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Prescribed-time stabilization and inverse optimal control of stochastic high-order nonlinear systems

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Abstract This paper investigates the prescribed-time state-feedback stabilization and prescribed-time inverse optimality problems for stochastic high-order nonlinear systems. First, a time-varying controller is designed by developing scaled quartic Lyapunov functions, which can guarantee that the system has a unique strong solution almost surely on the prescribed interval for any system initial conditions and that the states and the control converge to the origin in a mean-square form within the prescribed time. Then, the controller is redesigned to address the problem of prescribed-time inverse optimal mean-square stabilization. Finally, a concrete example is provided to confirm the efficiency of the proposed design schemes.

Keywords stochastic high-order nonlinear systems, prescribed-time stabilization, the scaled design, inverse optimality

1 Introduction

Stochastic nonlinear systems have widespread applications in many fields, such as finance and engineering [1, 2].

The stochastic nonlinear design has gained significant attention [3, 4] since Krstic and Deng presented a global asymptotic stabilization control design by developing quartic Lyapunov functions [5–7]. Tremendous advancements have been made in asymptotic stabilization designs for a class of important stochastic systems called stochastic high-order nonlinear systems, whose Jacobian linearizations are likely to possess unstable structures [8–12].

In recent decades, stochastic finite-time control has attracted considerable interest due to its advantages, such as faster convergence rates and improved stability. Refs. [13, 14] first defined finite-time stability and established the stability theory in stochastic finite-time control. Successful treatments of finite-time stabilization problems of stochastic high-order systems are presented in [15,16]. Unfortunately, in the results presented in [13–16], the settling time is uncertain and relies on the initial conditions of a system, which makes it difficult to apply the results in practice.

Prescribed-time control, namely the settling time of a controlled system being deterministic and exactly assignable, is uniquely superior in that it allows the control designers to set the convergence time in advance regardless of the initial conditions of the system. Although numerous solutions to the deterministic prescribed-time control have been reported [17–21], few studies have addressed stochastic settings. In the realm of stochastic prescribed-time control, Ref. [22] addressed the problems of stochastic prescribed-time stabilization together with prescribed-time inverse optimality with nonscaled new Lyapunov functions for the first time. The design employing the new scaled quartic Lyapunov functions proposed by [23] has a smaller control effort than the design proposed by [22]. Ref. [24] proposed two different output feedback control design solutions for the systems where unobservable states and unknown growth rates exist without or with sensor uncertainty to resolve the prescribed-time mean-square stabilization problems. Recently, Ref. [25] resolved mean-nonovershooting prescribed-time control problems in the presence of the finite-time vanishing noise. However, notably, Refs. [22–25] have not considered stochastic high-order nonlinear systems.

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In addition to stabilization, another important topic in stochastic control is inverse optimality. Ref. [6] published the first paper to present inverse optimal designs for general stochastic nonlinear systems. Subsequently, Refs. [26, 27] addressed the inverse optimal design problems of stochastic high-order systems. However, inverse optimal stabilization could only be achieved asymptotically in these studies. Furthermore, to the best of our knowledge, only Ref. [22] has considered the stochastic prescribed-time inverse optimal design but did not consider high-order systems.

Motivated by the above observations, this paper addresses the prescribed-time control problems in stochastic high-order nonlinear systems. The contributions comprise the following three components:

(1) This paper studies more general system models. Unlike the designs described by [28], where a system with only two integrators is investigated, this paper investigates more general stochastic high-order systems with an arbitrary number of integrators. Further, the system model studied herein is more general than those studied by [22–25] because it allows the Jacobian linearizations to have unstable modes.

(2) Unlike the designs described in [15, 16], where the convergence time depends on the initial conditions of a system, the designs developed herein have the significant advantage of the convergence time independent of the initial conditions of the system. The convergence time is deterministic, known, and can be arbitrarily preset as required.

(3) Unlike the inverse optimal designs described by [26, 27] where asymptotic stabilization controllers are constructed to minimize meaningful cost functionals, a new inverse optimal controller design in the prescribed-time sense is developed herein based on the results described by [22, 26].

The rest of the paper is structured as follows: Section 2 elaborates on the problems to be studied. Sections 3 and 4 focus on the design and analysis of the prescribed-time stabilization and prescribed-time inverse optimality, respectively. Section 5 provides a concrete example to demonstrate the validity of our designs. Finally, this study is summarized in Section 6.

Note. The implication of each notation, the requisite definitions, and lemmas are the same as those presented in [22] and have been omitted here.

2 Problem description

Consider the following stochastic high-order nonlinear systems:

$$\mathrm{d}x_i = x_{i+1}^p\mathrm{d}t + \varphi_i^{\mathrm{T}}(t,x)\mathrm{d}\omega,\ i=1,\ldots,n-1, \tag{1}$$

$$\mathrm{d}x_n = u^p\mathrm{d}t + \varphi_n^{\mathrm{T}}(t,x)\mathrm{d}\omega, \tag{2}$$

where $p \geqslant 1$ is an odd integer, $x=(x_1,\ldots,x_n)^{\mathrm{T}} \in \mathbb{R}^n$ is the system state vector, and $u \in \mathbb{R}$ is the control input. $\varphi_i : \mathbb{R}^+ \times \mathbb{R}^n \to \mathbb{R}^m$ are smooth functions and satisfy $\varphi_i(t,0)=0$. ω is an m-dimensional independent standard Wiener process defined on the complete filtered probability space $(\Omega,\mathcal{F},\mathcal{F}_t,P)$ with a filtration $\mathcal{F}_t$ satisfying the usual conditions (i.e., it is increasing and right continuous while $\mathcal{F}_0$ contains all P-null sets).

For the system (1) and (2), the following assumption is made.

