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• **Supplementary File** •

Event-triggered adaptive output feedback control for hyperbolic PDEs: swapping-based design

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Appendix A The proof of Proposition 1

Proof. First, applying the first equality of (6) to compute $\hat{\beta}_t(\varkappa,t)$, then utilizing (4) and the integration by parts, we deduce

$$\begin{aligned}\hat{\beta}_t(\varkappa,t)=&\,\mu\hat{\alpha}_\varkappa(\varkappa,t)+\hat{\theta}(\varkappa,t)\alpha(0,t)+\dot{\hat{\lambda}}(t)\omega(\varkappa,t)+r\int_\varkappa^1\hat{\theta}_t(\epsilon,t)\upsilon(1-(\epsilon-\varkappa),t)d\epsilon-\int_0^\varkappa\hat{k}_t(\varkappa-\epsilon,t)\hat{\alpha}(\epsilon,t)d\epsilon\\&-\mu\hat{k}(0,t)\hat{\alpha}(\varkappa,t)+\mu\hat{k}(\varkappa,t)\hat{\alpha}(0,t)-\mu\int_0^\varkappa\hat{k}_\varkappa(\varkappa-\epsilon,t)\hat{\alpha}(\epsilon,t)d\epsilon-\int_0^\varkappa\hat{k}(\varkappa-\epsilon,t)\hat{\theta}(\epsilon,t)d\epsilon\alpha(0,t)\\&-\dot{\hat{\lambda}}(t)\int_0^\varkappa\hat{k}(\varkappa-\epsilon,t)\omega(\epsilon,t)d\epsilon-r\int_0^\varkappa\hat{k}(\varkappa-\epsilon,t)\int_\epsilon^1\hat{\theta}_t(s,t)\upsilon(1-(s-\epsilon),t)dsd\epsilon.\end{aligned}\tag{A1}$$

Utilizing the first equation of (6) to compute $\hat{\beta}_\varkappa(\varkappa,t)$, we get

$$\hat{\beta}_\varkappa(\varkappa,t)=\hat{\alpha}_\varkappa(\varkappa,t)-\hat{k}(0,t)\hat{\alpha}(\varkappa,t)-\int_0^\varkappa\hat{k}_\varkappa(\varkappa-\epsilon,t)\hat{\alpha}(\epsilon,t)d\epsilon.\tag{A2}$$

Substituting (A2) into (A1), we obtain

$$\begin{aligned}\hat{\beta}_t(\varkappa,t)=&\,\mu\hat{\beta}_\varkappa(\varkappa,t)+\mu\hat{k}(\varkappa,t)\hat{\alpha}(0,t)+\Big(\hat{\theta}(\varkappa,t)-\int_0^\varkappa\hat{k}(\varkappa-\epsilon,t)\hat{\theta}(\epsilon,t)d\epsilon\Big)\alpha(0,t)+\dot{\hat{\lambda}}(t)\omega(\varkappa,t)\\&-\int_0^\varkappa\hat{k}_t(\varkappa-\epsilon,t)\hat{\alpha}(\epsilon,t)d\epsilon-\dot{\hat{\lambda}}(t)\int_0^\varkappa\hat{k}(\varkappa-\epsilon,t)\omega(\epsilon,t)d\epsilon+r\int_\varkappa^1\hat{\theta}_t(\epsilon,t)\upsilon(1-(\epsilon-\varkappa),t)d\epsilon\\&-r\int_0^\varkappa\hat{k}(\varkappa-\epsilon,t)\int_\epsilon^1\hat{\theta}_t(s,t)\upsilon(1-(s-\epsilon),t)dsd\epsilon.\end{aligned}\tag{A3}$$

Then, by means of $\hat{e}(\varkappa,t)=\alpha(\varkappa,t)-\hat{\alpha}(\varkappa,t)$ and the Volterra integral equation $\mu\hat{k}(\varkappa,t)=\int_0^\varkappa\hat{k}(\varkappa-\epsilon,t)\hat{\theta}(\epsilon,t)d\epsilon-\hat{\theta}(\varkappa,t)$, we obtain that the first equation of (7) holds. The second equation of (7) can be obtained by inserting $\varkappa=1$ into the first equation of (6).

Appendix B The proof of Theorem 1

Before giving the details of Theorem 1, we first give a few useful lemmas. In particular, we introduce some commonly used notations. For a signal $f(t)$ that varies with time, we define the vector space $f\in\mathcal{L}^p([a,b])\Leftrightarrow\big(\int_a^b|f(t)|^pdt\big)^{\frac{1}{p}}<+\infty$ for $p\geqslant1$, with the special case $f\in\mathcal{L}^\infty([a,b])\Leftrightarrow\sup_{a\leqslant t\leqslant b}|f(t)|<+\infty$. For the equivalence class of Lebesgue measurable functions $g:[0,1]\to\mathbb{R}$, its L^2-norm is represented as $\|g\|=\sqrt{\int_0^1g^2(\varkappa)d\varkappa}$, and the corresponding function space is represented as $L^2(0,1)=\{g(\varkappa)|\|g\|<+\infty\}$. For a function of several variables, $\cdot$ indicates the variable with respect to which the norm is taken, such as let $u(\varkappa,\cdot)\in\mathcal{L}^2([a,b])$ represent that $u(\varkappa,t)$ belongs to $\mathcal{L}^2([a,b])$ for any fixed $\varkappa$. In addition, $\|u(\cdot,t)\|^2$ denotes a compact notation of $\int_0^1u^2(\varkappa,t)d\varkappa$.

Lemma 1. Based on the event-triggered controller (8), the adaptive law (5) with initial data satisfying $\hat{\theta}(\varkappa,0)=\hat{\theta}_0(\varkappa)\leqslant\bar{\theta}$ and $\hat{\lambda}(0)=\hat{\lambda}_0\leqslant\bar{\chi}$ ensures the following inequalities:

$$|\hat{\theta}(\varkappa,t)|\leqslant\bar{\theta},\quad\forall(\varkappa,t)\in[0,1]\times[0,\mathcal{Y}_m),\tag{B1}$$

$$|\hat{\lambda}(t)|\leqslant\bar{\chi},\quad t\in[0,\mathcal{Y}_m),\tag{B2}$$

$$e(0,\cdot),\sigma\in\mathcal{L}^2([0,\mathcal{Y}_m))\cap\mathcal{L}^\infty([0,\mathcal{Y}_m)),\tag{B3}$$

$$\|\hat{\theta}_t\|,|\dot{\hat{\lambda}}|\in\mathcal{L}^2([0,\mathcal{Y}_m))\cap\mathcal{L}^\infty([0,\mathcal{Y}_m)),\tag{B4}$$

where $\bar{\chi}=\max\{|\bar{\lambda}|,|\underline{\lambda}|\}$ and

$$\sigma(t)=\frac{\hat{e}(0,t)}{\sqrt{1+\|\upsilon(\cdot,t)\|^2+\omega^2(0,t)}}.\tag{B5}$$

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Proof. Clearly, according to the properties of projection operator in [1, App. A] and initial data $\hat{\theta}(\varkappa,0)=\hat{\theta}_0(\varkappa)\leqslant\bar{\theta}$, $\hat{\lambda}(0)=\hat{\lambda}_0\leqslant\bar{\chi}$, it can be obtained properties (B1)-(B2).

