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Robustness of interdependent networks with weak dependency links and reinforced nodes

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Interdependent networks are more vulnerable than single, non-interacting networks under cascading failure [1–3]. Percolation theory provides a quantifiable theoretical framework to quantify and evaluate the robustness of networked systems. Most existing models [1–3] on the robustness of interdependent networks mainly focused on the giant connected component (GCC) to explore the structural robustness from the macro perspective, and only the nodes connected to GCC are considered to be functional.

However, some small or finite components (non-GCC) play a non-negligible role in the robustness and percolation transition behaviors of interdependent networks from a microscopic perspective, especially for large-scale network systems. Considering this situation, Yuan et al. [4] first introduced a certain fraction of reinforced nodes to prevent abrupt collapses in interdependent networks and found that the system becomes more robust as the fraction of reinforced nodes increases. Han et al. [5] deployed a fraction of multi-mode addressing nodes in interdependent networks, allowing finite components where the multi-mode addressed nodes are resided to re-establish a connection to the GCC through another addressing mode to enhance the robustness of the system. The above studies [4, 5] have mainly focused on "strong" dependency links in interdependent networks; i.e., the failure of one node in a network will immediately cause the complete failure of all connectivity links of its dependent partner in another network. In practice, interdependent systems with a weak dependency mechanism are more common [2, 3]. Most real interdependent systems have self-sustainability and functional independence, which can result in the failure of a node in one network destroying some connectivity links of its dependent partner in another network.

Considering the aforementioned issues, the combining effects of reinforced finite components and weak dependency links on the robustness of interdependent networks are missing and challenging existing theoretical methods. In this study, a novel generalized percolation model of interdependent networks with weak dependency links and reinforced nodes (WD-RN model) is investigated in theory and simulation. Considering the excessive cost of deploying reinforced nodes, this model can realize the trade-off between high robustness and cost efficiency by tuning weak dependency parameters. The main contributions of this study can be summarized as follows: (1) In contrast to [3–5], in general interdependent networks (Appendix B), by tuning weak dependency parameter and reinforced fraction, we find the existence of a hybrid percolation behavior in sub-network A and propose a theoretical framework to calculate phase transition thresholds and shift points of phase transition types. (2) Interestingly, in symmetric interdependent networks (Appendix C), there is a minimal weak dependency parameter α_c^* or minimum fraction of reinforced nodes ρ_c^*, which can prevent the system from abrupt collapse and can also be used as a critical threshold to distinguish between discontinuous and continuous phase transitions in two interdependent Erdős-Rényi (ER) and scale-free (SF) networks. (3) Importantly, the upper bound $\alpha^*_{\max}$ of α_c^* is found to be related only to the average degree in ER-ER networks. In particular, it is found that the upper bound $\rho^*_{\max}$ of ρ_c^* is a constant 0.1756 regardless of the average degree in ER-ER networks. (4) Our theory agrees well with the numerical simulation results in two interdependent ER and SF networks. We further test our model in independent empirical networks consisting of the power grid and autonomous systems of the Internet and found that increasing weak dependency parameters and the fraction of reinforced nodes can obviously enhance the robustness of the interdependent networks (Appendix D).

WD-RN model. In Appendix A.1, our model is composed of two fully interdependent sub-networks A and B with degree distributions $p_A(k)$ and $p_B(k)$, respectively, containing the same N nodes. A node in a sub-network A depends on one and only one node in sub-network B by a weak dependency link, and vice versa. Here, a weak dependency link structurally means that when a node i (j) in sub-network A (B) fails, each connectivity link of its dependency partner j (i) in sub-network B (A) is disconnected from its neighbor nodes with a probability $1-\alpha_B$ ($1-\alpha_A$), where the parameters α_A and α_B are applied to measure node-dependence strength between networks. We randomly choose a fraction ρ_A and ρ_B of nodes as reinforced nodes in each sub-network, respectively. These reinforced nodes can maintain function of the finite components in which they are located, even if they are disconnected from the mutual giant (largest) con-

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nected component (MGCC). In interdependent networks, these finite components containing at least one reinforced node and the MGCC constitute the mutual generalized giant connected component (MGGCC). Algorithm A1 shows a cascading failure process of our simulation.