Assumption 1. There exists a positive constant c such that the smooth functions $\varphi_i, i=1,\ldots,n$ satisfy

$$|\varphi_i(t,x)| \leqslant c\left(|x_1|^{\frac{p+1}{2}} + \cdots + |x_i|^{\frac{p+1}{2}}\right). \tag{3}$$

Remark 1. Compared with [28], we consider a more general system model in (1) and (2). Specifically, the system (1) and (2) allows that the number of integrators is arbitrary. However, Ref. [28] only studied a system with two integrators. Hence, the system model in [28] is a special case of (1) and (2).

Remark 2. Assumption 1 is natural for the prescribed-time control of stochastic high-order nonlinear systems. When $p=1$, Assumption 1 reduces to Assumption 1 presented in [22]. In addition, Assumption 1 is important in ensuring the prescribed-time stability of (1) and (2). More details can be found in the design process.

Herein, we aim to address prescribed-time control problems for the high-order system (1) and (2). Specifically, we first designed state-feedback controllers to render the system (1) and (2) prescribed-time mean-square stable. Thereafter, a new controller was redesigned to force the system (1) and (2) to achieve inverse optimal stabilization in the prescribed time.

3 Prescribed-time mean-square stabilization

3.1 Controller design

To start with, a scaling function is introduced as follows:

$$\mu(t) = \left(\frac{T}{t_0 + T - t}\right)^m, \ \forall t \in [t_0, t_0 + T), \tag{4}$$

where $T > 0$ refers to any prescribed time and $m \geqslant 2$ is an integer.

Next, we implement the prescribed-time controller design for the system (1) and (2) based on the backstepping technique.

Step 1. We first construct the virtual controller x_2^*.

Define

$$z_1 = x_1, \tag{5}$$

and choose the 1st Lyapunov candidate function

$$V_1 = \frac{1}{4} z_1^4. \tag{6}$$

From (1), (6), and Lemma 1 in [22], we have

$$\begin{aligned} \mathcal{L}V_1 &= z_1^3 x_2^p + \frac{3}{2} z_1^2 |\varphi_1|^2 \\ &\leqslant z_1^3 x_2^p + \frac{3}{2} c^2 z_1^{p+3} \\ &\leqslant z_1^3 (x_2^p - x_2^{*p}) + z_1^3 x_2^{*p} + \frac{3}{2} c^2 \mu^p z_1^{p+3}. \end{aligned} \tag{7}$$

Choosing

$$x_2^* = -\alpha_1 \mu z_1, \tag{8}$$

$$\alpha_1 = \left(n + \frac{3}{2} c^2\right)^{1/p}, \tag{9}$$

which substitute into (7), we can get

$$\mathcal{L}V_1 \leqslant -n\mu^p z_1^{p+3} + z_1^3 \left(x_2^p - x_2^{*p}\right). \tag{10}$$

Deductive step: In this step, we aim to design the virtual controller x_l^*.

Assume that the design procedures from Step 1 to Step $l-1$ have been completed, and the virtual controllers x_j^* for Step $j-1$ $(j = 2, \ldots, l)$ have been constructed as follows:

$$x_j^* = -\alpha_{j-1} \mu z_{j-1}, \tag{11}$$

$$z_{j-1} = x_{j-1} - x_{j-1}^*, \tag{12}$$

where α_{j-1} is a positive constant, and the $(j-1)$th Lyapunov candidate function $V_{j-1} = V_{j-2} + \frac{1}{4\mu^{4(j-2)}} z_{j-1}^4$ satisfies

$$\begin{aligned} \mathcal{L}V_{j-1} \leqslant &-(n-(j-2)) \sum_{k=1}^{j-1} \mu^{p-4(k-1)} z_k^{p+3} + \frac{1}{\mu^{4(j-2)}} z_{j-1}^3 \left(x_j^p - x_j^{*p}\right) \\ &+ \sum_{k=1}^{j-2} (j-k-1) \mu^{-4(k-1)+1} z_k^4 + \sum_{k=2}^{j-1} \Delta_{k2} \mu^{-4k+1}. \end{aligned} \tag{13}$$

Next, we verify that Eq. (13) is also valid for Step l.

Define

$$z_l = x_l - x_l^*, \tag{14}$$

and choose the lth Lyapunov candidate function

$$V_l = V_{l-1} + \frac{1}{4\mu^{4(l-1)}} z_l^4. \tag{15}$$

From (11) and (14) we get

$$z_l = x_l + \sum_{k=1}^{l-1} \beta_k(t) x_k, \tag{16}$$

$$\beta_k = \prod_{i=k}^{l-1} \mu\alpha_i. \tag{17}$$

Applying Itô differentiation rule, we have

$$\mathrm{d}z_l = \left(x_{l+1}^p + \sum_{k=1}^{l-1} \dot{\beta}_k(t) x_k + \sum_{k=1}^{l-1} \beta_k(t) x_{k+1}^p \right) \mathrm{d}t + \left(\varphi_l^{\mathrm{T}} + \sum_{k=1}^{l-1} \beta_k(t) \varphi_k^{\mathrm{T}} \right) \mathrm{d}\omega. \tag{18}$$

From (13), (15), (18), and Lemma 1 in [22], we get

$$\begin{aligned}
\mathcal{L}V_l \leqslant & - \big(n - (l-2)\big) \sum_{k=1}^{l-1} \mu^{p-4(k-1)} z_k^{p+3} + \frac{1}{\mu^{4(l-2)}} z_{l-1}^3 \big(x_l^p - x_l^{*p}\big) \\
& + \sum_{k=1}^{l-2} (l-k) \mu^{-4(k-1)+1} z_k^4 + \sum_{k=2}^{l-1} \Delta_{k2} \mu^{-4k+1} \\
& + \frac{1}{\mu^{4(l-1)}} z_l^3 \left(x_{l+1}^p + \sum_{k=1}^{l-1} \dot{\beta}_k(t) x_k + \sum_{k=1}^{l-1} \beta_k(t) x_{k+1}^p \right) \\
& - \frac{m}{4T} \mu^{-4(l-1)+\frac{1}{m}} z_l^4 + \frac{3}{2\mu^{4(l-1)}} z_l^2 \left| \varphi_l^{\mathrm{T}} + \sum_{k=1}^{l-1} \beta_k(t) \varphi_k^{\mathrm{T}} \right|^2.
\end{aligned} \tag{19}$$

