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Event-triggered adaptive output feedback control for hyperbolic PDEs: swapping-based design

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Partial differential equations (PDEs) characterize transport phenomena and fluid flow, and common cases include heat exchangers and road traffic [1]. Recently, scholars have become enthusiastic about event-triggered control (ETC) of hyperbolic PDEs, primarily because it can help conserve computing and communication resources. For example, Ref. [2] presented an output feedback ETC scheme for 2×2 hyperbolic PDEs.

Typically, in many cases, the system model is not entirely known. Adaptive control is an effective method of compensating for this parametric uncertainty. Currently, there are four commonly used adaptive control methods for PDEs, namely, Lyapunov-based design, identifier-based design, swapping-based design, and regulation-triggered batch least-square identifier (BaLSI) design. Based on state feedback, Ref. [3] studied adaptive ETC for first-order hyperbolic PDE-ordinary differential equation (PDE-ODE) systems using Lyapunov-based design, where parametric uncertainty occurs in the PDE subsystem. Based on output feedback, Ref. [4] investigated adaptive ETC for 2×2 hyperbolic PDE-ODE systems using Lyapunov-based design, where parametric uncertainty appears in the ODE subsystem. Then, Ref. [5] developed an adaptive state feedback ETC for 2×2 hyperbolic PDE-ODE systems using regulation-triggered BaLSI design. However, judging from the current results, Lyapunov-based design and regulation-triggered BaSLI design as used in [3, 5] cannot be extended directly to adaptive event-triggered output feedback stabilization of PDEs subject to parametric uncertainty. Thus, how to use other adaptive techniques to establish event-triggered output feedback controllers for PDEs subject to unknown parameters and states is worth studying.

In this paper, based on input and output filters, an adaptive event-triggered output feedback controller and a dynamic triggering condition for a first-order hyperbolic PDE are designed to ensure that all signals in the closed-loop system are bounded pointwise in space and time, and that the state of the original system is convergent pointwise in space. The following transport PDE with the uncertain spatially varying parameter and control coefficient is considered:

$$\begin{cases}\alpha_t(\varkappa,t)=\mu\alpha_\varkappa(\varkappa,t)+\theta(\varkappa)\alpha(0,t),\\ \alpha(1,t)=\lambda U(t),\end{cases}\tag{1}$$

where $\alpha:[0,1]\times[0,+\infty)\rightarrow\mathbb{R}$ is the system state with the initial data $\alpha(\varkappa,0)=\alpha_0(\varkappa)$, $\theta(\varkappa)\in\mathcal{C}([0,1];\mathbb{R})$ is an unknown function, μ is a known parameter, λ is designated as the control coefficient, and it is an unknown nonzero constant with a known sign, $U(t)$ denotes the input signal, and $y(t)=\alpha(0,t)$ is available for measurement.

Compared with the existing results, our contributions mainly include the following: (i) in contrast to [2] where plant parameters are completely known, this paper considers spatially varying parameters; (ii) unlike the adaptive event-triggered state feedback control scheme given in [3, 5], this paper deals with an adaptive output feedback ETC problem using only the boundary measurable signal $\alpha(0,t)$.

Remark 1. The system (1) can describe the evolution of substances in the transport pipeline, where the coefficient λ is the deviation between the input signal applied to the system and the designed one, which often occurs and is, in reality, somewhat uncertain due to aging of the equipment, external disturbance, or other factors, as described by [6].

Assumption 1. For all $\varkappa\in[0,1]$, there exists a positive number $\bar\theta$ such that $|\theta(\varkappa)|\leqslant\bar\theta$, where $\bar\theta$ is known.

Assumption 2. There are nonzero scalars $\underline{\lambda}$, $\bar\lambda$ such that $\underline{\lambda}\leqslant\lambda\leqslant\bar\lambda$, where $\underline{\lambda}$, $\bar\lambda$ are known and satisfy $\underline{\lambda}\bar\lambda>0$.