We discuss the following Lyapunov function

$$W(t)=r\int_0^1 e^2(\varkappa,t)d\varkappa+\frac{r}{2}\int_0^1\frac{\tilde{\theta}^2(\varkappa,t)}{\varsigma_1(\varkappa)}d\varkappa+\frac{\tilde{\lambda}^2(t)}{2\varsigma_2},\tag{B6}$$

where $\zeta_1(\varkappa),\zeta_2$ are defined in (5), and $\tilde{\theta}=\theta-\hat{\theta},\tilde{\lambda}=\lambda-\hat{\lambda}$ are parameter estimation errors. In fact, $W(t)$ can be restated as $W(e(\varkappa,t),\tilde{\theta}(\varkappa,t),\tilde{\lambda}(t))$. If $e(\varkappa,t)=\tilde{\theta}(\varkappa,t)=\tilde{\lambda}(t)=0$, then $W(0)=0$. If $e(\varkappa,t)$, $\tilde{\theta}(\varkappa,t)$, and $\tilde{\lambda}(t)$ are not equal to zero, $W(t)>0$. Thus, $W(t)$ is positive definite. Differentiating $W(t)$, utilizing $e_t(\varkappa,t)=\mu e_\varkappa(\varkappa,t)$ and inserting (5), it follows that

$$\dot{W}(t)=2\int_0^1 e(\varkappa,t)e_\varkappa(\varkappa,t)d\varkappa-r\int_0^1\frac{\tilde{\theta}(\varkappa,t)}{\varsigma_1(\varkappa)}\mathrm{proj}_{\bar{\theta}}\{\varsigma_1(\varkappa)\delta(t)\upsilon(1-\varkappa,t),\hat{\theta}(\varkappa,t)\}d\varkappa-\frac{\tilde{\lambda}(t)}{\varsigma_2}\mathrm{proj}_{[\underline{\lambda},\bar{\lambda}]}\{\varsigma_2\delta(t)\omega(0,t),\hat{\lambda}(t)\}.\tag{B7}$$

Utilizing $-\tilde{\theta}(\varkappa,t)\mathrm{proj}_{\bar{\theta}}\{\delta_1(\varkappa,t),\hat{\theta}(\varkappa,t)\}\leqslant-\tilde{\theta}(\varkappa,t)\delta_1(\varkappa,t)$, $-\tilde{\lambda}(t)\mathrm{proj}_{[\underline{\lambda},\bar{\lambda}]}\{\delta_2(t),\hat{\lambda}(t)\}\leqslant-\tilde{\lambda}(t)\delta_2(t)$, and then applying the integration by parts, we deduce

$$\dot{W}(t)\leqslant e^2(1,t)-e^2(0,t)-\delta(t)\Big(r\int_0^1\tilde{\theta}(\varkappa,t)\upsilon(1-\varkappa,t)d\varkappa+\tilde{\lambda}(t)\omega(0,t)\Big).\tag{B8}$$

Using $\hat{e}(\varkappa,t)=\alpha(\varkappa,t)-\hat{\alpha}(\varkappa,t),e(\varkappa,t)=\alpha(\varkappa,t)-\bar{\alpha}(\varkappa,t)$ and (3), one has

$$\hat{e}(0,t)=\tilde{\lambda}(t)\omega(0,t)+r\int_0^1\tilde{\theta}(\epsilon,t)\upsilon(1-\epsilon,t)d\epsilon+e(0,t).$$

Substituting the above equality into (B8) and utilizing $e(1,t)=0$ to yield

$$\dot{W}(t)\leqslant-e^2(0,t)-\sigma^2(t)+\delta(t)e(0,t),\tag{B9}$$

where $\sigma(t)$ is defined in (B5). Then, utilizing Young's inequality to yield

$$\dot{W}(t)\leqslant-\frac{1}{2}\sigma^2(t)-\frac{1}{2}e^2(0,t),\tag{B10}$$

which suggests that $W(t)$ is non-increasing and bounded. Integrating (B10) on $[0,\mathcal{Y}_m)$, we deduce $\frac{1}{2}\int_0^{\mathcal{Y}_m}e^2(0,\tau)d\tau\leqslant W(0)<+\infty$ and $\frac{1}{2}\int_0^{\mathcal{Y}_m}\sigma^2(\tau)d\tau\leqslant W(0)<+\infty$, i.e., $e(0,\cdot),\sigma\in\mathcal{L}^2([0,\mathcal{Y}_m))$. Owing to $e(0,t)=0$ for $t\geqslant r$, it can be easily obtained that

$$|\sigma(t)|\leqslant\frac{r\|\tilde{\theta}(\cdot,t)\|\|\upsilon(\cdot,t)\|+|\tilde{\lambda}(t)||\omega(0,t)|}{\sqrt{1+\|\upsilon(\cdot,t)\|^2+\omega^2(0,t)}}\leqslant r\|\tilde{\theta}(\cdot,t)\|+|\tilde{\lambda}(t)|.\tag{B11}$$

Together with properties (B1)-(B2), we get $\sigma\in\mathcal{L}^\infty([0,\mathcal{Y}_m))$, thus the property (B3) holds. Then, together with the adaptive law (5), we obtain

$$\|\hat{\theta}_t(\cdot,t)\|\leqslant\bar{\varsigma}_1\frac{|\hat{e}(0,t)|}{\sqrt{1+\|\upsilon(\cdot,t)\|^2+\omega^2(0,t)}}\frac{\|\upsilon(\cdot,t)\|}{\sqrt{1+\|\upsilon(\cdot,t)\|^2+\omega^2(0,t)}}\leqslant\bar{\varsigma}_1|\sigma(t)|,\tag{B12}$$

and

$$|\dot{\hat{\lambda}}(t)|\leqslant\bar{\varsigma}_2\frac{|\hat{e}(0,t)|}{\sqrt{1+\|\upsilon(\cdot,t)\|^2+\omega^2(0,t)}}\frac{|\omega(0,t)|}{\sqrt{1+\|\upsilon(\cdot,t)\|^2+\omega^2(0,t)}}\leqslant\bar{\varsigma}_2|\sigma(t)|.\tag{B13}$$

By means of (B3), it can be verified the property (B4).

Lemma 2. Based on the event-triggered controller (8), the first and the second equalities of (6) suggest that the following properties hold:

$$|\hat{k}(\varkappa,t)|\leqslant F_k,\quad\forall(\varkappa,t)\in[0,1]\times[0,\mathcal{Y}_m),\tag{B14}$$

$$\|\hat{\beta}(\cdot,t)\|\leqslant F_1\|\hat{\alpha}(\cdot,t)\|,\tag{B15}$$

$$\|\hat{\alpha}(\cdot,t)\|\leqslant F_2\|\hat{\beta}(\cdot,t)\|,\tag{B16}$$

$$\|\hat{k}_t\|\in\mathcal{L}^2([0,\mathcal{Y}_m))\cap\mathcal{L}^\infty([0,\mathcal{Y}_m)),\tag{B17}$$

for some positive scalars F_k,F_1 and F_2.

Proof. Because $\hat{\theta}$ is uniform bounded, the property (B14) can be obtained by utilizing the successive approximation method in [1, Appendix D] to the Volterra integral equation $\mu\hat{k}(\varkappa,t)=\int_0^\varkappa\hat{k}(\varkappa-\epsilon,t)\hat{\theta}(\epsilon,t)d\epsilon-\hat{\theta}(\varkappa,t)$. Then, recalling the first equation of (6), we obtain

$$\|\hat{\beta}(\cdot,t)\|=\sqrt{\int_0^1\Big(\hat{\alpha}(\varkappa,t)-\int_0^\varkappa\hat{k}(\varkappa-\epsilon,t)\hat{\alpha}(\epsilon,t)d\epsilon\Big)^2d\varkappa}.$$

Together with Minkowski's inequality and Cauchy-Schwarz inequality (see Appendix E) to yield

$$\|\hat{\beta}(\cdot,t)\|\leqslant\sqrt{\int_0^1\hat{\alpha}^2(\varkappa,t)d\varkappa}+\sqrt{\int_0^1\Big(\int_0^\varkappa\hat{k}(\varkappa-\epsilon,t)\hat{\alpha}(\epsilon,t)d\epsilon\Big)^2d\varkappa}\leqslant(1+\|K\|)\|\hat{\alpha}(\cdot,t)\|,\tag{B18}$$

where $K = \sqrt{\int_0^1 \int_0^{\varkappa} \hat{k}(\varkappa-\epsilon,t)^2 d\epsilon d\varkappa}$, which implies that the property (B15) holds with $F_1 = 1 + \|K\|$. Similarly, applying Minkowski's and Cauchy-Schwarz inequalities to the second equality of (6), we get the property (B16).