From (11), (12), (14), and Lemmas 2.1, 2.3, 2.5 in [29] we have

$$\begin{aligned}
& \frac{1}{\mu^{4(l-2)}} z_{l-1}^3 \left(x_l^p - x_l^{*p} \right) \\
& \leqslant \frac{1}{\mu^{4(l-2)}} p |z_{l-1}|^3 |z_l| \left(x_l^{p-1} + x_l^{*p-1} \right) \\
& \leqslant \frac{1}{\mu^{4(l-2)}} |z_{l-1}|^3 |z_l| \left((c_m+1) p \alpha_{l-1}^{p-1} \mu^{p-1} z_{l-1}^{p-1} + c_m p z_l^{p-1} \right) \\
& \leqslant (c_m+1) p \alpha_{l-1}^{p-1} \mu^{p-4(l-2)-1} |z_{l-1}|^{p+2} |z_l| + c_m p \mu^{-4(l-2)} |z_{l-1}|^3 |z_l|^p \\
& \leqslant \frac{1}{2} \mu^{p-4(l-2)} z_{l-1}^{p+3} + \Delta_{l1} \mu^{p-4(l-1)} z_l^{p+3},
\end{aligned} \tag{20}$$

where

$$c_m = \max\left\{1, 2^{p-2}\right\}, \tag{21}$$

$$\Delta_{l1} = \frac{1}{p+3} \left(\frac{p+3}{4(p+2)} \right)^{-(p+2)} \left((c_m+1) p \alpha_{l-1}^{p-1} \right)^{p+3} + \frac{p}{p+3} \left(\frac{p+3}{12} \right)^{-3/p} (c_m p)^{(p+3)/p}. \tag{22}$$

From (17) we obtain

$$\beta_k = \mu^{l-k} \prod_{i=k}^{l-1} \alpha_i, \tag{23}$$

$$\dot{\beta}_k = \frac{m(l-k)}{T}\prod_{i=k}^{l-1}\alpha_i\mu^{l-k+1/m} \leqslant \frac{m(l-k)}{T}\prod_{i=k}^{l-1}\alpha_i\mu^{l-k+1}. \tag{24}$$

By (14), (24), and Lemma 2.1 in [29], we obtain

$$\begin{aligned}
&\frac{1}{\mu^{4(l-1)}}z_l^3\sum_{k=1}^{l-1}\dot{\beta}_k(t)x_k\\
&\leqslant |z_l|^3\sum_{k=1}^{l-1}\frac{m(l-k)}{T}\left(\prod_{i=k}^{l-1}\alpha_i\right)\mu^{l-k-4(l-1)+1}(|z_k|+\alpha_{k-1}\mu|z_{k-1}|)\\
&\leqslant \sum_{k=1}^{l-1}\frac{m(l-k)}{T}\left(\prod_{i=k}^{l-1}\alpha_i\right)(1+\alpha_k)\mu^{l-k-4(l-1)+1}|z_k||z_l|^3\\
&\leqslant \sum_{k=1}^{l-1}\mu^{-4(k-1)+1}z_k^4+\frac{3}{4}\times 4^{-\frac{1}{3}}\sum_{k=1}^{l-1}\left(\frac{m(l-k)}{T}\left(\prod_{i=k}^{l-1}\alpha_i\right)(1+\alpha_k)\right)^{\frac{4}{3}}\mu^{-4l+5}z_l^4\\
&\leqslant \sum_{k=1}^{l-1}\mu^{-4(k-1)+1}z_k^4+\mu^{p-4(l-1)}z_l^{p+3}+\Delta_{l2}\mu^{-4l+1},
\end{aligned} \tag{25}$$

where

$$\Delta_{l2} = \frac{p-1}{p+3}\left(\frac{p+3}{4}\right)^{-\frac{4}{p-1}}\left[\frac{3}{4}\times 4^{-\frac{1}{3}}\sum_{k=1}^{l-1}\left(\frac{m(l-k)}{T}\left(\prod_{i=k}^{l-1}\alpha_i\right)(1+\alpha_k)\right)^{\frac{4}{3}}\right]^{\frac{p+3}{p-1}}. \tag{26}$$

By (11), (12), (23), and Lemmas 2.1, 2.3 in [29] we obtain

$$\begin{aligned}
&\frac{1}{\mu^{4(l-1)}}z_l^3\sum_{k=1}^{l-1}\beta_k(t)x_{k+1}^p\\
&\leqslant \frac{1}{\mu^{4(l-1)}}|z_l|^3\sum_{k=1}^{l-1}\mu^{l-k}\left(\prod_{i=k}^{l-1}\alpha_i\right)\left(2^{p-1}|z_{k+1}|^p+2^{p-1}\alpha_k^p\mu^p|z_k|^p\right)\\
&\leqslant \sum_{k=1}^{l-1}2^{p-1}(1+\alpha_k^p)\left(\prod_{i=k}^{l-1}\alpha_i\right)\mu^{p+l-k-4(l-1)}|z_k|^p|z_l|^3+2^{p-1}\alpha_{l-1}\mu^{-4l+3}z_l^{p+3}\\
&\leqslant \frac{1}{4}\sum_{k=1}^{l-1}\mu^{p-4(k-1)}z_k^{p+3}+\Delta_{l3}\mu^{p-4(l-1)}z_l^{p+3},
\end{aligned} \tag{27}$$

where

$$\Delta_{l3} = \frac{3}{p+3}\left(\frac{p+3}{4p}\right)^{-\frac{p}{3}}\sum_{k=1}^{l-1}\left[2^{p-1}(1+\alpha_k^p)\left(\prod_{i=k}^{l-1}\alpha_i\right)\right]^{\frac{p+3}{3}}+2^{p-1}\alpha_{l-1}. \tag{28}$$

