

A survey on the network models applied in the industrial network optimization

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Abstract Network architecture design is critical for optimizing industrial networks. Network architectures can be classified into small-scale networks and large-scale networks based on scale. Graph theory is an efficient mathematical tool for network topology modeling. For small-scale networks, their structure often has regular topology. For large-scale ones, the current body of work mainly focuses on random characteristics of network nodes and edges. Recently, widely used models include random networks, small-world networks, and scale-free networks. In this study, starting from the scale of the network, network modeling methods based on graph theory as well as their industrial applications, are summarized and analyzed. Moreover, a novel network performance metric, called system entropy, is proposed. From the perspective of mathematical properties, an analysis of its non-negativity and concavity is performed. The advantage of system entropy is that it can cover the existing regular networks, random networks, small-world networks, and scale-free networks, and has strong generality. The simulation results reveal that this proposed metric can achieve the comparison of various industrial networks under different models.

Keywords industrial network, small-scale network, large-scale network, graph theory, system entropy

1 Introduction

Industrial networks refer to networks that connect various devices, machines, sensors, and control systems within an industrial setting [1]. The abstraction of industrial networks facilitates the analysis of network performance. Consequently, network models play an important role in evaluating and optimizing industrial network performance including reliability, low latency, and robustness. A network can be categorized by scale; for example, it can be classified into large-scale networks and small-scale networks. To better describe network performance, graph theory [2–10] is introduced as an analysis tool.

For small-scale networks, the regularity of network topology is remarkable, such as ring networks and star networks. This network found real-world applications, such as redundant ethernet networks with ring topology [11] and gas pipeline monitoring systems [12] with linear topology.

For large-scale networks, the size of the matrix describing the network characteristics increases with the number of network nodes and edges. Moreover, the original small-scale network analysis method is not applicable; thus, studying the statistical characteristics of large-scale networks is vital. In large-scale networks, commonly used network models include random networks [13–15], small-world networks [16–19], and scale-free networks [20–25].

Random networks are a well-known classic model [14, 15] that introduces the probability of connection between nodes. At this point, the topology of the network is no longer fixed. In a random network, the average path length between nodes increases logarithmical with the node network size. This is a classical result of the asymptotic analysis of random networks.

In the 1990s, small-world networks were proposed as a special random network [19], in which the distance between any two nodes is small, and there is a relatively deterministic upper bound for the

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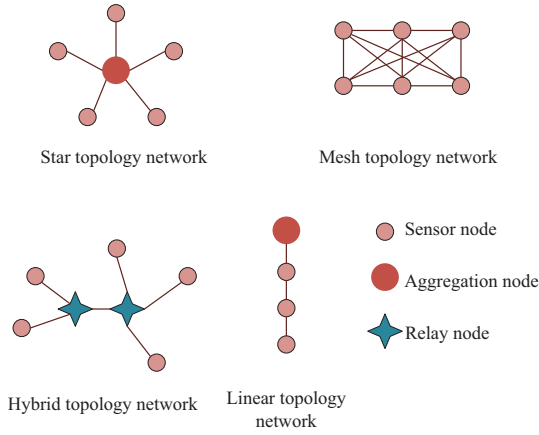


Figure 1 (Color online) Small scale network topology.

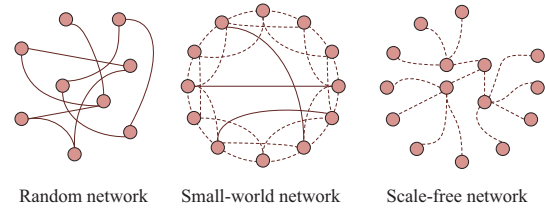


Figure 2 (Color online) Large scale network topology.

distance. This good connectivity relies on certain key nodes in the network. Furthermore, small-world networks have found real-world applications, such as social networks and the Internet [26]. In small-world networks, the clustering coefficient is a crucial performance metric. A higher clustering coefficient indicates that the network has better connectivity and the cost of establishing connections between nodes is lower.

Different from the above two networks, the scale-free network [24] is a network model in which the node degree follows the power law distribution and the distribution value depends on the parameter γ [27, 28]. In scale-free networks, nodes with higher degrees are more likely to obtain new connections. A widely used scale-free network model is called the *BA* model [24]. In this model, both the average path length and the clustering coefficient have closed-form expressions. The analysis results of the *BA* model reveal that its connectivity properties are better than those of small-world networks [25]. This type of network also has real-life applications, such as paper citation networks.