Remark 2. The known bounds $\bar\theta$, $\underline{\lambda}$, $\bar\lambda$ of unknown parameters $\theta(\varkappa),\lambda$ are used in the projection operator to ensure the boundedness of parameter estimations $\hat\theta(\varkappa,t)$ and $\hat\lambda(t)$.

Control design. The filters are designed as follows:

$$\begin{cases}\omega_t(\varkappa,t)=\mu\omega_\varkappa(\varkappa,t), & \omega(1,t)=U(t),\\ \upsilon_t(\varkappa,t)=\mu\upsilon_\varkappa(\varkappa,t), & \upsilon(1,t)=y(t),\end{cases}\tag{2}$$

with initial data $\omega(\varkappa,0)=\omega_0(\varkappa)$, $\upsilon(\varkappa,0)=\upsilon_0(\varkappa)$ for any $\varkappa\in[0,1]$. Analytically, the nonadaptive estimate of $\alpha(\varkappa,t)$ is built as $\bar\alpha(\varkappa,t)=\lambda\omega(\varkappa,t)+r\int_\varkappa^1\theta(\epsilon)\upsilon(1-(\epsilon-\varkappa),t)\mathrm{d}\epsilon$, with $r=1/\mu$. Then, define $e(\varkappa,t)=\alpha(\varkappa,t)-\bar\alpha(\varkappa,t)$, it is the nonadaptive estimation error. Together with (1), we find that e satisfies $e_t(\varkappa,t)-\mu e_\varkappa(\varkappa,t)=0$ with $e(1,t)=0$ and initial data $e(\varkappa,0)=e_0(\varkappa)$, for which $e\equiv 0$ for $t\geqslant r$.

From $\bar\alpha(\varkappa,t)$, we design an adaptive estimate of $\alpha(\varkappa,t)$ in the following manner:

$$\hat\alpha(\varkappa,t)=\hat\lambda(t)\omega(\varkappa,t)+r\int_\varkappa^1\hat\theta(\epsilon,t)\upsilon(1-(\epsilon-\varkappa),t)\mathrm{d}\epsilon,\tag{3}$$

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where $\hat{\lambda}$ and $\hat{\theta}$ are estimates of the unknown parameters λ and θ, respectively. Define $\hat{e}(\varkappa,t)=\alpha(\varkappa,t)-\hat{\alpha}(\varkappa,t)$, it is the adaptive estimation error. Using (2) and (3), we find that the dynamic $\hat{\alpha}(\varkappa,t)$ satisfies

$$\begin{cases}\hat{\alpha}_t(\varkappa,t)=\mu\hat{\alpha}_\varkappa(\varkappa,t)+\hat{\theta}(\varkappa,t)\alpha(0,t)+\dot{\hat{\lambda}}(t)\omega(\varkappa,t)\\ \qquad +r\displaystyle\int_\varkappa^1\hat{\theta}_t(\epsilon,t)\upsilon(1-(\epsilon-\varkappa),t)\mathrm{d}\epsilon,\\ \hat{\alpha}(1,t)=\hat{\lambda}(t)U(t),\end{cases}\tag{4}$$

with the initial data $\hat{\alpha}(\varkappa,0)=\hat{\alpha}_0(\varkappa)$ for all $\varkappa\in[0,1]$. The adaptive laws with normalization and projection operators [1, Appendix A] are, therefore, constructed as

$$\begin{cases}\hat{\theta}_t(\varkappa,t)=\mathrm{proj}_{\bar{\theta}}\{\varsigma_1(\varkappa)\delta(t)\upsilon(1-\varkappa,t),\hat{\theta}(\varkappa,t)\},\\ \dot{\hat{\lambda}}(t)=\mathrm{proj}_{[\underline{\lambda},\bar{\lambda}]}\{\varsigma_2\delta(t)\omega(0,t),\hat{\lambda}(t)\},\end{cases}\tag{5}$$