We take the time derivative of the Volterra integral equation $\mu\hat{k}(\varkappa,t) = \int_0^{\varkappa} \hat{k}(\varkappa-\epsilon,t)\hat{\theta}(\epsilon,t)d\epsilon - \hat{\theta}(\varkappa,t)$, which yields

$$\hat{k}_t(\varkappa,t) - \frac{1}{\mu}\int_0^{\varkappa} \hat{k}_t(\varkappa-\epsilon,t)\hat{\theta}(\epsilon,t)d\epsilon = -\frac{1}{\mu}\hat{\theta}_t(\varkappa,t) + \frac{1}{\mu}\int_0^{\varkappa} \hat{k}(\varkappa-\epsilon,t)\hat{\theta}_t(\epsilon,t)d\epsilon. \tag{B19}$$

In particular, we write (B19) as $G^{-1}[\hat{k}_t](\varkappa,t) = -\frac{1}{\mu}G[\hat{\theta}_t](\varkappa,t)$. Thus, we obtain $[\hat{k}_t](\varkappa,t) = -\frac{1}{\mu}G[G[\hat{\theta}_t]](\varkappa,t)$. According to (6) and (B15), it can be verified that $\|\hat{k}_t(\cdot,t)\| \leqslant \frac{1}{\mu}F_1^2\|\hat{\theta}_t(\cdot,t)\|$. Thanks to $\|\hat{\theta}_t\| \in \mathcal{L}^2([0,\mathcal{Y}_m)) \cap \mathcal{L}^\infty([0,\mathcal{Y}_m))$, it is easy to get the property (B17).

Lemma 3. Based on the triggering condition in Definition 1, the input holding error $d(t)$ and the event-triggered controller U_i defined in (8), there exists $d^2(t) \leqslant -\xi(t)$ with $\xi(t) < 0$ for any $t \in [0,\mathcal{Y}_m)$.

Proof. By means of the triggering condition in Definition 1, one has $d^2(t) \leqslant -\xi(t)$, $t \in [0,\mathcal{Y}_m)$. Recalling (9), we obtain

$$\dot{\xi}(t) \leqslant -(\rho+\nu)\xi(t) - \eta_1\|\hat{\alpha}(\cdot,t)\|^2 - \eta_2\hat{\alpha}^2(0,t) - \eta_3\alpha^2(0,t) - \eta_4\|\omega(\cdot,t)\|^2 - \eta_5\|\upsilon(\cdot,t)\|^2, \tag{B20}$$

for any $t \in (t_i,t_{i+1})$, $i \in \mathbb{N}$. Then, by means of the time continuity of $\xi(t)$, one has

$$\xi(t) \leqslant \xi(t_i)e^{-(\rho+\nu)(t-t_i)} - \int_{t_i}^{t} e^{-(\rho+\nu)(t-s)}\big(\eta_1\|\hat{\alpha}(\cdot,s)\|^2 + \eta_2\hat{\alpha}^2(0,s) + \eta_3\alpha^2(0,s) + \eta_4\|\omega(\cdot,s)\|^2 + \eta_5\|\upsilon(\cdot,s)\|^2\big)ds, \tag{B21}$$

for any $t \in [t_i,t_{i+1}]$, $i \in \mathbb{N}$. Since $\xi(t_0) = \xi(0) < 0$, one has $\xi(t) < 0$ for all $t \in [0,t_1]$. Then, utilizing (B21) on $[t_1,t_2]$, we can deduce that $\xi(t) < 0$ for any $t \in [t_1,t_2]$. Iterating multiple times in sequence, it follows that $\xi(t) < 0$ for any $t \in [0,\mathcal{Y}_m)$.

Lemma 4. For $d(t)$ given by the second equality of (8), the following inequality holds

$$|\dot{d}(t)|^2 \leqslant \tau_0 d^2(t) + \tau_1\|\hat{\alpha}(\cdot,t)\|^2 + \tau_2\hat{\alpha}^2(0,t) + \tau_3\alpha^2(0,t) + \tau_4\|\omega(\cdot,t)\|^2 + \tau_5\|\upsilon(\cdot,t)\|^2, \tag{B22}$$

for some scalars $\tau_j (j = 0,1,2,3,4,5) > 0$.

Proof. For all $t \in (t_i,t_{i+1})$, $i \in \mathbb{N}$, utilizing (8) and the boundary $\hat{\alpha}(1,t) = \hat{\lambda}(t)U_i$, we deduce

$$\hat{\alpha}(1,t) = \hat{\lambda}(t)d(t) + \int_0^1 \hat{k}(1-\epsilon,t)\hat{\alpha}(\epsilon,t)d\epsilon. \tag{B23}$$

We take the time derivative of the second equality of (8), inserting (4), utilizing integration by parts, and then inserting (B23), one has

$$\begin{aligned}\dot{d}(t) = {}& \frac{\dot{\hat{\lambda}}(t)}{\hat{\lambda}^2(t)}\int_0^1 \hat{k}(1-\epsilon,t)\hat{\alpha}(\epsilon,t)d\epsilon - \frac{1}{\hat{\lambda}(t)}\int_0^1 \hat{k}_t(1-\epsilon,t)\hat{\alpha}(\epsilon,t)d\epsilon - \mu\hat{k}(0,t)d(t) - \frac{\mu}{\hat{\lambda}(t)}\hat{k}(0,t)\int_0^1 \hat{k}(1-\epsilon,t)\hat{\alpha}(\epsilon,t)d\epsilon \\ &+ \frac{\mu}{\hat{\lambda}(t)}\hat{k}(1,t)\hat{\alpha}(0,t) + \frac{\mu}{\hat{\lambda}(t)}\int_0^1 \hat{k}_\epsilon(1-\epsilon,t)\hat{\alpha}(\epsilon,t)d\epsilon - \frac{1}{\hat{\lambda}(t)}\int_0^1 \hat{k}(1-\epsilon,t)\hat{\theta}(\epsilon,t)d\epsilon\alpha(0,t) \\ &- \frac{\dot{\hat{\lambda}}(t)}{\hat{\lambda}(t)}\int_0^1 \hat{k}(1-\epsilon,t)\omega(\epsilon,t)d\epsilon - \frac{r}{\hat{\lambda}(t)}\int_0^1 \hat{k}(1-\epsilon,t)\int_\epsilon^1 \hat{\theta}_t(s,t)\upsilon(1-(s-\epsilon),t)dsd\epsilon.\end{aligned} \tag{B24}$$

Utilizing Young's and Cauchy-Schwarz inequalities, we get

$$\begin{aligned}|\dot{d}(t)|^2 \leqslant {}& 9\mu^2\hat{k}^2(0,t)d^2(t) + 9\Big(\frac{\dot{\hat{\lambda}}^2(t)}{\hat{\lambda}^4(t)}\int_0^1 \hat{k}^2(1-\epsilon,t)d\epsilon + \frac{1}{\hat{\lambda}^2(t)}\int_0^1 \hat{k}_t^2(1-\epsilon,t)d\epsilon + \frac{\mu^2}{\hat{\lambda}^2(t)}\hat{k}^2(0,t)\int_0^1 \hat{k}^2(1-\epsilon,t)d\epsilon \\ &+ \frac{\mu^2}{\hat{\lambda}^2(t)}\int_0^1 \hat{k}_\epsilon^2(1-\epsilon,t)d\epsilon\Big)\|\hat{\alpha}(\cdot,t)\|^2 + \frac{9\mu^2}{\hat{\lambda}^2(t)}\hat{k}^2(1,t)\hat{\alpha}^2(0,t) + \frac{9}{\hat{\lambda}^2(t)}\int_0^1 \hat{k}^2(1-\epsilon,t)d\epsilon\int_0^1 \hat{\theta}^2(\epsilon,t)d\epsilon\alpha^2(0,t) \\ &+ \frac{9\dot{\hat{\lambda}}^2(t)}{\hat{\lambda}^2(t)}\int_0^1 \hat{k}^2(1-\epsilon,t)d\epsilon\|\omega(\cdot,t)\|^2 + \frac{9r^2}{\hat{\lambda}^2(t)}\int_0^1 \hat{k}^2(1-\epsilon,t)d\epsilon\int_0^1 \hat{\theta}_t^2(s,t)ds\|\upsilon(\cdot,t)\|^2.\end{aligned} \tag{B25}$$