Theoretical framework. The proposed model is solved by a series of self-consistent equations based on generating functions [1]. In each sub-network, the generating functions of degree distribution and the associated branching processes are $G_0(x) = \sum_k p(k)\, x^k$ and $G_1(x) = \sum_k \frac{p(k)k}{\langle k\rangle} x^{k-1}$, where $p(k)$ is degree distribution and $\langle k\rangle$ is the average degree in each sub-network. At the stable state, let f_A (f_B) denote the probability that a randomly chosen link belongs to the mutually generalized giant connected component (MGGCC) in sub-network A (B). Thus, f_A and f_B satisfy the following self-consistent equations (Appendix B):

$$f_A = p^2\left[1-(1-\rho_A)G_1^A(1-f_A)\right]\left[1-(1-\rho_B)G_0^B(1-f_B)\right] + p\alpha_A\left[1-(1-\rho_A)G_1^A(1-\alpha_A f_A)\right]\left\{1-p\left[1-(1-\rho_B)G_0^B(1-f_B)\right]\right\}, \quad (1)$$

$$f_B = p^2\left[1-(1-\rho_B)G_1^B(1-f_B)\right]\left[1-(1-\rho_A)G_0^A(1-f_A)\right] + p\alpha_B\left[1-(1-\rho_B)G_1^B(1-\alpha_B f_B)\right]\left\{1-p\left[1-(1-\rho_A)G_0^A(1-f_A)\right]\right\}. \quad (2)$$

Accordingly, the size of the mutually generalized giant connected components (MGGCC) can be expressed as

$$P_\infty^A = p^2\left[1-(1-\rho_A)G_0^A(1-f_A)\right]\left[1-(1-\rho_B)G_0^B(1-f_B)\right] + p\left[1-(1-\rho_A)G_0^A(1-\alpha_A f_A)\right]\left\{1-p\left[1-(1-\rho_B)G_0^B(1-f_B)\right]\right\}, \quad (3)$$

$$P_\infty^B = p^2\left[1-(1-\rho_B)G_0^B(1-f_B)\right]\left[1-(1-\rho_A)G_0^A(1-f_A)\right] + p\left[1-(1-\rho_B)G_0^B(1-\alpha_B f_B)\right]\left\{1-p\left[1-(1-\rho_A)G_0^A(1-f_A)\right]\right\}. \quad (4)$$

Similarly, let $\tilde{f}_A$ ($\tilde{f}_B$) denote the probability that a randomly chosen link reaches the MGCC in sub-network A (B), which can be written out as

$$\tilde{f}_A = p^2\left[1-G_1^A(1-\tilde{f}_A)\right]\left[1-(1-\rho_B)G_0^B(1-f_B)\right] + p\alpha_A\left[1-G_1^A(1-\alpha_A\tilde{f}_A)\right]\left\{1-p\left[1-(1-\rho_B)G_0^B(1-f_B)\right]\right\}, \quad (5)$$

and

$$\tilde{f}_B = p^2\left[1-G_1^B(1-\tilde{f}_B)\right]\left[1-(1-\rho_A)G_0^A(1-f_A)\right] + p\alpha_B\left[1-G_1^B(1-\alpha_B\tilde{f}_B)\right]\left\{1-p\left[1-(1-\rho_A)G_0^A(1-f_A)\right]\right\}. \quad (6)$$

Consequently, the size of the MGCC in sub-network A (B) can be computed as

$$U_\infty^A = p^2\left[1-G_0^A(1-\tilde{f}_A)\right]\left[1-(1-\rho_B)G_0^B(1-f_B)\right] + p\left[1-G_0^A(1-\alpha_A\tilde{f}_A)\right]\left\{1-p\left[1-(1-\rho_B)G_0^B(1-f_B)\right]\right\}, \quad (7)$$

and

$$U_\infty^B = p^2\left[1-G_0^B(1-\tilde{f}_B)\right]\left[1-(1-\rho_A)G_0^A(1-f_A)\right] + p\left[1-G_0^B(1-\alpha_B\tilde{f}_B)\right]\left\{1-p\left[1-(1-\rho_A)G_0^A(1-f_A)\right]\right\}. \quad (8)$$