Although research based on the above models has made substantial progress, future industrial networks need to cover a wider range, including not only communication networks represented by 5G/B5G network [29, 30], Internet-of-vehicles [31], and 6G network [32], but also traffic networks [33, 34] and power networks [19, 35–37]. It should be noted that networks between various industrial categories are quite different. For the comparison of different networks, a universal performance metric is required. Thus, in this study, a novel performance metric, called system entropy, is proposed, which is the function of the Laplacian matrix and can cover regular networks, random networks, small-world networks, and scale-free networks. Additionally, the non-negativity of system entropy, its concavity, and symmetry relative to the Laplacian matrix are given. The simulation results reveal that the metric has strong generality and can achieve comparisons of the performance of various industrial networks under different models.

This paper is outlined as follows. Section 2 presents the widely used definitions and performance metrics in graph theory and the calculation methods of the above metrics corresponding to various network models, including random networks, small-world networks, and scale-free networks. Section 3 gives examples of industrial networks based on the existing models as well as the optimization methods based on these models. Section 4 presents a detailed discussion of the properties of system entropy, including non-negativity, symmetry, and concavity relative to the Laplacian matrix. Section 5 highlights the simulation results of the performance of system entropy. Section 6 gives the conclusion and perspective.

2 Typical network topology theory

Based on scale, we classify the network architectures into small-scale networks and large-scale networks. For small-scale networks, the network topology tends to be regular, as shown in Figure 1. For large-scale networks, the network topology is more complex as shown in Figure 2.

Typically, networks consist of multiple nodes, and the connections between these nodes are represented by edges. Consequently, network topology can be abstracted as a graph, as depicted in Figures 1 and 2. Graph theory is employed to systematically model and analyze the properties of network topologies.

In Subsections 2.1 and 2.2, we provide basic definitions and performance metrics in graph theory in Subsection 2.1. Subsection 2.2 outlines the method for calculating performance indicators mapped to existing network models, which include regular networks, random networks, small-world networks, and scale-free networks.

2.1 Basic definitions and performance metrics in graph theory

From an application standpoint, graph theory [7–10] finds extensive use in modeling and analyzing network topology. Nodes and edges are the basic elements of a graph. In network modeling based on graph theory, nodes and edges always correspond to specific physical entities. For example, in communication networks, base stations (BS), user equipment, and sensors can all be modeled as nodes, while the communication links between these nodes can be modeled as edges. In the following, the basic definitions and performance metrics based on graph theory are given.

Definition 1. A graph containing a set of nodes V and a set of edges E can be written as $G = (V, E)$. Each edge $e \in E$ connects two nodes. For example, $e = \langle u, v \rangle$ denotes that e connects $u, v \in V$. $V(G)$ and $E(G)$ represent the set of nodes and edges in the graph G , respectively.

Definition 2. The node degree $\delta(v)$ represents the number of edges the node v is connected to.

Definition 3. For a graph G with n nodes, an $n \times n$ diagonal matrix is used to represent the degree matrix for G which can be defined as

$$D_{i,j} = \begin{cases} \delta(v_i), & \text{if } i = j, \\ 0, & \text{otherwise.} \end{cases} \quad (1)$$

Definition 4. For a graph G with n nodes, the $n \times n$ matrix A is used to represent the adjacency matrix of G and its entry $A_{i,j}$ denotes the weight of the edge connecting v_i and v_j . For an undirected graph, the matrix A is symmetric.

Definition 5. For a graph G with n nodes, the Laplacian matrix L is defined as

$$L = D - A. \quad (2)$$

The symmetric normalized Laplacian matrix is defined as

$$L^n := D^{-\frac{1}{2}} L D^{-\frac{1}{2}} = I - D^{-\frac{1}{2}} A D^{-\frac{1}{2}}. \quad (3)$$

In the following, building upon the aforementioned fundamental concepts, we will introduce several key performance metrics for evaluating graph features. These metrics include degree distribution, average path length, and clustering coefficient.

Definition 6. Let $p(\delta(v))$ denote the probability distribution of vertex degree $\delta(v)$.

Definition 7. For a graph G and $u, v \in V(G)$, $d(u, v)$ indicates the length of a shortest (u, v) -path. The diameter denotes the maximum shortest paths between any two nodes in the network.

