SCIENCE CHINA
Information Sciences

• LETTER •

February 2024, Vol. 67, Iss. 2, 129202:1–129202:2
https://doi.org/10.1007/s11432-022-3839-y

Event-triggered impulsive synchronization of heterogeneous neural networks

Zhengxin WANG[1,2], Chongfang JIN[1], Wangli HE[3], Min XIAO[2,4], Guo-Ping JIANG[2,4*] & Jinde CAO[5,6*]

[1]*College of Science, Nanjing University of Posts and Telecommunications, Nanjing* 210023*, China;*
[2]*Jiangsu Engineering Lab for IOT Intelligent Robots (IOTRobot), Nanjing University of Posts and Telecommunications, Nanjing* 210023*, China;*
[3]*Key Laboratory of Smart Manufacturing in Energy Chemical Process, Ministry of Education, East China University of Science and Technology, Shanghai* 200237*, China;*
[4]*School of Automation, Nanjing University of Posts and Telecommunications, Nanjing* 210023*, China;*
[5]*School of Mathematics, Southeast University, Nanjing* 210096*, China;*
[6]*Yonsei Frontier Lab, Yonsei University, Seoul* 03722*, Republic of Korea*

Received 31 December 2022/Revised 8 April 2023/Accepted 12 June 2023/Published online 5 January 2024

Synchronization is one of the most important dynamics in neural networks. Several scholars have studied the quasi-synchronization of heterogeneous neural networks [1], wherein external controllers play a significant role. Impulsive control, a kind of energy-saving control, is activated instantaneously only at specific discrete moments. To overcome the drawbacks of time-driven impulsive control, such as high cost and low efficiency, both centralized [2] and distributed event-triggered controls [3] are proposed. A review of the literature reveals that there are few studies on event-triggered impulsive control of heterogeneous neural networks, especially distributed event-triggered impulsive control. We aim to solve the quasi-synchronization problem of heterogeneous neural networks by applying event-triggered impulsive controls.

Notations. For a real symmetric matrix A, let $\lambda_{\min}(A)$ and $\lambda_{\max}(A)$ denote the minimum and maximum eigenvalues, respectively, and $A \succ 0$ indicates that A is positive definite. For an asymmetric matrix, we define $A_s = (A^{\mathrm{T}}+A)/2$. The terms E_n and 1_n represent an identity matrix with order n and a column vector with all n entries 1, respectively.

Model description. The sets of nodes and directed edges of a digraph $\mathcal{G}$ are recorded as $\mathcal{V} = \{1, 2, \ldots, N\}$ and $\mathcal{E} = \{(j, i) : i, j \in \mathcal{V} \text{ and } j \neq i\}$, respectively. Further, $(j, i) \in \mathcal{E}$ indicates an edge from the jth node to the ith node. $\mathcal{A} = [a_{ij}]_{N\times N}$ is the adjacency matrix, where the entry $a_{ij} = 1$ iff $(j, i) \in \mathcal{E}$. Now, suppose that the graph $\mathcal{G}$ contains a directed spanning tree with the first node as the root and does not contain self-loops. The Laplacian matrix of the subgraph from the 2nd node to the Nth node is defined as $L = [l_{ij}]_{(N-1)\times(N-1)}$. We define $\widetilde{L} = L + \mathrm{diag}\{a_{21}, a_{31}, \ldots, a_{N1}\}$.

Consider heterogeneous neural networks,

$$\dot{v}_i(t) = D_i v_i(t) + B_i f_i(v_i(t)) + c p_i(t) + I_i + u_i(t), \quad (1)$$

$i = 1, 2, \ldots, N$, where $v_i(t) = (v_{i1}(t), v_{i2}(t), \ldots, v_{in}(t))^{\mathrm{T}} \in \mathbb{R}^n$ denotes the state vector of the ith neural network, the matrix $D_i = \mathrm{diag}\{d_{i1}, d_{i2}, \ldots, d_{in}\} \in \mathbb{R}^{n\times n}$ is diagonal with the negative diagonal entries $d_{ij} < 0$, the activation function $f_i(v_i(t)) = [f_{i1}(v_i(t)), f_{i2}(v_i(t)), \ldots, f_{in}(v_i(t))]^{\mathrm{T}}$ is a continuous function with $f_i(0) = 0$, B_i is the constant coefficient matrix of the activation function, and the coefficient c represents the coupling strength for the neural networks. In addition, let $I_i = [I_{i1}, I_{i2}, \ldots, I_{in}]^{\mathrm{T}}$ denote the external input vector, $p_i(t)$ with $p_1(t) = 0$ denote the coupling, and $u_i(t)$ with $u_1(t) = 0$, $i = 1, 2, \ldots, N$ denote the control input. Further, let $\hat{I} = [I_2^{\mathrm{T}}, I_3^{\mathrm{T}}, \ldots, I_N^{\mathrm{T}}]^{\mathrm{T}}$ and $F(v(t)) = [f_2^{\mathrm{T}}(v_2(t)), f_3^{\mathrm{T}}(v_3(t)), \ldots, f_N^{\mathrm{T}}(v_N(t))]^{\mathrm{T}}$.