Thus, Eqs. (1)–(8) are proposed as the general theoretical framework of our model. For simplicity, Eqs. (1), (2), (5), and (6) can be transformed into $f_A = F_1(p, f_B)$, $f_B = F_2(p, f_A)$, $\tilde{f}_A = R_1(p, \tilde{f}_A, f_B)$, and $\tilde{f}_B = R_2(p, \tilde{f}_B, f_A)$. Combining (1), (2), (5), and (6), we can obtain the numerical solutions of f_A, f_B, $\tilde{f}_A$, and $\tilde{f}_B$. Substituting the solutions back into (3), (4), (7), and (8), the numerical solutions of P_∞^A, P_∞^B, U_∞^A, and U_∞^B can be obtained.

For the discontinuous (abrupt) phase transition, the size of the MGGCC abruptly increases at $p = p_c^{\mathrm{I}}$, and the function $f_A = F_1(p, f_B)$ and $f_B = F_2(p, f_A)$ at $p = p_c^{\mathrm{I}}$ satisfy the condition (Appendix B)

$$\frac{\partial F_1(p_c^I, f_B^{\mathrm{I}})}{\partial f_B^{\mathrm{I}}} \cdot \frac{\partial F_2(p_c^I, f_A^{\mathrm{I}})}{\partial f_A^{\mathrm{I}}} = 1, \quad (9)$$

where the curves $f_A = F_1(p, f_B)$ and $f_B = F_2(p, f_A)$ touch each other tangentially at $(f_A^{\mathrm{I}}, f_B^{\mathrm{I}})$. Combining (1), (2), and (9) together, the corresponding solutions of p_c^{I}, f_A^{I}, and f_B^{I} can be achieved.

If sub-network A has continuous phase transition at $p = p_c^{\mathrm{II}}$ for U_∞^A, $p_c^{\mathrm{II}}\,|_{U_\infty^A}$ satisfies the condition (Appendix B)

$$R_1'\left(p_c^{\mathrm{II}}\,|_{U_\infty^A}, 0, f_B\right) = 1. \quad (10)$$

Combining (1), (2), (6), and (10) together, $p_c^{\mathrm{II}}\,|_{U_\infty^A}$ can be obtained numerically.

If sub-network B has continuous phase transition at $p = p_c^{\mathrm{II}}$ for U_∞^B, $p_c^{\mathrm{II}}\,|_{U_\infty^B}$ satisfies the condition (Appendix B)

$$R_2'\left(p_c^{\mathrm{II}}\,|_{U_\infty^B}, 0, f_A\right) = 1. \quad (11)$$

Combining (1), (2), (5), and (11) together, $p_c^{\mathrm{II}}\,|_{U_\infty^B}$ can be obtained numerically.

Simulation and discussion. The extensive simulation results (Appendixes B–D). show that the type of the phase transitions of both sub-networks can be changed from discontinuous to continuous phase transition by tuning weak dependency parameters and reinforced fractions. In Appendixes B.3 and C.2, the simulation results (symbols) are in good agreement with the theoretical predictions (solid lines), indicating the rationality and validity of the WD-RN model.

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Supporting information Appendixes A–D. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

References

1 Buldyrev S V, Parshani R, Paul G, et al. Catastrophic cascade of failures in interdependent networks. Nature, 2010, 464: 1025–1028
2 Liu R R, Li M, Jia C X. Cascading failures in coupled networks: The critical role of node-coupling strength across networks. Sci Rep, 2016, 6: 35352
3 Cao Y Y, Liu R R, Jia C X, et al. Percolation in multilayer complex networks with connectivity and interdependency topological structures. Commun Nonlinear Sci Numer Simul, 2021, 92: 105492
4 Yuan X, Hu Y, Stanley H E, et al. Eradicating catastrophic collapse in interdependent networks via reinforced nodes. Proc Natl Acad Sci USA, 2017, 114: 3311–3315
5 Han W T, Tian L, Zhang F Y, et al. Robustness of interdependent multi-model addressing networks. Sci China Inf Sci, 2021, 64: 169304