Definition 8. For a connected graph G , the average shortest path length from node u to other nodes is calculated as

$$\bar{d}(u) \stackrel{\text{def}}{=} \frac{1}{|V(G)| - 1} \sum_{v \in V(G), v \neq u} d(u, v). \quad (4)$$

The average path length (APL) $d(G)$ is defined as

$$\bar{d}(G) \stackrel{\text{def}}{=} \frac{1}{|V(G)|} \sum_{u \in V(G)} \bar{d}(u) = \frac{1}{|V(G)|^2 - |V(G)|} \sum_{u, v \in V(G), u \neq v} d(u, v). \quad (5)$$

The average path length is an important performance metric for optimizing real-world networks. For instance, on the Internet, a shorter average path length implies lower transmission delays.

Another widely used metric for network analysis is the clustering coefficient. For a specific node v , the clustering coefficient measures the level of connectivity between its neighbor nodes.

Definition 9. For an undirected graph G , $N(v)$ denotes the neighbor set of the node $v \in V(G)$ and $k_v = |N(v)|$. In addition, $G[N(v)]$ refers to the subgraph induced by the neighbors of vertex v in the graph G , and m_v denotes the number of edges in the subgraph composed of the node set $N(v)$, i.e., $m_v = |E(G[N(v)])|$. The clustering coefficient $c(v)$ for vertex v with degree $\delta(v)$ is defined as

$$c(v) \stackrel{\text{def}}{=} \begin{cases} m_v / \binom{k_v}{2} = \frac{2 \cdot m_v}{k_v(k_v-1)}, & \text{if } \delta(v) > 1, \\ \text{undefined}, & \text{otherwise.} \end{cases} \quad (6)$$

Definition 10. For the above graph G , the set V^* consists of the vertices $\{v \in V(G) \mid \delta(v) > 1\}$. The average clustering coefficient (ACC) $C(G)$ for G is

$$C(G) \stackrel{\text{def}}{=} \frac{1}{|V^*|} \sum_{v \in V^*} c(v). \quad (7)$$

It should be noted that there is a difference between the node-based clustering coefficient as presented in (6) and the large-scale network clustering coefficient. The enumeration described in (7) is difficult to realize for large-scale networks. In Subsection 2.2, we will provide formulas for calculating the clustering coefficients of random networks, small-world networks, and scale-free networks.

2.2 Performance metrics in existing network models

While regular networks can be effectively described using graph theory tools and find wide practical applications, network randomness tends to increase with the network's size. Consequently, there arises a need for an efficient network model to describe large-scale networks. To account for this randomness, Erdős et al. [14, 15] proposed the ER network model to describe pure random networks. Building upon this foundation, Watts et al. [19] and Barabási et al. [24] introduced the small-world network model and the scale-free network model to better capture real-world large-scale networks. For networks of varying sizes, the calculation methods of performance indicators differ. In Subsubsection 2.2.1, we provide an analysis of small-scale networks. However, for large-scale networks, enumeration methods are replaced by statistical indicators due to their inherent randomness. In Subsubsections 2.2.2–2.2.4, we delve into the analysis of three types of large-scale network models, including the random network model, small-world network model, and scale-free network model.

2.2.1 Small-scale networks

Regular topologies are common in small-scale networks. Figure 1 shows the typical regular topologies including star network, mesh network, hybrid network, and linear network.

For example, small-scale wireless sensor networks are generally built according to regular topologies. The topologies of small-scale wireless sensor networks have been summarized in [38], encompassing triangular, square, pentagon, hexagon, heptagon, and octagon network topologies. It is worth noting that regular network topologies adhere to classical graph theory models. Consequently, their performance metrics, such as average path length and clustering coefficient, can be calculated using (5)–(7) presented in Subsection 2.1.

2.2.2 Random networks

One popular random network model was proposed by Erdős et al. [14, 15], known as the $ER(n, p_c)$ model. In this model, the network contains n nodes and any two nodes are connected with probability p_c . Therefore, the node degree in the $ER(n, p_c)$ model can only be calculated through its expectation, which is expressed as follows:

$$\bar{\delta}_{ER}(n, p_c) \stackrel{\text{def}}{=} \mathbb{E}[\delta] \stackrel{\text{def}}{=} \sum_{k=1}^{n-1} k \cdot \mathbb{P}[\delta = k] = p_c(n-1). \quad (8)$$

Consider the average path length for the $ER(n, p_c)$ model [25], its expression is given by

$$\bar{d}_{ER}(n, p_c) = \frac{\ln(n) - \gamma}{\ln(p_c n)} + 0.5, \quad (9)$$

where γ is the Euler constant (approximately equal to 0.5772).