By (11), (12), (23), and Lemmas 2.1, 2.3 in [29] we obtain

$$\begin{aligned}
&\frac{3}{2\mu^{4(l-1)}}z_l^2\left|\varphi_l^{\mathrm{T}}+\sum_{k=1}^{l-1}\beta_k(t)\varphi_k^{\mathrm{T}}\right|^2\\
&\leqslant \frac{3lc^2}{2\mu^{4(l-1)}}z_l^2\left(|x_l|^{p+1}+\sum_{k=1}^{l-1}\beta_k^2|x_k|^{p+1}\right)\\
&\leqslant \frac{3lc^2}{2\mu^{4(l-1)}}z_l^2\left(2^pz_l^{p+1}+2^p\alpha_{l-1}^{p+1}\mu^{p+1}z_{l-1}^{p+1}\right.
\end{aligned}$$

$$
\begin{aligned}
&+\sum_{k=1}^{l-1}\mu^{2(l-k)}\left(\prod_{i=k}^{l-1}\alpha_i^2\right)\left(2^p z_k^{p+1}+2^p\alpha_{k-1}^{p+1}\mu^{p+1}z_{k-1}^{p+1}\right)\Bigg)\\
\leqslant{}& \frac{3lc^2}{2\mu^{4(l-1)}}z_l^2\Bigg(2^p z_l^{p+1}+2^{p+1}\alpha_{l-1}^{p+1}\mu^{p+1}z_{l-1}^{p+1}\\
&+\sum_{k=1}^{l-2}2^p\left(1+\alpha_k^{p+1}\right)\left(\prod_{i=k}^{l-1}\alpha_i^2\right)\mu^{p+2(l-k)+1}z_k^{p+1}\Bigg)\\
\leqslant{}& \sum_{k=1}^{l-2}3lc^2 2^{p-1}\left(1+\alpha_k^{p+1}\right)\left(\prod_{i=k}^{l-1}\alpha_i^2\right)\mu^{p+5-2(l+k)}z_k^{p+1}z_l^2\\
&+3lc^2 2^p\alpha_{l-1}^{p+1}\mu^{p+5-4l}z_{l-1}^{p+1}z_l^2+3lc^2 2^{p-1}\mu^{-4(l-1)}z_l^{p+3}\\
\leqslant{}& \frac{1}{4}\sum_{k=1}^{l-1}\mu^{p-4(k-1)}z_k^{p+3}+\Delta_{l4}\mu^{p-4(l-1)}z_l^{p+3},
\end{aligned}\tag{29}
$$

where

$$
\begin{aligned}
\Delta_{l4}={}&\frac{2}{p+3}\left(\frac{p+3}{4(p+1)}\right)^{-\frac{p+1}{2}}\sum_{k=1}^{l-2}\left[3lc^2 2^{p-1}\left(1+\alpha_k^{p+1}\right)\left(\prod_{i=k}^{l-1}\alpha_i^2\right)\right]^{\frac{p+3}{2}}\\
&+\frac{2}{p+3}\left(\frac{p+3}{4(p+1)}\right)^{-\frac{p+1}{2}}\left(3lc^2 2^p\alpha_{l-1}^{p+1}\right)^{\frac{p+3}{2}}+3lc^2 2^{p-1}.
\end{aligned}\tag{30}
$$

Substituting (20)–(29) into (19) results in

$$
\begin{aligned}
\mathcal{L}V_l\leqslant{}&-(n-(l-1))\sum_{k=1}^{l-1}\mu^{p-4(k-1)}z_k^{p+3}+\frac{1}{\mu^{4(l-1)}}z_l^3\left(x_{l+1}^p-x_{l+1}^{*p}\right)\\
&+\frac{1}{\mu^{4(l-1)}}x_{l+1}^{*p}z_l^3+(\Delta_{l1}+\Delta_{l3}+\Delta_{l4}+1)\mu^{p-4(l-1)}z_l^{p+3}\\
&+\sum_{k=2}^{l}\Delta_{k2}\mu^{-4k+1}+\sum_{k=1}^{l-1}(l-k-1)\mu^{-4(k-1)+1}z_k^4.
\end{aligned}\tag{31}
$$

Choosing

$$
x_{l+1}^*=-\alpha_l\mu z_l,\tag{32}
$$

$$
\alpha_l=\left(\Delta_{l1}+\Delta_{l3}+\Delta_{l4}+(n-(l-1))+1\right)^{1/p},\tag{33}
$$

which substitute into (31), we can get

$$
\begin{aligned}
\mathcal{L}V_l\leqslant{}&-(n-(l-1))\sum_{k=1}^{l}\mu^{p-4(k-1)}z_k^{p+3}+\frac{1}{\mu^{4(l-1)}}z_l^3\left(x_{l+1}^p-x_{l+1}^{*p}\right)\\
&+\sum_{k=1}^{l-1}(l-k)\mu^{-4(k-1)+1}z_k^4+\sum_{k=2}^{l}\Delta_{k2}\mu^{-4k+1}.
\end{aligned}\tag{34}
$$

Step n: We now design the actual controller u.