In this study, the leader of the neural network systems is defined as $v_1(t)$ and is assumed to be bounded.

Assumption 1. For the activation function $f_i(v_i(t))$,

$$[f_i(x) - f_i(y)]^{\mathrm{T}}[f_i(x) - f_i(y)] \leqslant \xi_i (x - y)^{\mathrm{T}}(x - y),$$

holds for any $x, y \in \mathbb{R}^n$, where $\xi_i > 0$, $i = 1, 2, \ldots, N$. Let $\Xi = \mathrm{diag}\{\xi_2, \xi_3, \ldots, \xi_N\}$.

The measurement error is defined as

$$e_i(t) = v_i(t_r) - v_i(t),\ i = 2, 3, \ldots, N, \quad (2)$$

for $t \in [t_r, t_{r+1})$ and the synchronization error is defined as

$$\epsilon_i(t) = v_i(t) - v_1(t),\ i = 2, 3, \ldots, N. \quad (3)$$

Quasi-synchronization under centralized event-triggered impulsive control. $\kappa = \{t_0, t_1, t_2, \ldots\}$ is defined as the impulsive sequence satisfying $\lim_{r\to\infty} t_r = \infty$ and $0 = t_0 < t_1 < t_2 < \cdots$. In addition, the coupling $p_i(t)$ is defined as

$$p_i(t) = \sum_{j=1}^{N} a_{ij}\Gamma(v_j(t_r) - v_i(t_r)), \quad t \in [t_r, t_{r+1}), \quad (4)$$

$i = 2, 3, \ldots, N$, where $\Gamma = \mathrm{diag}\{\gamma_1, \gamma_2, \ldots, \gamma_n\}$ is the inner coupling matrix with $\gamma_k > 0$, $k = 1, 2, \ldots, n$.

* Corresponding author (email: jianggp@njupt.edu.cn, jdcao@seu.edu.cn)

A centralized event-triggered impulsive control (CETIC) is proposed as follows:

$$\text{CETIC}:\begin{cases} u_i=-\bar{c}a_{i1}(v_i(t)-v_1(t))\sum_{r=1}^{\infty}\delta(t-t_r),\\ t_{r+1}=\min\{t:\psi(t)>\varrho\},t_0=0,\end{cases}\tag{5}$$

where $\psi(t)$ will be designed later, $\bar{c}\in(0,1)$, $a_{i1}\geqslant 0$ denotes the pinning strength between node i and the leader, $\delta(\cdot)$ is the Dirac function, and ϱ is a positive constant. In addition, $N_0(t,s)$ is defined as the impulsive number during time interval $[s,t)$. In this study, $v_i(t)$ is assumed to be right-continuous at each impulsive instant $t=t_r$.

Combining (1) and (3),

$$\begin{aligned}\dot{\epsilon}_i(t)=&\ D_i\epsilon_i(t)+B_ih_i(\epsilon_i(t))-c\sum_{j=1}^{N}l_{ij}\Gamma\epsilon_j(t_r)\\ &+I_i-I_1+u_i(t)+G_i(v_1(t)),\end{aligned}\tag{6}$$

where $h_i(\epsilon_i(t))=f_i(v_i(t))-f_i(v_1(t))$, $G_i(v_1(t))=D_iv_1(t)-D_1v_1(t)+B_if_i(v_1(t))-B_1f_1(v_1(t))$, $i=2,3,\dots,N$.