It is evident that the average path length in an ER(n, p_c) model depends on its parameters rather than a specific network implementation.

For the clustering coefficient, it is worth noting that the expected clustering coefficient of an ER(n, p) model is equal to p_c , which is also independent of a specific network implementation.

2.2.3 Small-world networks

The small-world network model was introduced by Watts et al. [19], called WS(n, k, p_r) model, where the network contains n nodes, the average node degree is equal to k , and the rewiring probability is equal to p_r .

This small-world model is achieved by incorporating a small number of long-range edges into a regular network, where the diameter is proportional to the network's size. The small-world property is characterized by a very short average path length between any two nodes while maintaining a very high clustering coefficient.

When the rewiring probability p_r is equal to 0, the cluster coefficient of WS($n, k, 0$) model is given by

$$C_{\text{WS}}(k) = \frac{3(k-2)}{4(k-1)}. \quad (10)$$

It can be seen that the clustering coefficient of the WS($n, k, 0$) network model is independent of its scale and approaches 3/4 for large values of k .

Furthermore, the average path length in small-world networks depends on the presence of long-range connecting edges. We will explore the crucial role of long-distance connecting edges in real-world networks in Section 3.

2.2.4 Scale-free networks

In scale-free networks, the distribution of vertex degrees follows a power law. Specifically, the probability $P(k)$ that the degree of a node is equal to k is given by

$$P(k) \sim k^{-\gamma}, \quad (11)$$

where the range of γ is generally $2 < \gamma < 3$ [27, 28]. This implies that in a scale-free network, only a few nodes have a high degree of connectivity, and these nodes with high connectivity can be regarded as key nodes in the network.

Although the ER model and the WS model provide strong guidance, some practical networks still cannot be fully described by these two models. In [24], an efficient scale-free model is proposed and many real-world networks, such as literature citations and actor collaboration, can be accurately described by this model.

According to the analysis in [39], a scale-free network can be achieved through a growth process combined with an operation called preferential attachment. In [24], the construction procedure for scale-free networks is first initially proposed. The construction process combines the growth of the network with the attachment of new nodes to existing nodes with certain preferences (that is, the probability of selecting vertex u is proportional to its degree). This model is referred to as the BA(n, n_0, m) model, where n , n_0 , and m represent the number of complete graph nodes, the initial number of nodes, and the initial number of edges of new nodes, respectively.

The results of the average path length and clustering coefficient of the BA model are given below.

As regards to the average path lengths, the authors in [25] derived the expression for the BA(n, n_0, m) model as follows:

$$\bar{d}_{\text{BA}}(n, m) = \frac{\ln(n) - \ln(m/2) - 1 - \gamma}{\ln(\ln(n)) + \ln(m/2)} + 1.5, \quad (12)$$

where γ is the Euler constant, as defined in (9). Similar to the WS model, the BA model exhibits a short average path length.

The analytical formula for the clustering coefficient of the BA(n, n_0, m) model can be given by

$$C_{\text{BA}}(m, t) = \frac{m^2(m+1)^2}{4(m-1)} \left[\ln\left(\frac{m+1}{m}\right) - \frac{1}{m+1} \right] \frac{[\ln t]^2}{t}, \quad (13)$$

where $t = n - n_0$ denotes that the network is generated by the t -th step from the initial n_0 nodes. This formula demonstrates that as the network scales up, the clustering coefficient of the BA model decreases gradually.

3 Industrial network topology

Recently, as the promotion of 5G [40, 41] and the revolution of the Internet of Things and the industrial Internet [42], intelligent inter-connection and massive access have become the salient features of novel industrial networks [43–47]. As network scales expand, optimizing network architecture becomes crucial for large-scale networks.

A reasonable topology for a specific industrial network scenario should be well configured based on network functions, such as data flow behavior, power consumption, and latency. With increasing network scale, scalability and robustness against malicious attacks have emerged as critical evaluation indicators for network design. Network scalability concerns whether the addition of new nodes can minimize the impact on network performance. In addition, robustness against malicious attacks considers whether the addition of a large number of new nodes will lead to a significant increase in pressure on hubs, thereby impacting performance.

Based on the topological types introduced in Subsection 2, this section summarizes commonly used network models. Through these examples, it discusses how to select appropriate topological structures in real-world scenarios based on specific requirements and structural characteristics.