Analogous to (32) and (33), by designing the actual controller

$$
u=-\alpha_n\mu z_n,\tag{35}
$$

where $\alpha_n>0$ is constant, we can acquire

$$
\mathcal{L}V_n\leqslant-\sum_{k=1}^{n}\mu^{p-4(k-1)}z_k^{p+3}+\sum_{k=1}^{n-1}(n-k)\mu^{-4(k-1)+1}z_k^4+\sum_{k=2}^{n}\Delta_{k2}\mu^{-4k+1},\tag{36}
$$

where $V_n=\sum_{k=1}^{n}\frac{1}{4\mu^{4(k-1)}}z_k^4$.

3.2 Stability analysis

Here we present the primary stability results on the system (1) and (2).

Theorem 1. For the system (1) and (2), if Assumption 1 is satisfied, under the controls (8), (11), and (35), the system has a unique solution on $[t_0, t_0+T)$ almost surely for all $x(t_0) \in \mathbb{R}^n$. Furthermore, the equilibrium point of the closed-loop system is prescribed-time mean-square stable with $\lim_{t\to t_0+T} Eu^2 = \lim_{t\to t_0+T} E|x|^2 = 0$.

Proof. From (5), (11), and (12) we have

$$x = \begin{bmatrix} 1 & 0 & \cdots & 0 & 0 \\ -\alpha_1\mu & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \\ 0 & 0 & \cdots & -\alpha_{n-1}\mu & 1 \end{bmatrix} z, \tag{37}$$

where $z = (z_1, \dots, z_n)^{\mathrm{T}}$. From (37) we get

$$|x| \leqslant \sqrt{n + \sum_{i=1}^{n-1} \mu^2 \alpha_k^2}\, |z|, \tag{38}$$

which is equivalent to

$$|z| \geqslant \frac{1}{\sqrt{n + \sum_{k=1}^{n-1} \mu^2 \alpha_k^2}}\, |x|. \tag{39}$$

From (15) and (39), it is easy to see that the first condition (9) in Lemma 1 [22] holds.

From Lemma 2.1 in [29] we have

$$(n-k+1/4)\mu^{-4(k-1)+1} z_k^4 \leqslant \mu^{p-4(k-1)} z_k^{p+3} + \Delta_{n,k+4}\mu^{-4k+1}, \quad k = 1, \dots, n, \tag{40}$$

where

$$\Delta_{n,k+4} = \frac{p-1}{p+3}\left(\frac{p+3}{4}\right)^{-\frac{4}{p-1}} (n-k+1/4)^{\frac{p+3}{p+1}}. \tag{41}$$

Substituting (40) into (36), $\mathcal{L}V_n$ in (36) becomes

$$\mathcal{L}V_n \leqslant -\frac{1}{4}\sum_{k=1}^{n} \mu^{-4(k-1)+1} z_k^4 + \sum_{k=1}^{n} \Delta_k \mu^{-4k+1}, \tag{42}$$

where $\Delta_k = \Delta_{k2} + \Delta_{n,k+4}$ and $\Delta_{12} = 0$. This term indicates the second condition (10) of Lemma 1 in [22] is satisfied.

Therefore, the plant consisting of (1), (2), and (35) satisfies the locally Lipschitz condition. Hence, according to Lemma 1 in [22], the conclusion (1) is valid.

We then verify the conclusion (2).

Defining

$$\rho_k = \inf\{t : t_0 \leqslant t < t_0 + T, |x(t)| \geqslant k\}, \tag{43}$$

where $k \geqslant 0$ is an integer, choosing

$$\bar{V} = \mathrm{e}^{\int_{t_0}^{t} \mu(s)\mathrm{d}s} V_n, \tag{44}$$

and employing Itô's formula over $[t_0, \rho_k \wedge t]$, we arrive at

$$\bar{V}(\rho_k \wedge t, x(\rho_k \wedge t)) = V(t_0, x(t_0)) + \int_{t_0}^{\rho_k \wedge t} \mathcal{L}\bar{V}(x(\tau), \tau)\mathrm{d}\tau + \int_{t_0}^{\rho_k \wedge t} \frac{\partial \bar{V}}{\partial x} g^{\mathrm{T}}(\tau, x(\tau))\mathrm{d}\omega(\tau). \tag{45}$$

Taking expectation on the both sides of (45) and using Dynkin's formula in [1] and (42), we have

$$\begin{aligned}&\mathrm{E}\bar{V}(\rho_k \wedge t, x(\rho_k \wedge t))\\&= V_n(t_0, x(t_0)) + \mathrm{E}\left\{\int_{t_0}^{\rho_k\wedge t} \mathcal{L}\bar{V}(x(\tau),\tau)\mathrm{d}\tau\right\}\\&= V_n(t_0, x(t_0)) + \mathrm{E}\left\{\int_{t_0}^{\rho_k\wedge t} \mathrm{e}^{\int_{t_0}^{\tau}\mu(s)\mathrm{d}s}(\mathcal{L}V_n + \mu V_n)\mathrm{d}\tau\right\}\\&\leqslant V_n(t_0, x(t_0)) + \sum_{i=1}^{n}\int_{t_0}^{\rho_k\wedge t} \Delta_i \mu^{-4i+1}(\tau)\mathrm{e}^{\int_{t_0}^{\tau}\mu(s)\mathrm{d}s}\mathrm{d}\tau. \end{aligned} \tag{46}$$

According to Fatou Lemma, by (44) and (46) we reach

$$\mathrm{E}\bar{V}(t, x(t)) \leqslant V_n(t_0, x(t_0)) + \sum_{i=1}^{n}\int_{t_0}^{t} \Delta_i \mu^{-4i+1}(\tau)\mathrm{e}^{\int_{t_0}^{\tau}\mu(s)\mathrm{d}s}\mathrm{d}\tau. \tag{47}$$