Let $\epsilon(t)=[\epsilon_2^{\mathrm{T}}(t),\epsilon_3^{\mathrm{T}}(t),\dots,\epsilon_N^{\mathrm{T}}(t)]^{\mathrm{T}}$. Thus, the error system can be further expressed in a vector form:

$$\begin{cases}\dot{\epsilon}(t)=D\epsilon(t)+BH(\epsilon(t))-c(\widetilde{L}\otimes\Gamma)\epsilon(t_r)+I+G(v_1(t)),t\neq t_r,\\ \epsilon(t_r)=\big((E_{N-1}-\bar{c}\hat{D})\otimes E_n\big)\epsilon(t_r^-),\end{cases}\tag{7}$$

where $D=\mathrm{diag}\{D_2,D_3,\dots,D_N\}$, $\hat{D}=\mathrm{diag}\{a_{21},a_{31},\dots,a_{N1}\}$, $B=\mathrm{diag}\{B_2,B_3,\dots,B_N\}$, $H(\epsilon(t))=[h_2^{\mathrm{T}}(\epsilon_2(t)),h_3^{\mathrm{T}}(\epsilon_3(t)),\dots,h_N^{\mathrm{T}}(\epsilon_N(t))]^{\mathrm{T}}$, $I=[(I_2-I_1)^{\mathrm{T}},(I_3-I_1)^{\mathrm{T}},\dots,(I_N-I_1)^{\mathrm{T}}]^{\mathrm{T}}$, and $G(v_1(t))=[G_2^{\mathrm{T}}(v_1(t)),G_3^{\mathrm{T}}(v_1(t)),\dots,G_N^{\mathrm{T}}(v_1(t))]^{\mathrm{T}}$.

The event-triggered function is designed as

$$\psi(t)=e^{\mathrm{T}}(t)e(t)-v(t_r)^{\mathrm{T}}Qv(t_r),\tag{8}$$

where $Q=[a_1(\Theta\widetilde{L}\otimes\Gamma)_s-a_1^2(\Theta\widetilde{L}\otimes\Gamma)^{\mathrm{T}}(\Theta\widetilde{L}\otimes\Gamma)]$, $a_1\in(0,1)$, $e(t)=[e_2^{\mathrm{T}}(t),e_3^{\mathrm{T}}(t),\dots,e_N^{\mathrm{T}}(t)]^{\mathrm{T}}$, and $\Theta=\mathrm{diag}\{\mu_2,\mu_3,\dots,\mu_N\}$ is a positive diagonal matrix.

Theorem 1. Zeno behavior can be ruled out under the CETIC (5) and the event-triggered function (8), that is, there is a constant $T_{\min}>0$ such that the interval time $t_{r+1}-t_r>T_{\min}>0$, if

$$(\Theta\widetilde{L}\otimes\Gamma)_s-a_1(\Theta\widetilde{L}\otimes\Gamma)^{\mathrm{T}}(\Theta\widetilde{L}\otimes\Gamma)\succ 0.\tag{9}$$

Theorem 2. Supposing that the conditions of Theorem 1 and Assumption 1 hold, the heterogeneous neural networks (1) can achieve global exponential quasi-synchronization if $\sigma_1+\frac{\ln\rho_1}{T_{\min}}<0$, where $\sigma_1=\lambda_{\max}\{\tilde{D}+\frac{1}{2}(\Theta\otimes E_n)BB^{\mathrm{T}}(\Theta\otimes E_n)^{\mathrm{T}}+(\frac{1}{2}\Xi+E_{N-1})\otimes E_n\}$ with $\tilde{D}=\mathrm{diag}\{\mu_2D_2,\mu_3D_3,\dots,\mu_ND_N\}$, $\rho_1=\lambda_{\max}\{(E_{N-1}-\bar{c}\hat{D})^{\mathrm{T}}\Theta(E_{N-1}-\bar{c}\hat{D})\}/\lambda_{\min}\{\Theta\}$, and $T_{\min}$ is the length of the minimal impulsive interval.

Quasi-synchronization under distributed event-triggered impulsive control. $\kappa^i=\{t_0^i,t_1^i,t_2^i,\dots\},i=2,3,\dots$ is defined as the impulsive sequence of the ith neural network with $0=t_0^i<t_1^i<t_2^i<\cdots$ and $\lim_{r\to\infty}t_r^i=\infty$. The internal coupling is then designed as

$$p_i(t)=\sum_{j=2}^{N}a_{ij}\Gamma(v_j(t_r^i)-v_i(t_r^i)),\quad t\in[t_r^i,t_{r+1}^i),\tag{10}$$

$i=2,3,\dots,N$.