Subsections 3.1–3.3 provide details on industrial examples and optimization methods based on regular networks, small-world networks, and scale-free networks, respectively. Subsequently, the fourth subsection offers a summary of the current industrial network models.

3.1 Regular networks

Regular topologies find extensive applications in industrial networks, where various models are utilized to accommodate specific industrial networks. Common network performance indicators include network diameter, average path length, and clustering coefficient. Selecting an appropriate topology is crucial for achieving network optimization. Typical regular topology models include linear, mesh, tree, cluster, cluster tree, ring, star, and bus topologies. Specific cases regarding the application of these topologies are analyzed in the subsequent subsections. In practical industrial applications, networks can employ either a single-type topology or a multi-type topology according to requirements.

3.1.1 *Single-type topology*

A suitable network topology should well adapt to real-world application scenarios, maximizing its benefits. Here, we explore three application cases of single-type topologies.

Wireless sensor networks in the oil and gas industry utilize a linear topology aligned with the linear pipelines [12]. This choice reflects the linear nature of oil and gas pipelines, and if the sensors are monitoring the pipe at a safe distance, the sensor network will also have a linear topology.

In redundant ethernet, packets are guaranteed to reach their destination even if there is a link failure [11]. Traditional star topologies of common ethernet networks are neither flexible nor reliable enough due to their dependence on switches or hubs. Redundant ethernet systems, based on ring topologies without switches and hubs, effectively address this issue. A main problem in ring networks is that broadcast messages, if not blocked, fall into an infinite loop of ring networks. To prevent unexpected packet storms in ring networks, a logical polling mechanism was proposed [11].

The topologies of data center networks (DCNs) are mainly split into switch-centered and server-centered networks. Recently, due to the revolution of cloud computing applied in DCN, the traditional topology is unable to meet the requirements. Therefore, FAT-Tree topology was introduced in DCN [48] to handle the bandwidth bottlenecks and single failures.

3.1.2 *Multi-type topology*

In more complex application scenarios, the combination of multiple network topologies is often employed to leverage their respective advantages [49]. Here, we present two application cases of optimized multi-type topologies.

To deal with the time-limited traffic in remote sensing and actuator control in production automation and factory monitoring, a multi-hop cluster tree structure is introduced. The limited delay capability of cluster tree networks enables IEEE 802.15.4 to support time-sensitive traffic [50]. This topology features multiple levels of parent-child relationships among routers with the lowest level determining the height of the tree, allowing the network to calculate the delay from the source to the coordinator.

To ensure real-time characteristics and reliability, industrial-switched ethernet tends to adopt a simple network topology scheme with a third-order ring topology combined with a tree topology. From a performance perspective, the tree topology provides improved real-time performance due to its lower delay, while the ring topology enhances network redundancy due to its fast recovery capabilities. The ring-tree topology combines the structural advantages of both topologies to construct a hybrid ring-tree topology. It employs the ring topology in the upper layer to provide a reliable network and the tree topology in the lower layer to achieve shorter delays [51].

3.2 Small-world networks

As network scales expand, network complexity grows, making complex networks a significant research focus in recent years. One crucial property of complex networks is the small-world property [52]. Its application in industrial networks can give full play to its advantages, including a smaller average path length, larger clustering coefficient, and improved scalability [53, 54].

Small-world networks can be seen as a compromise between regular and random networks [55]. Numerous studies [56, 57] have demonstrated that implementing small-world networks can reduce energy consumption and enhance efficiency. In practical applications, a high number of end-to-end hops often leads to increased latency. Therefore, introducing a small number of long-range connections can transform a regular or random network into a small-world network, effectively reducing network latency [58].

Industrial networks have inherent requirements for low latency [59, 60] and high energy efficiency [61, 62]. Nowadays, small-world properties are observed in various industrial networks, including traffic networks, power networks, and wireless sensor networks (WSNs).

In the following, we will introduce commonly used optimization approaches that focus on long-range connection edges and discuss how network topologies inspired by the small-world model are generated.

3.2.1 The effect of long-range connected edges

The introduction of long-range connected edges is a popular optimization strategy for small-world networks. In the real world, such as various traffic networks [63–65], including university campus traffic networks and air traffic networks, the presence of small-world properties has been observed. The method of adding long-range connected edges can vary depending on the specific network type and engineering requirements.