with initial data $\hat{\theta}(\varkappa,0)=\hat{\theta}_0(\varkappa)\leqslant\bar{\theta}$, $\hat{\lambda}(0)=\hat{\lambda}_0\leqslant\max\{|\bar{\lambda}|,|\underline{\lambda}|\}$, and $\delta(t)=\frac{\hat{e}(0,t)}{1+\|\upsilon(\cdot,t)\|^2+\omega^2(0,t)}$. In addition, $0<\underline{\varsigma_1}\leqslant\varsigma_1(\varkappa)\leqslant\bar{\varsigma}_1$ and $0<\underline{\varsigma_2}\leqslant\varsigma_2\leqslant\bar{\varsigma}_2$ are design gains for all $\varkappa\in[0,1]$.

The following PDE backstepping is considered:

$$\begin{cases}\hat{\beta}(\varkappa,t)=\hat{\alpha}(\varkappa,t)-\displaystyle\int_0^\varkappa\hat{k}(\varkappa-\epsilon,t)\hat{\alpha}(\epsilon,t)\mathrm{d}\epsilon,\\ \hat{\alpha}(\varkappa,t)=\hat{\beta}(\varkappa,t)-r\displaystyle\int_0^\varkappa\hat{\theta}(\varkappa-\epsilon,t)\hat{\beta}(\epsilon,t)\mathrm{d}\epsilon,\end{cases}\tag{6}$$

with $\hat{k}$ satisfying the integral equation $\mu\hat{k}(\varkappa,t)=\int_0^\varkappa\hat{k}(\varkappa-\epsilon,t)\hat{\theta}(\epsilon,t)\mathrm{d}\epsilon-\hat{\theta}(\varkappa,t)=-G[\hat{\theta}](\varkappa,t)$. For convenience, we write $\hat{\beta}(\varkappa,t)=G[\hat{\alpha}](\varkappa,t)$, then $\hat{\alpha}(\varkappa,t)=G^{-1}[\hat{\beta}](\varkappa,t)$.

Proposition 1. The Volterra integral transformation (6) and the controller $U(t)=\frac{1}{\hat{\lambda}(t)}\int_0^1\hat{k}(1-\epsilon,t)\hat{\alpha}(\epsilon,t)\mathrm{d}\epsilon$, maps (4) into the following target system:

$$\begin{cases}\hat{\beta}_t(\varkappa,t)=\mu\hat{\beta}_\varkappa(\varkappa,t)-\mu\hat{k}(\varkappa,t)\hat{e}(0,t)+\dot{\hat{\lambda}}(t)\omega(\varkappa,t)\\ \qquad -\dot{\hat{\lambda}}(t)\displaystyle\int_0^\varkappa\hat{k}(\varkappa-\epsilon,t)\omega(\epsilon,t)\mathrm{d}\epsilon\\ \qquad +rG\left[\displaystyle\int_\varkappa^1\hat{\theta}_t(\epsilon,t)\upsilon(1-(\epsilon-\varkappa),t)\mathrm{d}\epsilon\right](\varkappa,t)\\ \qquad -\displaystyle\int_0^\varkappa\hat{k}_t(\varkappa-\epsilon,t)G^{-1}[\hat{\beta}](\epsilon,t)\mathrm{d}\epsilon,\\ \hat{\beta}(1,t)=0.\end{cases}\tag{7}$$

The detail of Proposition 1 is put in Appendix A.

