*(t)$ as $t \to \Lambda$ and $x(t) = x^*(t)$ as $t \geqslant \Lambda$, where Λ denotes a finite time.

Assumption 1 ([5]). The desired formation (G, x^*) can be uniquely determined by the bearing vectors $\{g_{ij}^*\}_{(i,j)\in E}$ and the leaders' positions $\{x_i^*\}_{i\in V_\mathrm{l}}$ if and only if $L_\mathrm{ff} > 0$.

Assumption 2 ([3]). Suppose G_f is the graph composed by followers, which has r disjoint strong components namely $G_1, \ldots, G_r$ with $V(G_i) \bigcap V(G_j) = \emptyset$, $i, j = 1, \ldots, r$, $i \neq j$, and $\bigcup_{i=1}^r V(G_i) \subseteq V(G)$. $V(G_i)$ denotes the vertex set of the i-th strong components. If $\bigcup_{i=1}^r V(G_i) \subset V(G)$, assume that each vertex in $V(G)\backslash \bigcup_{i=1}^r V(G_i)$ is reachable from at least one vertex in $\bigcup_{i=1}^r V(G_i)$.

Remark 1. It means that the leader-following formation under pinning control can be achieved when there is a directed spanning tree among the leaders and multiple followers.

* Corresponding author (email: zhuwei@cqupt.edu.cn)

Main results. Suppose that leaders move with the constant velocity $\bar{v} = [\bar{v}_1, \ldots, \bar{v}_d]^{\mathrm{T}}$. Define position differences as $\varepsilon_i(t) = x_i(t) - x_i(t_0)$ and $\varepsilon_i^*(t) = x_i^*(t) - x_i^*(t_0)$, where $t = t_0$ is the initial time. Then the control input $u_i(t)$ for each follower $i \in V_{\mathrm{f}}$ is designed as

$$u_i(t) = \left(\alpha + \gamma\frac{\dot{\varphi}(t)}{\varphi(t)}\right)\left[\sum_{j\in\mathcal{N}_i}(g_{ij}(t) - g_{ij}^*) + \sum_{j\in V_{\mathrm{l}}} d_{si}(g_{ij}(t) - g_{ij}^*)\right] - o_i(\varepsilon_i(t) - \varepsilon_i^*(t)), \quad (2)$$

where α, γ, $o_i = \mathrm{diag}(o_{i1}, o_{i2}, \ldots, o_{id}) \in \mathbb{R}^{d\times d}$ are positive control gains. $\varphi(t)$ is defined as a function satisfying $\varphi(t) = (\frac{\Lambda - t_0}{\Lambda - t})^q$ for $t \in [t_0, \Lambda)$ and $\varphi(t) = 1$ for $t \in [\Lambda, \infty)$, where $q \in \mathbb{R}_{>0}$ is the user-chosen parameter and $\mathbb{R}_{>0}$ denotes the set of positive real numbers. For $t \in [t_0, \Lambda)$, the derivative of $\varphi(t)$ is $\dot{\varphi}(t) = \frac{q}{\Lambda - t}(\frac{\Lambda - t_0}{\Lambda - t})^q$. Assuming that the right-hand derivative at $t = \Lambda$ is adopted as $\dot{\varphi}(t)$, then we have $\dot{\varphi}(t) = 0$ for $t \in [\Lambda, \infty)$. Obviously, $\varphi(t)$ satisfies the properties of $\psi(t)$ in Lemma 1 in Appendix D.

Then, using the control mechanism (2), the compact form of MASs (1) can be rewritten as

$$\dot{x}(t) = -\left(\alpha + \gamma\frac{\dot{\varphi}}{\varphi}\right)\begin{bmatrix}\mathbf{0}_{dn_{\mathrm{l}}\times dn_{\mathrm{l}}} & \mathbf{0}_{dn_{\mathrm{l}}\times dn_{\mathrm{f}}}\\ \mathbf{0}_{dn_{\mathrm{f}}\times dn_{\mathrm{l}}} & I_{dn_{\mathrm{f}}}\end{bmatrix}(\bar{H}^{\mathrm{T}} + \bar{D}^{\mathrm{T}}) \cdot (g(t) - g^*) + \begin{bmatrix}\mathbf{1}_{n_{\mathrm{l}}}\otimes\bar{v}\\ \mathbf{0}_{dn_{\mathrm{f}}}\end{bmatrix} - O(\varepsilon(t) - \varepsilon^*(t)), \quad (3)$$

where $O = \mathrm{diag}(O_{\mathrm{l}}, O_{\mathrm{f}}) = \mathrm{diag}(o_1, \ldots, o_n)$, $\varepsilon(t) = [\varepsilon_{\mathrm{l}}^{\mathrm{T}}(t), \varepsilon_{\mathrm{f}}^{\mathrm{T}}(t)]^{\mathrm{T}} = [\varepsilon_1^{\mathrm{T}}(t), \ldots, \varepsilon_n^{\mathrm{T}}(t)]^{\mathrm{T}}$, and $\varepsilon^*(t)$ is the similar compact form.