For instance, in the modeling of university campus traffic networks in [33], five universities are considered, including Huazhong University of Science and Technology (HUST), Wuhan University (WU), Wuhan University of Technology (WUT), Central South University (CSU), and Sun Yat-sen University (SYSU). In the proposed university campus traffic network model, road intersections are represented as nodes, and roads as links. Based on the analysis of degree distribution, clustering coefficient, average path length, network density, and network tightness, it is shown that the university campus traffic network reveals small-world property [33, 66]. In this context, wide trunk roads can be regarded as equivalent long-distance connecting edges, which prove pivotal in improving the traffic situation.

Another example is the global air traffic network [67], where airports serve as nodes and air routes between them form the connecting edges. This network exhibits a short path length and high clustering coefficient. With the escalating demand for air travel, challenges such as increased delays, delay propagation, and airspace congestion emerge in this network [34].

To address these challenges, Boeing's new generation of passenger aircraft 787 adopts multiple technologies to reduce unit energy consumption and increase flight range, thus improving the transport capability of the air traffic network. The main reason for this is that long-haul flights, which are equivalent to long-distance connection edges, significantly enhance connectivity between any two points in the network [67].

In addition to the above traffic networks, adding long-range connected edges also plays a significant role in emergency response networks and tactical wireless networks [58].

3.2.2 Generation of network topology inspired by small-world model

In the real world, certain industrial networks exhibit characteristics inspired by the small-world property, such as power networks. On this basis, it becomes an effective network optimization strategy to explore and apply small-world principles in the design of network topologies. In addition to power networks, we will also analyze the role of the small-world model when building wireless sensor networks (WSN).

Power networks. Drawing inspiration from the small-world model, power networks are constructed according to the methods in [68–73]. From the perspective of voltage levels, the power supply networks for middle and low voltages within cities are closely connected, while the transmission networks for high voltages are sparser. From the perspective of geographical distribution, networks within individual regions are closely connected, while networks spanning different regions exhibit sparse connections. This implies that power networks possess high local clustering and low global interconnectivity, reflecting the influence of the small-world property. This property is observable in existing power networks, such as East China Power Network [36], West China Power Network [74], and North China Power Network [37].

At the same time, it should be noted that there are some special performance requirements of power networks. For example, cascading faults should be avoided in power networks [37, 75]. Therefore, modifications to the small-world model of power networks are necessary to enhance their resilience against cascading faults [35].

Wireless sensor networks. Besides power networks, wireless sensor networks (WSN) can also be analyzed using small-world models [76–83]. Average path length and energy consumption are critical metrics in the performance evaluation of WSN. Different small-world models focus on different performance metrics. Existing research [84] indicates that the Newman-Watts model has superior path length, clustering coefficient, and data communication delay, while the Kleinberg model excels in reducing energy consumption during data communication.

The analysis shows that in wireless sensor networks with small-world properties, long wireless links consume the most energy for packet transmission. To mitigate energy consumption, wired transmission for long links is introduced as illustrated in Figure 3 [85]. The advantage of wired transmission lies in better channel quality and lower transmit power requirements, leading to reduced data retransmission caused by low channel quality. The existing analysis [86] suggests that adding a small number of wired long connections to WSN not only reduces the average end-to-end hops but also balances network energy consumption, thereby extending network lifetime.

In the practical application of WSN, in addition to the average path length and energy consumption, the following performance metrics hold significant importance [87].

- (1) Network dynamics. The ability of small-world WSN to adapt to node mobility.
- (2) Node deployment. The ability of small-world WSN to efficiently generate routes for ad-hoc networks.
- (3) Compatibility of multi-functional nodes. The ability of small-world WSN to accommodate nodes with multiple functions and enable them to coexist within the network.
- (4) Control package overhead. The ability of small-world WSN to use a minimum number of control packets to achieve synchronization and other control operations to reduce energy consumption.
- (5) Quality of service (QoS). The ability of small-world WSN to achieve the compromise between QoS and node lifetime.

To meet the above performance requirements, the method of constructing wireless sensor networks needs further refinement. Ref. [85] introduced a topology planning approach where the small-world properties are fully exploited. The results in [85] demonstrate that the proposed approach can reduce the network diameter by nearly 50% and average path length by 47%.