Therefore, we obtain

$$|\dot{d}(t)|^2 \leqslant \tau_0 d^2(t) + \tau_1\|\hat{\alpha}(\cdot,t)\|^2 + \tau_2\hat{\alpha}^2(0,t) + \tau_3\alpha^2(0,t) + \tau_4\|\omega(\cdot,t)\|^2 + \tau_5\|\upsilon(\cdot,t)\|^2, \tag{B26}$$

with $\tau_0 = 9\mu^2F_k^2, \tau_1 = 9\Big(\frac{F_{\dot{\hat{\lambda}}}^2F_k^2}{\underline{\chi}^4} + \frac{F_{\hat{k}_t}^2}{\underline{\chi}^2} + \frac{\mu^2F_k^4}{\underline{\chi}^2} + \frac{\mu^2F_{\hat{k}_\epsilon}^2}{\underline{\chi}^2}\Big), \tau_2 = \frac{9\mu^2F_k^2}{\underline{\chi}^2}, \tau_3 = \frac{9F_k^2\bar{\theta}^2}{\underline{\chi}^2}, \tau_4 = \frac{9F_{\dot{\hat{\lambda}}}^2F_k^2}{\underline{\chi}^2}, \tau_5 = \frac{9r^2F_k^2F_{\hat{\theta}_t}^2}{\underline{\chi}^2}$, where $\underline{\chi} = \min\{|\bar{\lambda}|, |\underline{\lambda}|\}$, $\bar{\theta}$ and F_k are given by (B1) and (B14) respectively, $F_{\dot{\hat{\lambda}}}$, $F_{\hat{\theta}_t}$, $F_{\hat{k}_t}$ are positive constants such that $\sup_{t\geqslant0}|\dot{\hat{\lambda}}(t)| \leqslant F_{\dot{\hat{\lambda}}}$, $\sup_{t\geqslant0}\|\hat{\theta}_t(\cdot,t)\| \leqslant F_{\hat{\theta}_t}$, $\sup_{t\geqslant0}\|\hat{k}_t(\cdot,t)\| \leqslant F_{\hat{k}_t}$, which can be guaranteed by (B4) and (B17), and $F_{\hat{k}_\epsilon} > 0$ is a scalar such that $\sup_{0\leqslant\epsilon\leqslant\varkappa\leqslant1,t\geqslant0}|\hat{k}_\epsilon(\varkappa,\epsilon,t)| \leqslant F_{\hat{k}_\epsilon}$, which can be guaranteed by the fact that $\hat{k}(\varkappa,\epsilon,t)$ is continuously differentiable with respect to its arguments.

Proof of Theorem 1. Utilizing Lemma 3, one has $d^2(t) \leqslant -(1-\varpi)\xi(t) - \varpi\xi(t)$ with $\varpi \in (0,1)$ and $\xi(t) < 0$ for all $t \in [0,\mathcal{Y}_m)$. Let us employ the auxiliary function $\Omega(t) := \frac{d^2(t)+(1-\varpi)\xi(t)}{-\varpi\xi(t)}$. It is worthy noting that $\Omega(t)$ is continuous on $[t_i,t_{i+1})$, $i \in \mathbb{N}$ and $\Omega(t_{i+1}^-) = 1$. Moreover, it can be given from $d(t_i) = 0$ that $\Omega(t_i) < 0$. For the function Ω, a lower bound for the minimum dwell-time is determined by the time from $\Omega(t_i)$ to $\Omega(t_{i+1}^-)$. Utilizing the intermediate value theorem, it can be given that there is a constant $t' > t_i$ such that $\Omega(t_i) = 0$ and $\Omega(t) \in [0,1]$ for all $t \in [t', t_{i+1}^-]$.

Go through the same methods as in the proof of [2, Theorem 1] and selecting $\eta_j = \frac{\tau_j}{1-\varpi}(j = 1, 2, 3, 4, 5)$, it is easy to give that the time from $\Omega(t') = 0$ to $\Omega(t_{i+1}^-) = 1$ is

$$t^* = \int_0^1 \frac{1}{\pi_1 s^2 + \pi_2 s + \pi_3} ds > 0, \tag{B27}$$

with $\pi_1 = \nu\varpi$, $\pi_2 = 1+\tau_0+2\nu(1-\varpi)+\rho$, and $\pi_3 = \frac{1-\varpi}{\varpi}(1+\tau_0+\nu(1-\varpi)+\rho)$ being positive constants, where $\tau_j(j = 1, 2, \ldots, 5)$ are defined in Lemma 4. Hence, we obtain $t_{i+1} - t' \geqslant t^*$. Owing to $t_{i+1} - t_i \geqslant t_{i+1} - t'$, we infer $t_{i+1} - t_i \geqslant t^*$. Hence, we write t^* as a lower-bound for the minimum dwell-time. From (B27), we can conclude that t^* depends solely on the system parameter and design parameter, and is independent of initial data.

Appendix C The proof of Theorem 2

Proof. According to Theorem 1, we know that there is a scalar $t^* > 0$ such that $\inf_{\{i\}}\{t_{i+1}-t_i\} \geqslant t^*$, that is, No Zeno phenomenon occurs. Thus, $\mathcal{Y}_m = +\infty$. Together with Proposition 2, we conclude that for given initial data $\alpha_0(\varkappa), \hat{\alpha}_0(\varkappa), \omega_0(\varkappa), \upsilon_0(\varkappa) \in L^2(0, 1)$ compatible with the boundary condition, the closed-loop system composed of the original system (1), filter (2), state estimate (3), and controller (8) exists a unique solution $\alpha, \hat{\alpha}, \omega, \upsilon \in \mathcal{C}([0, +\infty); L^2(0, 1))$.

Next, we show that all signals in the closed-loop system are bounded, and the state of the original system is convergent. The following Lyapunov functions are discussed:

$$V_1(t) = -\xi(t), \tag{C1}$$
$$V_2(t) = \int_0^1 (1+\varkappa)\hat{\beta}^2(\varkappa, t)d\varkappa, \tag{C2}$$
$$V_3(t) = \int_0^1 (1+\varkappa)\omega^2(\varkappa, t)d\varkappa, \tag{C3}$$
$$V_4(t) = \int_0^1 (1+\varkappa)\upsilon^2(\varkappa, t)d\varkappa. \tag{C4}$$

Utilizing (C2) to compute $\dot{V}_2(t)$, substituting the first equality of (7), using the integration by parts, and utilizing the fact $\hat{\beta}(1, t) = \hat{\lambda}(t)d(t)$, it is easy to verify that

$$\begin{aligned}\dot{V}_2(t) = {} & 2\mu\hat{\lambda}^2(t)d^2(t) - \mu\hat{\beta}^2(0, t) - \mu\|\hat{\beta}(\cdot, t)\|^2 - 2\mu\int_0^1 (1+\varkappa)\hat{\beta}(\varkappa, t)\hat{k}(\varkappa, t)d\varkappa\hat{e}(0, t) + 2\dot{\hat{\lambda}}(t)\int_0^1 (1+\varkappa)\hat{\beta}(\varkappa, t)\omega(\varkappa, t)d\varkappa \\ & - 2\dot{\hat{\lambda}}(t)\int_0^1 (1+\varkappa)\hat{\beta}(\varkappa, t)\int_0^\varkappa \hat{k}(\varkappa-\epsilon, t)\omega(\epsilon, t)d\epsilon d\varkappa + 2r\int_0^1 (1+\varkappa)\hat{\beta}(\varkappa, t)G\Big[\int_\varkappa^1 \hat{\theta}_t(\epsilon, t)\upsilon(1-(\epsilon-\varkappa), t)d\epsilon\Big](\varkappa, t)d\varkappa \\ & - 2\int_0^1 (1+\varkappa)\hat{\beta}(\varkappa, t)\int_0^\varkappa \hat{k}_t(\varkappa-\epsilon, t)G^{-1}[\hat{\beta}](\epsilon, t)d\epsilon d\varkappa.\end{aligned} \tag{C5}$$