From (44) and (47) we obtain

$$\mathrm{E}V_n \leqslant \mathrm{e}^{-\int_{t_0}^{t}\mu(s)\mathrm{d}s}\left(V_n(t_0, x(t_0)) + \sum_{i=1}^{n}\int_{t_0}^{t} \Delta_i \mu^{-4i+1}(\tau)\mathrm{e}^{\int_{t_0}^{\tau}\mu(s)\mathrm{d}s}\mathrm{d}\tau\right), \quad \forall t \in [t_0, t_0+T). \tag{48}$$

Using Schwarz's inequality and (12), we can acquire

$$\begin{aligned}\mathrm{E}|z|^2 &\leqslant (\mathrm{E}|z|^4)^{1/2}\\&= \big(\mathrm{E}(z_1^2 + \cdots + z_n^2)^2\big)^{1/2}\\&\leqslant \sqrt{n}\big(\mathrm{E}(z_1^4 + \cdots + z_n^4)\big)^{1/2}\\&\leqslant 2\sqrt{n}\mu^{2(n-1)}\big(\mathrm{E}V_n\big)^{1/2}, \quad \forall t \in [t_0, t_0+T). \end{aligned} \tag{49}$$

From (38) and (49) we have

$$\begin{aligned}\mathrm{E}|x|^2 &\leqslant \left(n + \sum_{k=1}^{n-1}\mu^2\alpha_k^2\right)\mathrm{E}|z|^2\\&\leqslant 2\sqrt{n}\mu^{2(n-1)}\left(n + \sum_{k=1}^{n-1}\mu^2\alpha_k^2\right)\big(\mathrm{E}V_n\big)^{1/2}, \quad \forall t \in [t_0, t_0+T). \end{aligned} \tag{50}$$

By (12), (35), and (49) we obtain

$$\begin{aligned}\mathrm{E}u^2 &= \alpha_n^2\mu^2\mathrm{E}|z_n|^2\\&\leqslant \alpha_n^2\mu^2\mathrm{E}|z|^2\\&\leqslant 2\sqrt{n}\alpha_n^2\mu^{2n}\big(\mathrm{E}V_n\big)^{1/2}, \quad \forall t \in [t_0, t_0+T). \end{aligned} \tag{51}$$

From (4), (48), (50), and (51), we can obtain

$$\lim\nolimits_{t\to t_0+T}\mathrm{E}u^2 = \lim\nolimits_{t\to t_0+T}\mathrm{E}|x|^2 = 0. \tag{52}$$

Hence, the theorem is proven.

Remark 3. The value of the growth order $\frac{p+1}{2}$ in Assumption 1 is important. If this order is greater than $\frac{p+1}{2}$, then some terms such as $d_i z_i^k$ $(d_i > 0, k > p+3)$ are generated, which cannot be counteracted by $-\alpha_i z_i^{p+3}$. So Assumption 1 is crucial in ensuring the prescribed-time stability.

Remark 4. In this part, a new design method is proposed for stochastic high-order nonlinear systems (1) and (2) such that the mean-square stabilization within the prescribed time can be achieved. Unlike the asymptotic stabilization control designs in [8–12] where the convergence time is infinite and in the finite-time stabilization designs in [15, 16] wherein the settling time is unknown, stochastic, and reliant on the initial conditions of a system, the superiority of our designs stems from the fact that they enable the control workers to preset the convergence time $t_0 + T$ on demand regardless of the initial conditions of a system. Additionally, from (35), the controller designed herein is time-varying.

4 Prescribed-time inverse optimality

To begin with, we rewrite (1) and (2) as

$$\mathrm{d}x = \begin{bmatrix} x_2^p \\ \vdots \\ x_n^p \\ 0 \end{bmatrix} \mathrm{d}t + \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} u^p \mathrm{d}t + \begin{bmatrix} \varphi_1^{\mathrm{T}} \\ \vdots \\ \varphi_{n-1}^{\mathrm{T}} \\ \varphi_n^{\mathrm{T}} \end{bmatrix} \mathrm{d}\omega$$
$$\triangleq F\mathrm{d}t + G_1 u^p \mathrm{d}t + G_2 \mathrm{d}\omega; \tag{53}$$

the controller

$$u = -\alpha_n \mu z_n \tag{54}$$

leads to

$$\mathcal{L}V_n \leqslant -\mu V_n + \Delta_n \mu^{-3}, \tag{55}$$

where $\Delta_n = \sum_{k=2}^n \Delta_{k2} + \sum_{k=1}^n \Delta_{n,k+4}$ and the meaning of each symbol can be found in Section 3.

We then present the primary results for the prescribed-time inverse optimality problem as follows.

Theorem 2. For the system (1) and (2), if Assumption 1 is satisfied, then the controller

$$u^* = -\left(\frac{2}{3}\beta\right)^{\frac{1}{p}} \alpha_n \mu z_n, \quad \beta \geqslant 2, \quad \forall t \in [t_0, t_0 + T), \tag{56}$$

achieves the prescribed-time mean-square stabilization and also minimizes the cost functional

$$J(u) = \limsup_{r\to\infty} \mathrm{E}\left[2\beta V_n(\tau_r, x(\tau_r)) + \int_{t_0}^{\tau_r} \left(l(t, x(t)) + \frac{27}{16\beta^2 \alpha_n^{3p} \mu^{4(n-1)+3p} z_n^{3p-3}} u^{4p}\right) \mathrm{d}t\right], \tag{57}$$

where

$$\begin{aligned} l(t,x) =& 2\beta\left(\frac{1}{\mu^{4(n-1)-p}} \alpha_n^p z_n^{p+3} - \frac{\partial V_n}{\partial t} - \frac{\partial V_n}{\partial x} F \right. \\ & \left. - \frac{1}{2}\mathrm{Tr}\left\{G_2^{\mathrm{T}} \frac{\partial^2 V_n}{\partial x^2} G_2\right\} + \Delta_n \mu^{-3}\right) + \frac{1}{\mu^{4(n-1)-p}} \beta(\beta-2)\alpha_n^p z_n^{p+3} \end{aligned} \tag{58}$$

is radially unbounded, positive definite, and not necessarily decrescent.