3.3 Scale-free networks

As described in Subsection 2.2.4, the remarkable property of a scale-free network lies in the power law distribution of its node degree. Consequently, the number of key nodes is relatively small. There are many examples of scale-free networks in the real world, including the Internet and industrial networks. In the process of generating a scale-free network, newly added nodes tend to follow the growth and preferred attachment mechanism [88], connecting to the key nodes with higher degree. Therefore, the imbalance of degree distribution in scale-free networks will continue to amplify. For scale-free networks, with the expansion of the network scale, the average path length needs to be paid attention to. Additionally, it is crucial to consider that the key nodes in the scale-free network will become the primary targets of

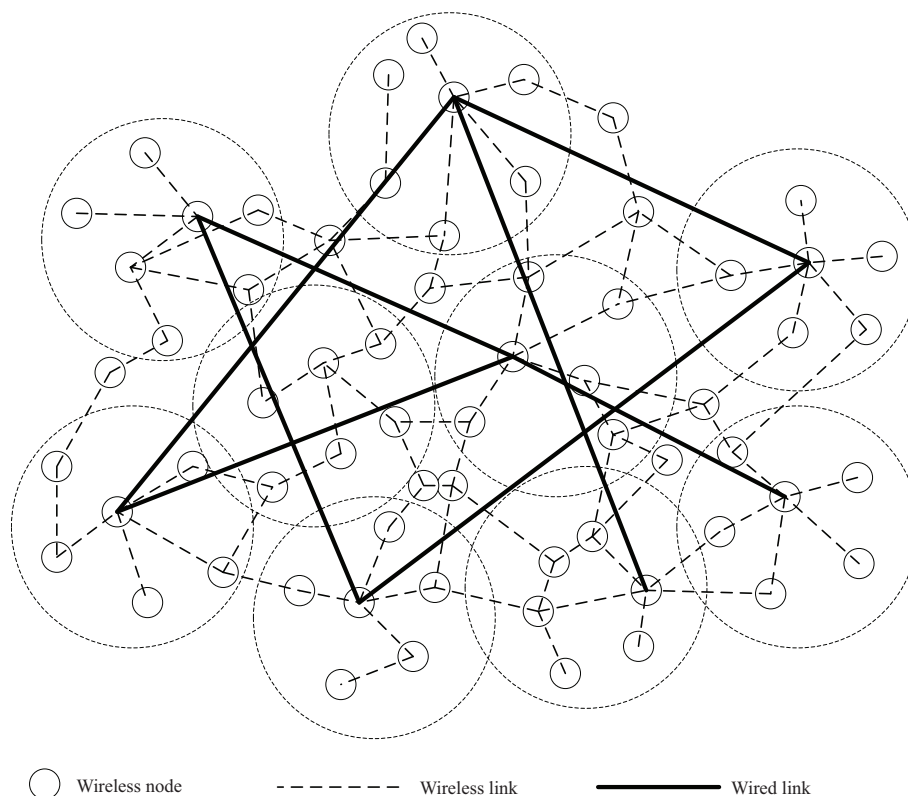


Figure 3 Wired and wireless hybrid structures [85].

network attacks. Hence, the robustness of scale-free networks in the face of attacks is a focus of network optimization. In the following, the average path length and robustness of scale-free networks are analyzed.

3.3.1 Reducing the average path length

Scale-free architectures can improve end-to-end performance by reducing the average path length and minimizing traffic on bottleneck links.

All kinds of scale-free industrial Internet benefit from small-world characteristics. For instance, in the case of the World Wide Web (WWW) with approximately 800 million pages in 1999, the average shortest path between any two pages was only around 19 [89,90]. This phenomenon is primarily attributed to the presence of hubs in the network, and associated with this feature, end-to-end data transmission requires only a handful of transfers. From the above description, it is not difficult to find that scale-free networks exhibit small-world properties to some extent due to the existence of these hubs [89].

Inspired by the properties of scale-free networks, researchers have attempted to introduce scale-free topologies into existing networks to enhance their performance. For example, Ref. [91] analyzed the end-to-end performance of TCP streams over a scale-free network, which benefits from decreasing the maximum length of edges. For the same reason, an arbitrary weight-based scale-free topology control algorithm (AWSF) was proposed in [92], which incorporates scale-free characteristics into WSN to reduce delay. In addition, Refs. [21,93] have also explored the introduction of scale-free characteristics to enhance efficiency in their respective networks.

3.3.2 Improvement of network robustness

Similar to the small-world feature that brings energy savings to WSNs, the performance advantage of scale-free networks comes from the presence of hubs that greatly reduce the average distance. Consequently, in the scale-free WSN, data can quickly reach its destination through several hub nodes.