Further, a distributed event-triggered impulsive control (DETIC) is designed as

$$\text{DETIC}:\begin{cases} u_i=-\bar{c}a_{i1}(v_i(t)-v_1(t))\sum_{k=1}^{\infty}\delta(t-t_r^i),\\ t_{r+1}^i=\min\{t:\psi_i(t)>\varrho_2^i\},t_0^i=0,\end{cases}\tag{11}$$

where the event-triggered function $\psi_i(t)=e_i^{\mathrm{T}}(t)e_i(t)+(\frac{a_2^i}{2}+(a_2^i)^2)(\sum_{j=2}^{N}l_{ij}\Gamma v_j(t_r^i))^{\mathrm{T}}\sum_{j=2}^{N}l_{ij}\Gamma v_j(t_r^i)+\frac{a_2^i\chi^2}{2}-a_2^iv_i^{\mathrm{T}}(t_r^i)\sum_{j=2}^{N}l_{ij}\Gamma v_j(t_r^i)$ with $a_2^i\in(0,1)$, $\varrho_2^i>0$, and $\bar{c}\in(0,1)$. In addition, $N_0^i(t,s)$ is defined as the impulsive number of the ith neural network during time interval $[s,t)$.

Different from (2), the measurement error of the ith neural network for $t\in[t_r^i,t_{r+1}^i)$ should be

$$e_i(t)=v_i(t_r^i)-v_i(t),\ i=2,\dots,N.\tag{12}$$

The synchronization error system for the ith neural network can be further expressed as

$$\begin{cases}\dot{\epsilon}_i(t)=D_i\epsilon_i(t)+B_ih_i(\epsilon_i(t))-c\sum_{j=2}^{N}l_{ij}\Gamma\epsilon_j(t_r^i)\\ \qquad\quad+I_i-I_1+G_i(v_1(t)),\ t\neq t_r^i,\\ \epsilon_i(t_r^i)=(1-\bar{c}a_{i1})\epsilon_i(t_r^{i\,-}),\end{cases}\tag{13}$$

where $h_i(\epsilon_i(t))=f_i(v_i(t))-f_i(v_1(t))$, and $G_i(v_1(t))=D_iv_1(t)-D_1v_1(t)+B_if_i(v_1(t))-B_1f_1(v_1(t))$.

Theorem 3. Under the distributed event-triggered impulsive scheme (11), Zeno behavior can be ruled out, that is, there is a constant $T_{\min}^i>0$ such that the interval time $t_{r+1}^i-t_r^i>T_{\min}^i>0,i=2,3,\dots,N$.

Theorem 4. Suppose that the condition of Theorem 3 and Assumption 1 hold. The error system (13) is globally exponentially stable, if $\sigma_2^i+\frac{2\ln(1-\bar{c}a_{i1})}{T_{\min}^i}<0$, where $\sigma_2^i=\mu_i\lambda_{\max}\{D_i+\frac{1}{2}B_iB_i^{\mathrm{T}}+\frac{\xi_i+2}{2}E_n\}$ and $T_{\min}^i$ denotes the length of the minimal impulsive interval of the ith neural network $i=2,3,\dots,N$.

Conclusion. Both centralized and distributed event-triggered impulsive strategies were first designed, and Zeno behaviors were further ruled out. Thereafter, quasi-synchronization of neural networks under the two control strategies was discussed, and several sufficient criteria were derived by utilizing the comparison principle and stability theory. In addition, the upper bounds of the quasi-synchronization errors were presented by sets. Finally, two types of controllers were applied to numerical examples to illustrate the theoretical results (in supplementary).

Acknowledgements This work was supported by Qing Lan Project of Jiangsu Province, Key Project of Natural Science Foundation of China (Grant No. 61833005), National Natural Science Foundation of China (Grant Nos. 42375016, 61873326, 62073172), and Shanghai International Science & Technology Cooperation Program (Grant No. 21550712400).

Supporting information Appendixes A–F. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

References

1 Tang Z, Xuan D, Park J H, et al. Impulsive effects based distributed synchronization of heterogeneous coupled neural networks. IEEE Trans Netw Sci Eng, 2021, 8: 498–510
2 Sun W, Yuan Z, Lu Z, et al. Quasisynchronization of heterogeneous neural networks with time-varying delays via event-triggered impulsive controls. IEEE Trans Cybern, 2022, 52: 3855–3866
3 Tan X, Cao M, Cao J, et al. Event-triggered synchronization of multiagent systems with partial input saturation. IEEE Trans Control Netw Syst, 2021, 8: 1406–1416