For the fourth term of (C5), utilizing Cauchy-Schwarz and Young's inequalities to yield

$$\Big| - 2\mu\int_0^1 (1+\varkappa)\hat{\beta}(\varkappa, t)\hat{k}(\varkappa, t)d\varkappa\hat{e}(0, t)\Big| \leqslant \varepsilon_1\|\hat{\beta}(\cdot, t)\|^2 + \frac{4\mu^2F_k^2}{\varepsilon_1}\hat{e}^2(0, t), \tag{C6}$$

with $\varepsilon_1 > 0$ being an arbitrary constant. For the fifth term of (C5), we deduce

$$\Big|2\dot{\hat{\lambda}}(t)\int_0^1 (1+\varkappa)\hat{\beta}(\varkappa, t)\omega(\varkappa, t)d\varkappa\Big| \leqslant \varepsilon_2\|\hat{\beta}(\cdot, t)\|^2 + \frac{4F_{\dot{\hat{\lambda}}}^2}{\varepsilon_2}\|\omega(\cdot, t)\|^2, \tag{C7}$$

with $\varepsilon_2 > 0$ being an arbitrary constant. For the sixth term of (C5), we have

$$\Big| - 2\dot{\hat{\lambda}}(t)\int_0^1 (1+\varkappa)\hat{\beta}(\varkappa, t)\int_0^\varkappa \hat{k}(\varkappa-\epsilon, t)\omega(\epsilon, t)d\epsilon d\varkappa\Big| \leqslant \varepsilon_3\|\hat{\beta}(\cdot, t)\|^2 + \frac{4F_{\dot{\hat{\lambda}}}^2F_k^2}{\varepsilon_3}\|\omega(\cdot, t)\|^2, \tag{C8}$$

with $\varepsilon_3 > 0$ being an arbitrary scalar. For the seventh term of (C5), utilizing Cauchy-Schwarz and Young's inequalities, it follows from (6) and (B15) that

$$\begin{aligned} & \Big|2r\int_0^1 (1+\varkappa)\hat{\beta}(\varkappa, t)G\Big[\int_\varkappa^1 \hat{\theta}_t(\epsilon, t)\upsilon(1-(\epsilon-\varkappa), t)d\epsilon\Big](\varkappa, t)d\varkappa\Big| \\ & \leqslant \varepsilon_4\int_0^1 \hat{\beta}^2(\varkappa, t)d\varkappa + \frac{4r^2F_1^2}{\varepsilon_4}\int_0^1 \Big(\int_\varkappa^1 \hat{\theta}_t(\epsilon, t)\upsilon(1-(\epsilon-\varkappa), t)d\epsilon\Big)^2 d\varkappa \\ & \leqslant \varepsilon_4\|\hat{\beta}(\cdot, t)\|^2 + \frac{4r^2F_1^2F_{\hat{\theta}_t}^2}{\varepsilon_4}\|\upsilon(\cdot, t)\|^2,\end{aligned} \tag{C9}$$

where ε_4 is an arbitrary positive constant. In a similar manner, for the last term of (C5), it can be obtained from the second equality of (6) and (B16) that

$$\begin{aligned} & \Big| - 2\int_0^1 (1+\varkappa)\hat{\beta}(\varkappa, t)\int_0^\varkappa \hat{k}_t(\varkappa-\epsilon, t)G^{-1}[\hat{\beta}](\epsilon, t)d\epsilon d\varkappa\Big| \\ & \leqslant \varepsilon_5\int_0^1 \hat{\beta}^2(\varkappa, t)d\varkappa + \frac{4}{\varepsilon_5}\int_0^1 \Big(\int_0^\varkappa \hat{k}_t(\varkappa-\epsilon, t)G^{-1}[\hat{\beta}](\epsilon, t)d\epsilon\Big)^2 d\varkappa\end{aligned}$$

$$\leqslant \varepsilon_5\|\hat{\beta}(\cdot,t)\|^2 + \frac{4}{\varepsilon_5}\Big(\int_0^1 \hat{k}_t(1-\epsilon,t)G^{-1}[\hat{\beta}](\epsilon,t)d\epsilon\Big)^2$$
$$\leqslant \varepsilon_5\|\hat{\beta}(\cdot,t)\| + \frac{4F_{\hat{k}_t}^2F_2^2}{\varepsilon_5}\|\hat{\beta}(\cdot,t)\|^2, \tag{C10}$$

where $\varepsilon_5 > 0$ is an arbitrary scalar. Taking $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = \varepsilon_4 = \varepsilon_5 = \frac{\mu}{10}$, inserting (C6)-(C10) into (C5), one has

$$\dot{V}_2(t) \leqslant 2\mu\bar{\chi}^2 d^2(t) - \mu\hat{\beta}^2(0,t) - \frac{\mu}{2}\|\hat{\beta}(\cdot,t)\|^2 + 40\mu F_k^2\hat{e}^2(0,t) + \frac{40F_{\hat{\lambda}}^2}{\mu}(1+F_k^2)\|\omega(\cdot,t)\|^2$$
$$+ \frac{4F_1^2F_{\hat{\theta}_t}^2}{\mu^3}\|\upsilon(\cdot,t)\|^2 + \frac{40F_{\hat{k}_t}^2F_2^2}{\mu}\|\hat{\beta}(\cdot,t)\|^2. \tag{C11}$$

Utilizing (C3) to compute $\dot{V}_3(t)$, inserting the first equality of (2), using the integration by parts, and then applying the second equality of (8), it can be verified that

$$\dot{V}_3(t) = 2\mu\omega^2(1,t) - \mu\omega^2(0,t) - \mu\|\omega(\cdot,t)\|^2 \leqslant 4\mu d^2(t) + \frac{4\mu F_{\hat{k}}^2}{\underline{\chi}^2}\|\hat{\alpha}(\cdot,t)\|^2 - \mu\omega^2(0,t) - \mu\|\omega(\cdot,t)\|^2. \tag{C12}$$

We take the time derivative of (C4), and then insert the second equation of (2), one has

$$\dot{V}_4(t) = 2\mu\upsilon^2(1,t) - \mu\upsilon^2(0,t) - \mu\|\upsilon(\cdot,t)\|^2. \tag{C13}$$

From $\hat{e}(\varkappa,t) = \alpha(\varkappa,t) - \hat{\alpha}(\varkappa,t)$ and (6), we get

$$\upsilon(1,t) = \hat{\alpha}(0,t) + \hat{e}(0,t) = \hat{\beta}(0,t) + \hat{e}(0,t). \tag{C14}$$

Inserting (C14) into (C13) yields

$$\dot{V}_4(t) \leqslant 4\mu\hat{\beta}^2(0,t) + 4\mu\hat{e}^2(0,t) - \mu\upsilon^2(0,t) - \mu\|\upsilon(\cdot,t)\|^2. \tag{C15}$$

According to (B5), it can be obtained that

$$\hat{e}^2(0,t) \leqslant \sigma^2(t)(1 + \|\upsilon(\cdot,t)\|^2 + \omega^2(0,t)). \tag{C16}$$

The following Lyapunov function is discussed:

$$V(t) = a_1V_1(t) + a_2V_2(t) + a_3V_3(t) + a_4V_4(t), \tag{C17}$$

where a_1, a_2, a_3, and a_4 are positive scalars to be chosen later. We differentiate (C17) with respect to time t, and then insert (9), (C11), (C12), and (C15). After that, we apply (B16), (C14), and (C16), it follows that

$$\dot{V}(t) \leqslant -(a_1\nu - 2a_2\bar{\chi}^2\mu - 4a_3\mu)d^2(t) + a_1\rho\xi(t) - (a_2\mu - 2a_1\eta_3 - a_1\eta_2 - 4a_4\mu)\hat{\beta}^2(0,t) - \frac{a_2\mu}{2}\|\hat{\beta}(\cdot,t)\|^2$$
$$+ \Big(a_1\eta_1F_2^2 + \frac{4a_3\mu F_2^2F_k^2}{\underline{\chi}^2} + \frac{40a_2F_{\hat{k}_t}^2F_2^2}{\mu}\Big)\|\hat{\beta}(\cdot,t)\|^2 - a_3\mu\|\omega(\cdot,t)\|^2 + \Big(a_1\eta_4 + \frac{40a_2F_{\hat{\lambda}}^2}{\mu}(1+F_k^2)\Big)\|\omega(\cdot,t)\|^2$$
$$+ \Big(a_1\eta_5 + \frac{40a_2F_1^2F_{\hat{\theta}_t}^2}{\mu^3} + \sigma^2(t)\big(2a_1\eta_3 + 4a_4\mu + 40a_2\mu F_k^2\big)\Big)\|\upsilon(\cdot,t)\|^2 - a_4\mu\|\upsilon(\cdot,t)\|^2 - a_4\mu\upsilon^2(0,t)$$
$$- \big(a_3\mu - (2a_1\eta_3 + 4a_4\mu + 40a_2\mu F_k^2)\sigma^2(t)\big)\omega^2(0,t) + \big(2a_1\eta_3 + 4a_4\mu + 40a_2\mu F_k^2\big)\sigma^2(t). \tag{C18}$$