Let the initial states be $x(t_0) = [(x_{\mathrm{l}}^*(t_0))^{\mathrm{T}}, x_{\mathrm{f}}^{\mathrm{T}}(t_0)]$ and $\dot{x}(t_0) = [(v_{\mathrm{l}}^*(t_0))^{\mathrm{T}}, v_{\mathrm{f}}^{\mathrm{T}}(t_0)] = [\mathbf{1}_{n_{\mathrm{l}}}\otimes\bar{v}, v_{\mathrm{f}}^{\mathrm{T}}(t_0)]$. Then, the position error is $\delta_i(t) = x_i(t) - x_i^*(t) = [\delta_{i1}^{\mathrm{T}}(t), \ldots, \delta_{id}^{\mathrm{T}}(t)]^{\mathrm{T}}$ and $\delta(t) = [\delta_{\mathrm{l}}^{\mathrm{T}}(t), \delta_{\mathrm{f}}^{\mathrm{T}}(t)] = [\delta_1^{\mathrm{T}}(t), \ldots, \delta_n^{\mathrm{T}}(t)]^{\mathrm{T}}$.

Next, the following result can be obtained for avoiding collision between agents.

Theorem 1. Under Assumption 1, if

$$\|\delta(t)\| \leqslant \theta := \frac{1}{\sqrt{2}}\left(\min_{i,j\in V}\|x_i^* - x_j^*\| - \xi\right), \quad (4)$$

for any $\xi \in (0, \min_{i,j\in V}\|x_i^* - x_j^*\|)$, then $\|x_i(t) - x_j(t)\| \geqslant \xi$ for any $i, j \in V$ and $t > t_0$. Namely, a collision-free path can be produced for each agent.

Furthermore, if $\|\delta(t)\| \leqslant \|\delta(t_0)\|$ for any $t > t_0$, the condition can be replaced by $\|\delta(t_0)\| \leqslant \theta$.

Remark 2. Different from the previous studies, such as [4, 5], θ does not depend on the number n, which implies a larger initial position error can be realized in large-scale networks. Note that $\|\delta(t_0)\| \leqslant \theta$ is only a sufficient condition, which is also confirmed by the simulation example in Appendix F.1.

To establish the relationship between bearing error and position error, we obtain Lemma 2 in Appendix D by slightly revising Lemmas 2 and 3 in [5].

Theorem 2. Suppose Assumptions 1 and 2 hold. Consider the first-order MAS (1) with the control input (2). If

$$o_w = \mathrm{diag}\left(\frac{\bar{v}_1}{\delta_{w1}(t_0)}, \ldots, \frac{\bar{v}_d}{\delta_{wd}(t_0)}\right), \quad (5)$$

where $\frac{\bar{v}_k}{\delta_{wk}(t_0)} > 0$, $w = n_{\mathrm{l}}+1, \ldots, n$, and $k = 1, \ldots, d$, then x converges to x^* as $t \to \Lambda$ and $x = x^*$ for $t \geqslant \Lambda$. Moreover, the control input $u_{\mathrm{f}}(t) = [u_{n_{\mathrm{l}}+1}^{\mathrm{T}}(t), \ldots, u_n^{\mathrm{T}}(t)]^{\mathrm{T}}$ holds C^1 smooth and bounded for $[t_0, +\infty)$ with the condition that

$$q\gamma > \frac{4\|\bar{H}\|(\|\delta(t_0)\| + \|\widetilde{x}^*\|)}{\min_s\{1 + b_{si}\}\lambda_{\min}(L_{\mathrm{ff}})}. \quad (6)$$

Remark 3. The control gain parameter O is designed based on the system environment, including the desired velocity and the initial positions. The position differences $\varepsilon_i(t)$ and $\varepsilon_i^*(t)$ are both known for the i-th agent. In fact, the follower should be aware of the leader's control input.

Remark 4. It should be noted that $\varphi^{-q}(t)$ decreases from 1 to 0 in $[t_0, \Lambda)$, which enables the implement of formation gaol within $t = \Lambda$. Even if $\lim_{t\to\Lambda^-}\varphi(t) = +\infty$, the condition (5) indicates that large γ makes $\|u\|$ always bounded. More importantly, the finite time Λ is preset by users without the initial conditions, which is better than non-smooth controllers, such as [1], in practical complex tasks.

The proof of Theorems 1 and 2 can be found in Appendix E. The validity of the theoretical results is discussed in Examples 1 and 2 of Appendix F. Then, a comparison with existing results is also given in Appendix F.3.

Conclusion. The finite-time bearing-only formation problem of first-order MASs under pinning control has been discussed in this study. Considering the situation of tracking moving formation in finite time, the validity of the pinning controller is verified by Lyapunov stability theory and numerical simulations. Besides, using the time-varying function with certain properties, a finite-time control mechanism is proposed for the preset formation stabilization time and the smooth bounded controller. Our results extend the finite-time bearing-only formation studies based on the time-varying function from stationary to tracking formation. In practice, MASs may be described by nonlinear dynamics and the communication topology may be directed. Thus, it will be investigated in future work.

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Supporting information Appendixes A–F. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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