Proof. By incorporating (55) into (58), we get

$$l(t,x) \geqslant 2\beta\mu V_n + \frac{1}{\mu^{4(n-1)-p}} \beta(\beta-2)\alpha_n^p z_n^{p+3}. \tag{59}$$

From (4) and (39) together with the definition of V_n in (15), we have

$$V_n \geqslant \frac{1}{4n\mu^{4(n-1)}} |z|^4 \geqslant \frac{1}{4n\mu^{4(n-1)}(n + \sum_{k=1}^{n-1} \mu^2 \alpha_k^2)^2} |x|^4. \tag{60}$$

Noting $\beta \geqslant 2$, from (59) and (60), we have

$$l(t,x) \geqslant 2\beta\mu V_n \geqslant \frac{\beta\mu}{2n\mu^{4(n-1)}(n + \sum_{k=1}^{n-1} \mu^2 \alpha_k^2)^2} |x|^4. \tag{61}$$

From (61), it is evident that $l(t,x)$ is enough defined well enough. Thereby, $J(u)$ represents a meaningful cost functional.

At first, we verify that Eq. (54) is a stabilizing controller of system (53). From (54) and (55), noting $\beta \geqslant 2$, we obtain

$$\begin{aligned}\mathcal{L}V_n|_{(53)} &= -\frac{2\beta}{3\mu^{4(n-1)-p}}\alpha_n^p z_n^{p+3} + \frac{\partial V_n}{\partial t} + \frac{\partial V_n}{\partial x}F + \frac{1}{2}\mathrm{Tr}\left\{G_2^{\mathrm{T}}\frac{\partial^2 V_n}{\partial x^2}G_2\right\} \\ &\leqslant -\frac{1}{\mu^{4(n-1)-p}}\alpha_n^p z_n^{p+3} + \frac{\partial V_n}{\partial t} + \frac{\partial V_n}{\partial x}F + \frac{1}{2}\mathrm{Tr}\left\{G_2^{\mathrm{T}}\frac{\partial^2 V_n}{\partial x^2}G_2\right\} \\ &\leqslant -\mu V_n + \Delta_n \mu^{-3}. \end{aligned} \tag{62}$$

It can be deduced from (62) and Theorem 1 that the system (53) achieves prescribed-time mean-square stable under the controller (54).

Next, we prove the inverse optimality. From Dynkin's formula in [1] we can acquire

$$\mathrm{E}\left\{V(\tau_r, x) - V(t_0, x) - \int_{t_0}^{\tau_r}\mathcal{L}V(t,x)\mathrm{d}t\right\} = 0. \tag{63}$$

From (57), (58), and (63), we have

$$\begin{aligned} J(u) =& \limsup_{r\to\infty} \mathrm{E}\left\{2\beta V_n(\tau_r, x(\tau_r)) + \int_{t_0}^{\tau_r}\left(l(t,x(t)) + \frac{27}{16\beta^2\alpha_n^{3p}\mu^{4(n-1)+3p}z_n^{3p-3}}u^{4p}\right)\mathrm{d}t\right\} \\ =& \limsup_{r\to\infty} \mathrm{E}\left\{2\beta V_n(t_0, x(t_0)) + \int_{t_0}^{\tau_r}\left(2\beta\mathcal{L}V_n|_{(53)} + l(t,x(t)) \right.\right. \\ &\left.\left. + \frac{27}{16\beta^2\alpha_n^{3p}\mu^{4(n-1)+3p}z_n^{3p-3}}u^{4p}\right)\mathrm{d}t\right\} \\ =& \limsup_{r\to\infty} \mathrm{E}\left\{2\beta V_n(t_0, x(t_0)) + \int_{t_0}^{\tau_r}\left(\frac{2\beta}{\mu^{4(n-1)}}z_n^3 u^p + \frac{\beta^2}{\mu^{4(n-1)-p}}\alpha_n^p z_n^{p+3}\right.\right. \\ &\left.\left. + \frac{27}{16\beta^2\alpha_n^{3p}\mu^{4(n-1)+3p}z_n^{3p-3}}u^{4p}\right)\mathrm{d}t\right\}. \end{aligned} \tag{64}$$

By using Lemma A.3 in [22], we obtain

$$\begin{aligned} -\frac{2\beta}{\mu^{4(n-1)}}z_n^3 u^p &= \beta^2\left(-\frac{1}{\mu^{3(n-1)-\frac{3}{4}p}}\alpha_n^{\frac{3p}{4}}z_n^{\frac{3p}{4}+\frac{9}{4}}\right)\left(\frac{2}{\beta\alpha_n^{\frac{3p}{4}}\mu^{(n-1)+\frac{3}{4}p}z_n^{\frac{3p}{4}-\frac{3}{4}}}u^p\right) \\ &\leqslant \frac{\beta^2}{\mu^{4(n-1)-p}}\alpha_n^p z_n^{p+3} + \frac{27}{16\beta^2\alpha_n^{3p}\mu^{4(n-1)+3p}z_n^{3p-3}}u^{4p}. \end{aligned} \tag{65}$$

The equality in (65) holds if and only if

$$u^* = -\left(\frac{2}{3}\beta\right)^{\frac{1}{p}}\alpha_n\mu z_n, \quad \beta \geqslant 2. \tag{66}$$

Consequently, the minimum of (57) is acquired by (66), and accordingly we have

$$\min_u J(u) = 2\beta V_n(t_0, x(t_0)). \tag{67}$$

Thus, this theorem is proven.