We choose a_1 and a_4 to satisfy $a_2 > \frac{2a_1\eta_3 + a_1\eta_2}{\mu} + 4a_4$ and $a_3 > (\frac{2a_1\eta_3}{\mu} + 4a_4 + 40a_2F_k^2)\sigma^2(t)$. Meanwhile, we take the design parameter $\nu > \frac{2a_2\bar{\chi}^2\mu + 4a_3\mu}{a_1}$. Together with (C17), it can be deduced that

$$\dot{V}(t) \leqslant a_1\rho\xi(t) - \frac{a_2\mu}{2}\|\hat{\beta}(\cdot,t)\|^2 - a_3\mu\|\omega(\cdot,t)\|^2 - a_4\mu\|\upsilon(\cdot,t)\|^2$$
$$+ \hbar_1(t)\|\hat{\beta}(\cdot,t)\|^2 + \hbar_2(t)\|\omega(\cdot,t)\|^2 + \hbar_3(t)\|\upsilon(\cdot,t)\|^2 + \hbar_4(t), \tag{C19}$$

where $\hbar_1(t) = a_1\eta_1F_2^2 + \frac{4a_3\mu F_2^2F_k^2}{\underline{\chi}^2} + \frac{40a_2F_2^2F_{\hat{k}_t}^2}{\mu}$, $\hbar_2(t) = a_1\eta_4 + \frac{40a_2F_{\hat{\lambda}}^2}{\mu}(1+F_k^2)$, $\hbar_3(t) = a_1\eta_5 + \frac{40a_2F_1^2F_{\hat{\theta}_t}^2}{\mu^3} + \sigma^2(t)\big(2a_1\eta_3 + 4a_4\mu + 40a_2\mu F_k^2\big)$, and $\hbar_4(t) = \big(2a_1\eta_3 + 4a_4\mu + 40a_2\mu F_k^2\big)\sigma^2(t)$ are non-negative, bounded and integrable functions. By means of (C1)-(C4), we obtain

$$\dot{V}(t) \leqslant -a_1\rho V_1(t) - \frac{a_2\mu}{4}V_2(t) - \frac{a_3\mu}{2}V_3(t) - \frac{a_3\mu}{2}V_4(t) + \hbar_1(t)V_2(t) + \hbar_2(t)V_3(t) + \hbar_3(t)V_4(t) + \hbar_4(t). \tag{C20}$$

Utilizing (C17) yields

$$\dot{V}(t) \leqslant -\delta V(t) + \hbar(t)V(t) + \hbar_4(t), \tag{C21}$$

with $\delta = \min\{\rho, \frac{\mu}{4}\}$ being a positive constant and $\hbar(t) = \frac{\hbar_1(t)}{a_2} + \frac{\hbar_2(t)}{a_3} + \frac{\hbar_3(t)}{a_4}$ being a non-negative, bounded and integrable function.

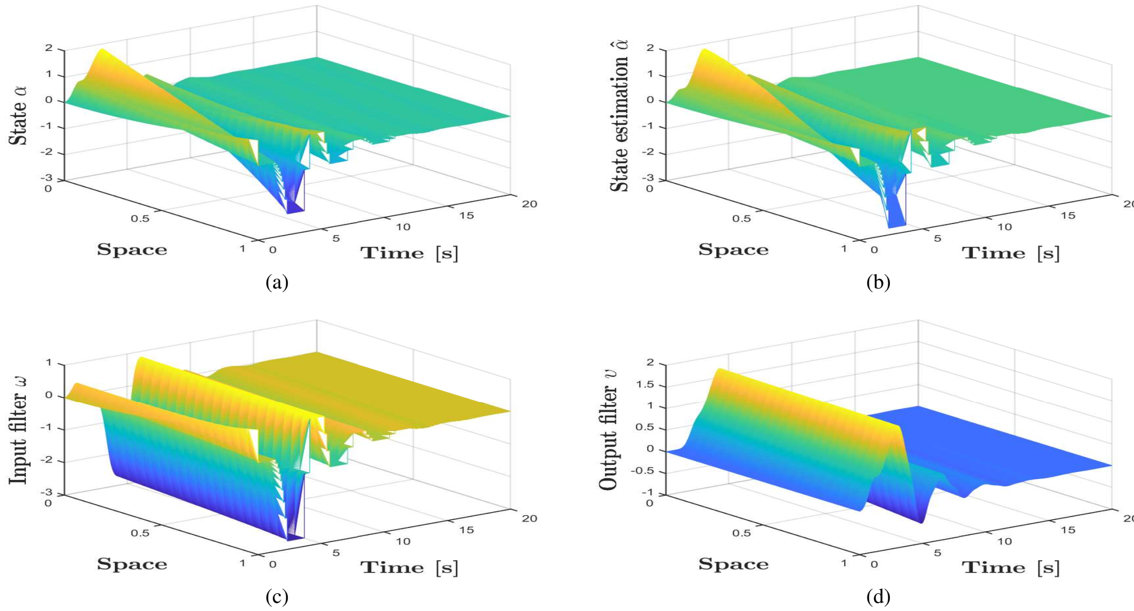

Figure D1 The trajectories of the states in Case 1. (a) State $\alpha(\varkappa, t)$; (b) State estimation $\hat{\alpha}(\varkappa, t)$; (c) Input filter $\omega(\varkappa, t)$; (d) Output filter $\upsilon(\varkappa, t)$.

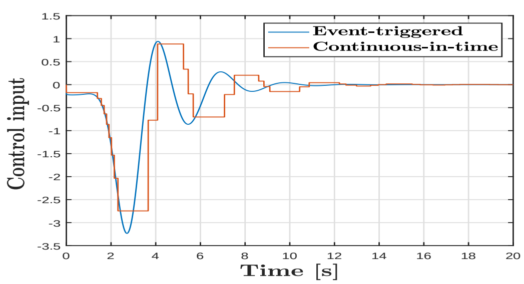

Figure D2 The evolution of the control input in Case 1.

According to Lemma 5 in Appendix E, it can be verified that $V(t)$ is integrable and bounded on $t \in [0, +\infty)$ and $\lim_{t\to+\infty} V(t) = 0$, which means that $|\xi(t)|$, $\|\hat{\beta}(\cdot, t)\|$, $\|\omega(\cdot, t)\|$, and $\|\upsilon(\cdot, t)\|$ are square integrable and bounded on $t \in [0, +\infty)$ and $\lim_{t\to+\infty} |\xi(t)| = 0$, $\lim_{t\to+\infty} \|\hat{\beta}(\cdot, t)\| = 0$, $\lim_{t\to+\infty} \|\omega(\cdot, t)\| = 0$, and $\lim_{t\to+\infty} \|\upsilon(\cdot, t)\| = 0$.

Utilizing the second equation of (6), it can be given that $\|\hat{\alpha}(\cdot, t)\|$ is square integrable and bounded on $t \in [0, +\infty)$ and $\lim_{t\to+\infty} \|\hat{\alpha}(\cdot, t)\| = 0$. From (B4) and $e \equiv 0$ for $t \geqslant r$, we infer that $\|\alpha(\cdot, t)\|$ is square integrable and bounded on $t \in [0, +\infty)$ and $\lim_{t\to+\infty} \|\alpha(\cdot, t)\| = 0$. Proposition 1 means that $U(t)$ is square integrable and bounded on $t \in [0, +\infty)$ and $\lim_{t\to+\infty} U(t) = 0$, so is the event-triggered controller U_i. From the first equation of (2), we obtain that $\sup_{\varkappa\in[0,1]} |\omega(\varkappa, t)|$ is square integrable and bounded on $t \in [0, +\infty)$ and $\lim_{t\to+\infty} \sup_{\varkappa\in[0,1]} |\omega(\varkappa, t)| = 0$. Thus, it can be deduced that $\lim_{t\to+\infty} \sup_{\varkappa\in[0,1]} |\bar{\alpha}(\varkappa, t)| = 0$. Because $e \equiv 0$ for $t \geqslant r$, we obtain that $\lim_{t\to+\infty} \sup_{\varkappa\in[0,1]} |\alpha(\varkappa, t)| = 0$. Due to $\alpha(0, \cdot) \in \mathcal{L}^2([0, +\infty)) \cap \mathcal{L}^\infty([0, +\infty))$ and $\lim_{t\to+\infty} |\alpha(0, t)| = 0$, it follows from the second equation of (2) that $\sup_{\varkappa\in[0,1]} |\upsilon(\varkappa, t)|$ is square integrable and bounded on $t \in [0, +\infty)$ and $\lim_{t\to+\infty} \sup_{\varkappa\in[0,1]} |\upsilon(\varkappa, t)| = 0$. Then, according to (3), it is easy to obtain that $\sup_{\varkappa\in[0,1]} |\hat{\alpha}(\varkappa, t)|$ is square integrable and bounded on $t \in [0, +\infty)$ and $\lim_{t\to+\infty} \sup_{\varkappa\in[0,1]} |\hat{\alpha}(\varkappa, t)| = 0$. Together with the first equation of (6), we obtain that $\sup_{\varkappa\in[0,1]} |\hat{\beta}(\varkappa, t)|$ is square integrable and bounded on $t \in [0, +\infty)$ and $\lim_{t\to+\infty} \sup_{\varkappa\in[0,1]} |\hat{\beta}(\varkappa, t)| = 0$.

