SCIENCE CHINA
Information Sciences

• LETTER •

April 2024, Vol. 67, Iss. 4, 140204:1–140204:2
https://doi.org/10.1007/s11432-023-3983-7

Special Topic: Analysis and Control of Stochastic Systems

Hybrid stochastic control strategy by two-layer networks for dissipating urban traffic congestion

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Received 18 October 2023/Revised 8 January 2024/Accepted 23 February 2024/Published online 27 March 2024

With the rapid development of mobile devices and geographic information systems, social network data have become part of the traffic condition dissemination system, thereby influencing traffic conditions. Current research has indicated that the dissemination of traffic congestion information can partially alleviate congestion problems and thus enhance road transportation efficiency [1]. Therefore, incorporating an information propagation layer and its coupling relationships into the existing single-layer model for propagating traffic congestion [2] is deemed essential. Furthermore, while random control strategies have seen significant development in various domains [3–5], their exploration in the context of traffic congestion issues warrants further investigation. Building on this analysis, this study focuses on the coupling relationship between congestion conditions and information propagation, constructing a stochastic heterogeneous traffic congestion control model. The specific framework is illustrated in Figure 1.

Model formulation. The links in the urban road network are categorized into three states: free-flow links (F), representing road segments where drivers can maintain their desired speeds; congested links (C), representing road segments where vehicle speeds are limited by the speed of the lead vehicle in the platoon; and recovery links (R), representing road segments where vehicle speeds transition from restricted flow to smooth flow. The congestion propagation mechanisms of the FCR model are as follows. During rush hour, a constant number of Λ_2 links join F per minute. The emigration for each node is μ_2. In the event of increased numbers of vehicles during rush hour, C contacts with upstream and downstream F and α is a rate of F becoming C. When the number of vehicles decreases within C, the congestion state returns to normal and γ is, therefore, the rate of C becoming R.

Hybrid control strategy. In the context of managing congestion, we consider the social network to be the primary control strategy. Three control parameters are introduced to propagate congestion information propagation: Some travelers possess knowledge of road conditions, allowing them to choose alternate routes. By reporting congestion information and guidance by traffic management, certain measures are implemented, termed "Immune measures". Additionally, travelers at congested points can share road conditions spontaneously with other motorists through the social network, referred to as "Crowdsourcing measures".

In social networks, the population is divided into three groups. Travelers who have not encountered the congestion information are designated as ignoramuses (I), travelers who propagate the information by sharing it are labeled as sharers (S), and travelers who have no response to the information are called stiflers (T). The mechanisms for information propagation involve travelers transitioning between these categories based on their awareness of road conditions and their interest levels.

Furthermore, a random control strategy, driven by white noise, is proposed to enhance control performance. In managing congestion at the road layer, two random controllers are designed:

$$
\begin{aligned}
g_1(t) &= \sigma_1[-F(t)\dot{B}_1(t), -C(t)\dot{B}_1(t), -R(t)\dot{B}_1(t), 0, 0, 0]^{\mathrm{T}},\\
g_2(t) &= \sigma_2[-F(t)C(t)\dot{B}_2(t), F(t)C(t)\dot{B}_2(t), 0, 0, 0, 0]^{\mathrm{T}}.
\end{aligned}
$$

The corresponding parameter perturbations are

$$\mu_2 \to \mu_2 + \sigma_1 \mathrm{d}B_1(t),\ \alpha \to \alpha + \sigma_2 \mathrm{d}B_2(t).$$

Using the aforementioned method, the controlled FCR-IST model can be described as

$$
\begin{cases}
\mathrm{d}F(t) = [\Lambda_2 - \alpha F(t)C(t) - \beta F(t)S(t) - \mu_2 F(t)]\,\mathrm{d}t\\
\qquad\quad - \sigma_1 F(t)\mathrm{d}B_1(t) - \sigma_2 F(t)C(t)\mathrm{d}B_2(t),\\
\mathrm{d}C(t) = [\alpha F(t)C(t) - \gamma C(t) - \xi C(t)S(t) - \mu_2 C(t)]\,\mathrm{d}t\\
\qquad\quad - \sigma_1 C(t)\mathrm{d}B_1(t) + \sigma_2 F(t)C(t)\mathrm{d}B_2(t),\\
\mathrm{d}R(t) = [\beta F(t)S(t) + \gamma C(t) + \xi C(t)S(t) - \mu_2 R(t)]\,\mathrm{d}t\\
\qquad\quad - \sigma_1 R(t)\mathrm{d}B_1(t),\\
\mathrm{d}I(t) = [\Lambda_1 - \delta I(t)S(t) - \mu_1 I(t)]\,\mathrm{d}t,\\
\mathrm{d}S(t) = [\delta I(t)S(t) - \theta S(t) - \mu_1 S(t)]\,\mathrm{d}t,\\
\mathrm{d}T(t) = [\theta S(t) - \mu_1 T(t)]\,\mathrm{d}t,
\end{cases}
\tag{1}
$$

with initial value $(F(0), C(0), R(0), I(0), S(0), T(0)) \in R^6_+$ where $R^6_+ = \{(x_1, x_2, x_3, x_4, x_5, x_6) | x_k > 0, k = 1, 2, \ldots, 6\}$. All parameters' values are non-negative.