Remark 5. In comparison to those in [26, 27] where the inverse optimal asymptotic stabilization in probability can be ensured, the controller (56) we constructed achieves inverse optimal mean-square stabilization within prescribed time.

Remark 6. As described by Definition 2 in [22], Theorem 2 addresses the inverse optimal mean-square stabilization within prescribed time by selecting $S(t,x) = 2\beta V_n(t,x)$, $k = -4(n-1) - 3p$, $R(x) = (\frac{27}{16\beta^2\alpha_n^{3p}})^{1/4p}$, $\gamma(r) = r^{4p}$, and $l(t,x)$ in (58).

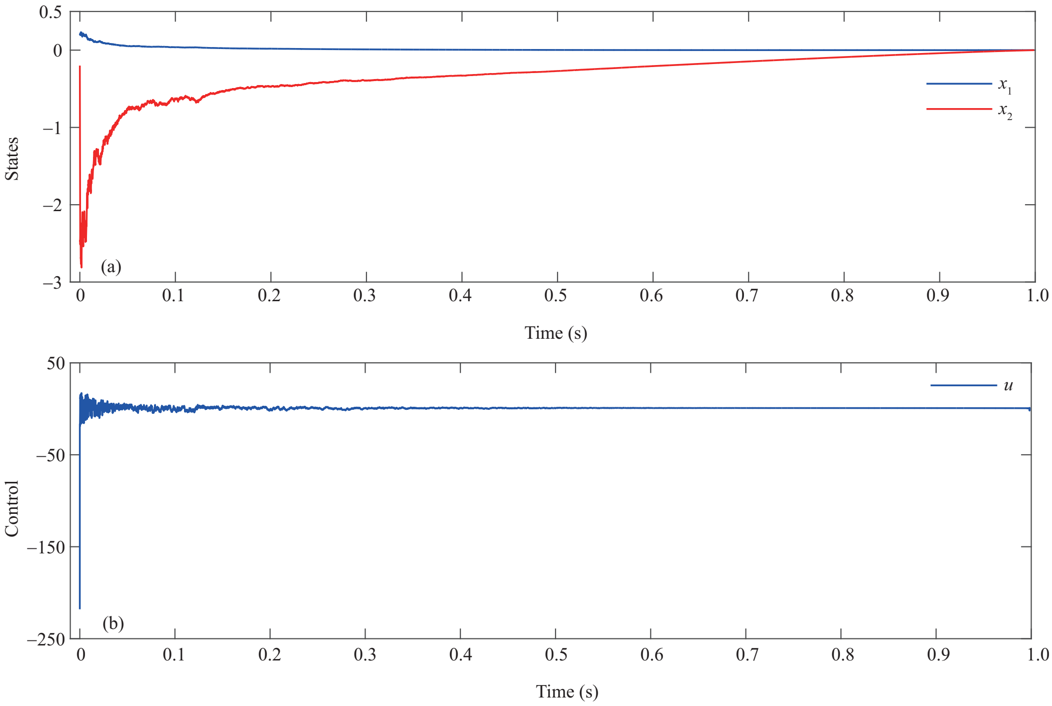

Figure 1 (Color online) Response of (a) the states and (b) the controller of the system (68) and (69).

5 A simulation example

Consider a concrete high-order system

$$\mathrm{d}x_1 = x_2^3\mathrm{d}t + 0.1x_1^2\mathrm{d}\omega, \tag{68}$$

$$\mathrm{d}x_2 = u^3\mathrm{d}t + 0.2x_1x_2\mathrm{d}\omega. \tag{69}$$

Apparently, Assumption 1 can be satisfied by $c = 0.1$.

In simulation, when $l = n = 2$, by choosing $t_0 = 0$, $T = 1$, and $m = 2$, and following (20)–(33), we can acquire the controller as follows:

$$u = -844\mu x_2 - 1069\mu^2 x_1, \tag{70}$$

where $\mu(t) = (\frac{1}{1-t})^2$, $t \in [0, 1)$. Furthermore, if we randomly select the initial states of the system as $x_1(0) = 0.2$, $x_2(0) = -0.2$, the performance of the states and the controller of the system (68) and (69) is given by Figure 1. From Figure 1(a), we obtain $\lim_{t\to 1}\mathrm{E}|x_1|^2 = \lim_{t\to 1}\mathrm{E}|x_2|^2 = 0$, which shows that $\lim_{t\to 1}\mathrm{E}|x|^2 = 0$. From Figure 1(b), we get $\lim_{t\to 1}\mathrm{E}u^2 = 0$.

Thus, the validity of the prescribed-time state-feedback designs developed in Section 3 is verified.

6 Conclusion

This paper investigates the problems of the prescribed-time state-feedback stabilization together with prescribed-time inverse optimality for stochastic high-order nonlinear systems. A new time-varying controller is designed by developing the new scaled quartic Lyapunov functions, which can guarantee that the system has a unique strong solution almost surely on the prescribed interval for any initial conditions and the states and the control converge to the origin in mean-square within the prescribed time. In addition, we redesigned the controller to stabilize the system within the prescribed time and concurrently minimized the meaningful cost functional.

Many significant control problems can be studied in the future. These include finding ways to generalize the results presented herein for adaptive control [30, 31], distributed control [32], privacy security control [33], quantized control [34], sampled-data control [35, 36], Markovian switching [37], and systems with full-state constraints [38]. In addition, noting Ref. [39] proposed an interesting improved optimized backstepping control via reinforcement learning, another interesting topic is generalizing the results we achieved herein to more general systems via reinforcement learning.

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