Note that the Lyapunov functions $V_2(t) = \int_0^1 \hat{\beta}^2(\varkappa, t)d\varkappa$, $V_3(t) = \int_0^1 \omega^2(\varkappa, t)d\varkappa$, and $V_4(t) = \int_0^1 \upsilon^2(\varkappa, t)d\varkappa$ are not chosen mainly because some favorable terms $-\mu \int_0^1 \hat{\beta}^2(\varkappa, t)d\varkappa$, $-\mu \int_0^1 \omega^2(\varkappa, t)d\varkappa$, and $-\mu \int_0^1 \upsilon^2(\varkappa, t)d\varkappa$ cannot be obtained in derivative calculation, that is, the stability analysis of the closed-loop system cannot be obtained.

Appendix D Simulation

In this part, we take two cases where λ is positive and negative respectively to validate the availability of the theoretical analysis. The finite difference method is utilized to discrete time step and space step as $dt = 0.002$ and $d\varkappa = 0.05$.

Case 1. The simulation model is (1) subject to parameters as follows: $\mu = 0.75$, $\theta(\varkappa) = \frac{1}{2}(1 + e^{-\varkappa}\cosh(\pi\varkappa))$, and $\lambda = 0.8$. The bounds of the unknown function $\theta(\varkappa)$ are set to: $\underline{\theta} = -10$ and $\bar{\theta} = 10$. The bounds of the uncertain parameter λ are set to: $\underline{\lambda} = 0.2$ and $\bar{\lambda} = 100$. The initial data are selected as: $\alpha(\varkappa, 0) = \varkappa\sin(\varkappa)$, $\hat{\theta}(\varkappa, 0) = 0.5$, and $\hat{\lambda}(0) = 0.6$. The gains are selected as $\varsigma_1(\varkappa) = 10\varkappa$ and $\varsigma_2 = 0.5$. The remaining parameters are determined as follows: $\xi(0) = -0.1$, $\rho = 80$, $\nu = 154.3$, $\eta_1 = 12.5$, $\eta_2 = 0.9375$, $\eta_3 = 0.3625$, $\eta_4 = 125.75$, and $\eta_5 = 0.0125$.

The simulation results are obtained and presented in Figures D1-D5. Figure D1 shows the response of states α, $\hat{\alpha}$, ω, and υ

under ETC. The trajectories of the event-triggered and continuous-in-time control inputs are given in Figure D2. Figure D3(a) illustrates the evolution of the dynamic triggering condition, if the trajectory $d^2(t)$ intersects with $-\xi(t)$, an event is triggered. From Figure D3(b), we can observe that the execution count is 34 with a minimal inter-execution time being 0.106s, substantially exceeding the highly conservative minimal dwell-time estimate $4.5 \times 10^{-5}s$ obtained from (B27). It can be seen from Figure D4 that the estimated parameters are both bounded and convergent. At last, simulations are conducted for 100 different initial data determined by $\alpha_0(\varkappa) = \varkappa\sin(n\varkappa), n = 1, 2, \ldots, 100$, and inter-execution times among two triggering moments are computed. The positive constant ρ, characterizing the decay rate of $\xi(t)$ described by (9), is employed to regulate the sampling speed of ETC. Thus, a comparison is drawn between cases of slow sampling and fast sampling, i.e., when $\rho = 1$ and $\rho = 80$, respectively. Figure D5 presents the density of the inter-execution times, demonstrating that when ρ is small, the inter-execution times are larger, and the sampling is less often.

Case 2. Consider the simulation model (1) with $\lambda = -0.8$, the bounds of the uncertain parameter λ are set to: $\underline{\lambda} = -100$ and $\bar{\lambda} = -0.2$, the initial condition is defined as $\hat{\lambda}(0) = -0.6$. The remaining parameters and initial data are consistent with Case 1.

The simulation results are shown in Figures D6-D10. From Figure D6, we can observe that the states α, $\hat{\alpha}$, ω, and υ converge to zero under ETC. The trajectories of the event-triggered and continuous-in-time control inputs are shown in Figure D7. Figure D8(a) illustrates the evolution of the dynamic triggering condition. It can be seen from Figure D8(b) that the execution count is 43 with the minimal inter-execution time being 0.062s, substantially exceeding the highly-conservative minimal dwell-time estimate $1.2\times10^{-5}s$ obtained from (B27). It can be seen from Figure D9 that the estimated parameters θ and λ are bounded and convergent. Figure D10 displays the density of the inter-execution times for 100 distinct initial data. It is worth pointing out that it is just a coincidence that negative λ is required more execution and less minimal dwell-time than positive λ.

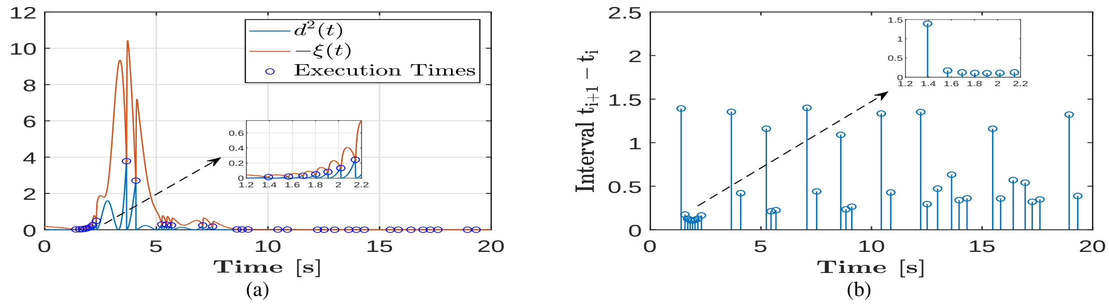

Figure D3 (a) and (b) denote the responses involved in the triggering condition and triggering interval in Case 1, respectively.

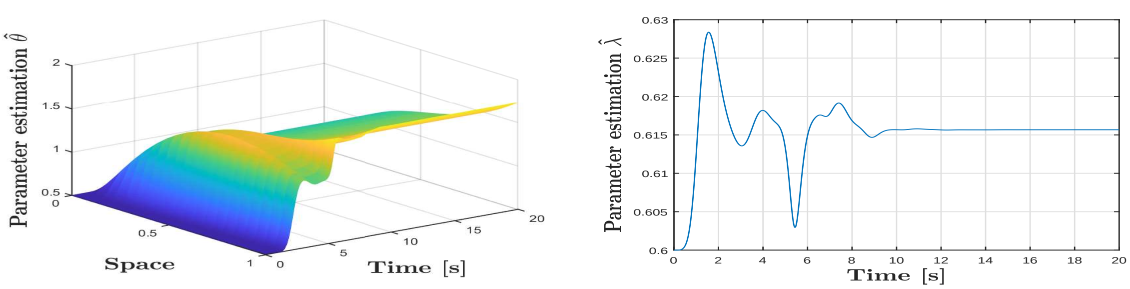

Figure D4 The estimates $\hat{\theta}$ and $\hat{\lambda}$ of the unknown parameters θ and λ in Case 1.

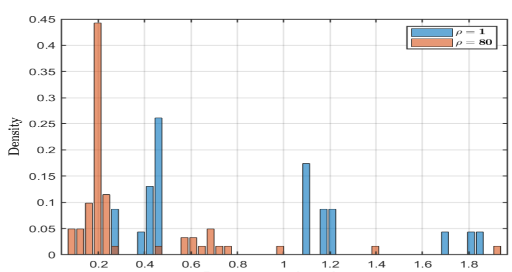

Figure D5 Distribution of the inter-execution times based on 100 distinct initial data: $\alpha_0(\varkappa) = \varkappa\sin(n\varkappa), n = 1, 2, \ldots, 100$ in Case 1.