Theorem 1. For any given initial value, the model (1) has a unique global positive solution with probability one,

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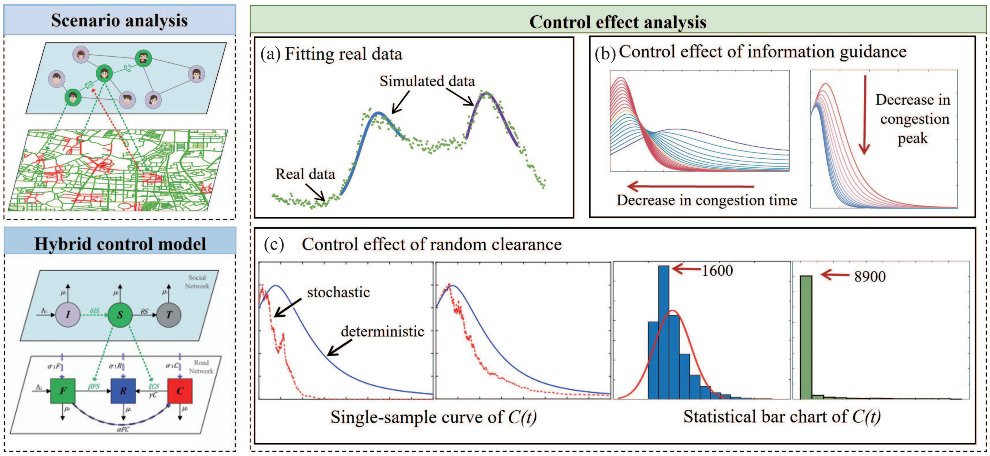

Figure 1 (Color online) Methodology and numerical simulation flow.

namely, $P\{(F(t), C(t), R(t), I(t), S(t), T(t)) \in R_+^6, \forall t \geqslant 0\} = 1$.

The proof of the theorem is presented in Appendix A.

Theorem 2. For the model (1), if the control intensities σ_1 and σ_2 satisfy the condition that

$$\min\left\{\frac{\alpha\Lambda_2}{\mu_2}, \frac{\alpha^2}{2{\sigma_2}^2}\right\} < \max\left\{0, \frac{\xi[\Lambda_1\delta - \mu_1(\theta+\mu_1)]}{(\theta+\mu_1)\delta}\right\} + \gamma + \mu_2 + \frac{1}{2}{\sigma_1}^2,$$

then for any initial value, $C(t)$ obeys

$$\limsup_{t\to\infty} \frac{1}{t}\log C(t) \leqslant \min\left\{\frac{\alpha\Lambda_2}{\mu_2}, \frac{\alpha^2}{2{\sigma_2}^2}\right\} - \frac{\xi[\Lambda_1\delta - \mu_1(\theta+\mu_1)]}{(\theta+\mu_1)\delta} - \gamma - \mu_2 - \frac{{\sigma_1}^2}{2} < 0 \quad \text{a.s.}$$

In other words, the road link will almost certainly be restored to smooth flow.

The proof of the theorem is presented in Appendix B.

Corollary 1. For the model (1), if the condition ${\sigma_1}^2 > \frac{2\alpha\Lambda_2}{\mu_2} - 2\gamma - 2\mu_2$ is satisfied, then for any initial value, it holds that $\limsup_{t\to\infty} \frac{1}{t}\log C(t) < 0$ a.s.

Corollary 2. For the model (1), if the condition ${\sigma_2}^2 > \frac{\alpha^2}{2(\gamma+\mu_2)}$ is satisfied, then for any initial value, it holds that $\limsup_{t\to\infty} \frac{1}{t}\log C(t) < 0$ a.s.

Corollary 3. For the model (1), if the conditions $\Lambda_1\delta > \mu_1(\theta+\mu_1)$ and $\frac{\xi[\Lambda_1\delta-\mu_1(\theta+\mu_1)]}{(\theta+\mu_1)\delta} > \frac{\alpha\Lambda_2}{\mu_2} - \gamma - \mu_2$ are satisfied, then for any initial value, it holds that $\limsup_{t\to\infty} \frac{1}{t}\log C(t) < 0$ a.s.

The proof of the three corollaries is presented in Appendix C.

In the absence of control strategies within the road network, model (1) reverts to the classical susceptible-infectious-recovered (SIR) model. The condition $R_0 = \frac{\alpha\Lambda_2}{\mu_2(\gamma+\mu_2)} < 1$ indicates that traffic congestion will not spread across the road network, as corroborated by [2]. This assertion is further validated by Corollary 1, assuming $\sigma_1 = 0$. Moreover, Corollary 1 establishes that congestion will not proliferate even when R_0 exceeds 1, provided that $\frac{{\sigma_1}^2}{2(\gamma+\mu_2)} > R_0 - 1$. Corollaries 1 and 2 reveal the efficacy of stochastic stabilization in managing traffic congestion. Additionally, Corollary 3 elucidates the impact of informational guidance in mitigating traffic congestion. Specifically, if the number of continuous information spreaders S_1^* is positive, and this information influences the road network, then the synergistic effect of both these factors can mitigate traffic congestion effectively, even when the stochastic intensities σ_1 and σ_2 are null.

Simulation. Through numerical simulation experiments, we validated the accuracy of the model (1) and the effectiveness of the hybrid control approach. Specifically, by comparing actual data with the simulated data generated by the model (1), it was observed that the newly established model aligns well with real-world data, see Figure 1(a). Additionally, we confirmed the efficacy of the hybrid control strategy by incorporating control intensities of varying magnitudes; see Figures 1(b) and (c). Detailed results of this simulation can be seen in Appendix D.

Conclusion. This work proposed stochastic network control strategies for traffic congestion on roads from the perspectives of internal guidance and external intervention. Using three steps: establishing the controlled model, analyzing control effects theoretically, and conducting simulation experiments, the effectiveness of information guidance and random guidance is verified from both the theoretical and simulation perspectives, thereby offering a novel avenue for addressing traffic congestion.

Acknowledgements This work was supported in part by National Natural Science Foundation of China (Grant Nos. 62273157, 62322310, 62333006), Guangdong Basic and Applied Basic Research Foundation (Grant No. 2023B1515020091).

Supporting information Appendixes A–D. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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