The adaptive event-triggered controller U_i and the input holding error $d(t)$ are determined respectively as

$$\begin{cases}U_i:=U(t_i)=\dfrac{1}{\hat{\lambda}(t_i)}\displaystyle\int_0^1\hat{k}(1-\epsilon,t_i)\hat{\alpha}(\epsilon,t_i)\mathrm{d}\epsilon,\\ d(t)=U_i-U(t),\end{cases}\tag{8}$$

for $t\in[t_i,t_{i+1}),i\in\mathbb{N}$. Using the first equation of (8), the second equation of (1) is written as $\alpha(1,t)=\lambda U_i$. Using (2) and (6), it can, therefore, be found that the boundary of the target system (7) satisfies $\hat{\beta}(1,t)=\hat{\lambda}(t)d(t)$ for $t\in[t_i,t_{i+1}),i\in\mathbb{N}$.

Definition 1. Let ρ,ν,η_i $(i=1,2,3,4,5)$ be several positive numbers, the event times $t_i\geqslant 0$ $(i\in\mathbb{N})$ subject to $t_0=0$ consist of a finite increasing sequence according to the following criteria: (i) if $\mathcal{S}_d=\{t\in\mathbb{R}^+|t>t_i\wedge d^2(t)>-\xi(t)\}=\emptyset$, then the set of the times of the event is $\{t_0,t_1,\ldots,t_i\}$; (ii) if $\mathcal{S}_d\neq\emptyset$, then the next event time is determined as $t_{i+1}=\inf\{t\in\mathbb{R}^+|t>t_i\wedge d^2(t)>-\xi(t)\}$, where $d(t)$ is defined in (8) for $t\in[t_i,t_{i+1})$ and $\xi(t)$ satisfies

$$\dot{\xi}(t)=-\rho\xi(t)+\nu d^2(t)-\eta_1\|\hat{\alpha}(\cdot,t)\|^2-\eta_2\hat{\alpha}^2(0,t)$$
$$-\eta_3\alpha^2(0,t)-\eta_4\|\omega(\cdot,t)\|^2-\eta_5\|\upsilon(\cdot,t)\|^2,\tag{9}$$

for $t\in(t_i,t_{i+1})$, $i\in\mathbb{N}$ with $\xi(t_0)=\xi(0)<0$ and $\xi(t_i^-)=\xi(t_i)=\xi(t_i^+)$.

Proposition 2. For given initial data $\alpha_0(\varkappa)$, $\hat{\alpha}_0(\varkappa)$, $\omega_0(\varkappa)$, and $\upsilon_0(\varkappa)$ belong to $L^2(0,1)$ compatible with the boundary condition, the closed-loop system composed of the system (1), filter (2), state estimate (3), and controller (8) exists a unique solution $\alpha,\hat{\alpha},\omega,\upsilon\in\mathcal{C}([0,\mathcal{Y}_m);L^2(0,1))$, where $\mathcal{Y}_m=\lim_{i\to\infty}(t_i)=+\infty$ or $\mathcal{Y}_m<+\infty$.

Proof. The details are closely analogous to that of [3, Proposition 4] and are, therefore, omitted.

Theorem 1. Based on the triggering condition in Definition 1 and the event-triggered controller (8), there does exist a positive scalar t^* such that $t_{i+1}-t_i\geqslant t^*,i\in\mathbb{N}$.

Theorem 2. Under Assumptions 1 and 2, the closed-loop system composed of the system (1), filter (2), state estimate (3) with (5), controller (8), and triggering condition in Definition 1 satisfies:

(i) No Zeno behavior happens, i.e., $\mathcal{Y}_m=+\infty$, and the closed-loop system is well-posed;

(ii) All signals of the closed-loop system are bounded in their respective domains of definition and the state of the original system converges to zero.

The details of Theorems 1 and 2 are put in Appendixes B and C, respectively, and simulation examples are provided in Appendix D to verify the theoretical results.

Concluding remarks. A notable advantage of using a swapping-based design, is that it transforms the system (1) into a linear parametric form (2) concerning uncertain parameters. This facilitates the use of various well-established adaptive laws, such as the least-squares update law and the gradient law (5). Additionally, it permits normalization, which ensures the boundedness of the update law irrespective of the boundedness of the system state.

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Supporting information Appendixes A–E. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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