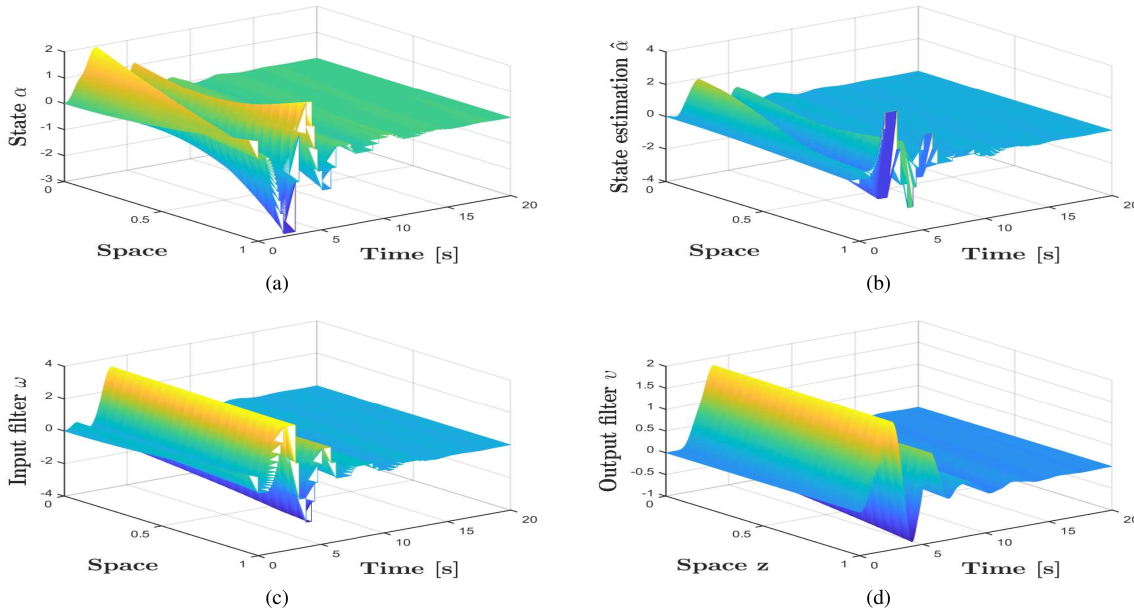

Figure D6 The trajectories of the states in Case 2. (a) State $\alpha(\varkappa, t)$; (b) State estimation $\hat{\alpha}(\varkappa, t)$; (c) Input filter $\omega(\varkappa, t)$; (d) Output filter $\upsilon(\varkappa, t)$.

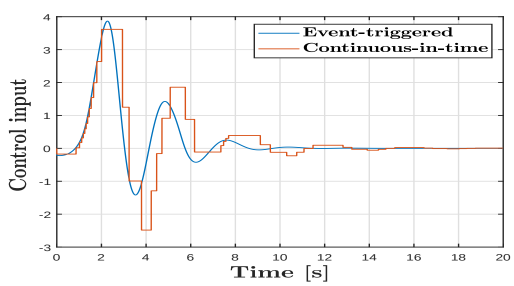

Figure D7 The evolution of the control input in Case 2.

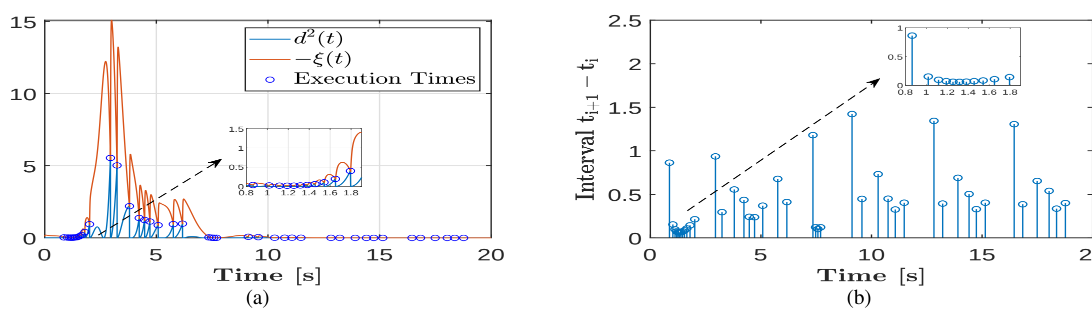

Figure D8 (a) and (b) denote the responses involved in the triggering condition and triggering interval in Case 2, respectively.

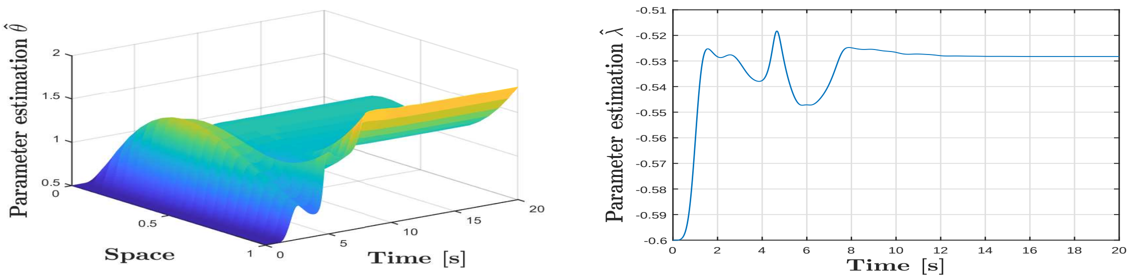

Figure D9 The estimates $\hat{\theta}$ and $\hat{\lambda}$ of the unknown parameters θ and λ in Case 2.

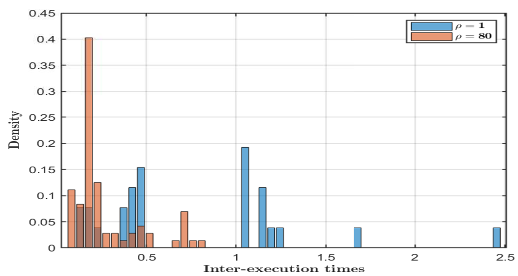

Figure D10 Distribution of the inter-execution times based on 100 distinct initial data: $\alpha_0(\varkappa) = \varkappa\sin(n\varkappa), n = 1, 2, \ldots, 100$ in Case 2.

Appendix E Some Useful Criterion and Inequalities

Minkowski's inequality (*[1]*). For any $\varkappa \in [a, b]$, the following inequality holds:

$$\sqrt{\int_a^b (f(\varkappa) + g(\varkappa))^2 d\varkappa} \leqslant \sqrt{\int_a^b f^2(\varkappa) d\varkappa} + \sqrt{\int_a^b g^2(\varkappa) d\varkappa}, \tag{E1}$$

where $f(\varkappa)$ and $g(\varkappa)$ are two scalar functions.

Cauchy-Schwarz inequality (*[1]*). For any $\varkappa \in [a, b]$, the following inequality holds:

$$\int_a^b f(\varkappa) g(\varkappa) d\varkappa \leqslant \sqrt{\int_a^b f^2(\varkappa) d\varkappa} \sqrt{\int_a^b g^2(\varkappa) d\varkappa}, \tag{E2}$$

where $f(\varkappa)$ and $g(\varkappa)$ are two scalar functions.

Lemma 5 (*[1]*). Suppose ϱ, $\hbar_1$, and $\hbar_2$ are real valued, nonnegative functions defined over $\mathbb{R}^+$, and let $c > 0$ be a scalar. If $\hbar_1$ and $\hbar_2$ belong to $\mathcal{L}^1([0, +\infty))$ and ϱ satisfies $\dot{\varrho}(t) \leqslant -c\varrho(t) + \hbar_1(t)\varrho(t) + \hbar_2(t)$, then

$$\varrho \in \mathcal{L}^1([0, +\infty)) \cap \mathcal{L}^\infty([0, +\infty)), \quad \lim_{t \to +\infty} \varrho(t) = 0. \tag{E3}$$

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2 Rathnayake B, Diagne M, Espitia N, et al. Observer-based event-triggered boundary control of a class of reaction-diffusion PDEs. IEEE Trans Autom Control, 2022, 67 (6): 2905-2917