However, the reliance on hubs can also be a serious disadvantage of the network [94]. In the existing research, it was found that simply removing a few key hubs from the Internet would split the entire system into isolated sets. The analysis in [89] shows that eliminating 5%–15% of the hub nodes could crash a system.

In addition to the destruction of the key hub nodes, the energy depletion of nodes is also an important reason for the failure of the original network topology. In WSN, sensor nodes are typically powered by limited-capacity batteries. In most cases, they are deployed in unattended and harsh environments, where it is often difficult to recharge or replace their batteries. Therefore, when the power is exhausted, the node fails permanently. In addition, the more important a node plays in the network (represented as a hub node), the more traffic it can relay or aggregate, and the faster it can drain the battery. If a hub node fails, the links connected to it become unavailable and the network connectivity is reduced. Therefore, the design of efficient and robust wireless sensor networks becomes very important, especially for large-scale wireless sensor networks (LS-WSNs), which often cover extensive monitoring areas and handle substantial packet loads.

In [95], a structure in which the degree of nodes decreases from the center to the boundary is proposed. It was named the onion structure. This gradually decentralized structure is more robust.

To further improve the performance of scale-free networks, some novel methods are proposed. The scheme called energy-aware low potential-degree common neighbor (ELDCN) was proposed in [22]. As the network expands, ELDCN avoids new nodes establishing edges to hubs with high potential connectivity, which takes both energy efficiency and robustness into account. The improved scale-free network (ISFN) [96] relieves the vulnerability of the topology through link-based and eccentricity-ratio-based swap operations that maintain the scale-free attribute. A more flexible approach with tunable coefficients was proposed in [97] to improve the network structure and achieve a better balance between connectivity and consumption. This way of design can better match real-world scenarios with requirements of concrete compromise.

3.4 Summary of existing network models

In Subsections 3.1–3.3, the applications of regular networks, small-world networks, and scale-free networks are introduced in industrial scenarios. It can be seen that the regular network model is suitable for small-scale networks. For large-scale networks, small-world network and scale-free network models are preferred.

Table 1 summarizes various industrial Internet scenarios and topologies. It classifies them based on network scale and topology type, and highlights the optimization methods relevant to each network instance.

For small-scale regular networks, optimization methods are closely related to the specific network topology. For example, ring-tree networks combine the advantages of both ring and tree networks and are used in switched industrial ethernet [51]. Similarly, cluster-trees [50], fat-tree [48], and ring topologies [11] each have their associated optimization methods. In addition, for some specific industrial applications, the topology of the network depends on the deployment of industrial equipment. For example, for the gas pipeline monitoring system [12], the monitoring network is designed as a linear topology due to the deployment characteristics of gas pipelines.

In small-world networks, optimizing edge connections, such as incorporating long-range and wired connections, is an effective technique for improving performance. For example, adding long-range connections can improve the performance of traffic networks [33, 34, 67], power networks [35, 36], and communication networks [58, 85]. Moreover, to optimize the performance for specific small-world network applications, various methods can be employed, including utilizing the information sharing of small-world network [60], optimizing the relay nodes for delay tolerant networks (DTNs) [59], implementing topology creation mechanism based on leveling for network routing [87], employing small world characteristics by in-network caching for information center network [52], enabling multi-dimensional switching capability for all-optical or hybrid optical/electric switching architecture [53], reducing heterogeneity and improving the spreading speed for network connection information dissemination [56], and introducing multi-objective firefly algorithm for job scheduling [62].

In scale-free networks, common optimization methods include reducing network energy consumption [22, 97] and swapping connection edges to enhance robustness [96]. For community detection, it is an effective method to seek the similarity between nodes [88]. Similar to small-world networks, connectivity between nodes should be fully considered. Obtaining a strong connection between nodes [92] and reducing the average path length [91] are important ways to enhance scale-free network traffic. In addition, the evolutionary game is an effective method [93] for the application of scale-free networks to big data processing.

Table 1 Summary of industrial Internet scenarios & topologies

Network scale	Topology type	Idea of optimization	Network instances
Small scale	Ring-tree	Combine the advantages of ring and tree network	Switched industrial Ethernet [51]
	Cluster-tree	Multi-channel capability	Delay-limited network [50]
	Fat-tree	Load balancing of links and switches	Data center network (DCN) [48]
	Ring	Simple hops between nodes	Redundant Ethernet networking [11]
	Linear	Adapting to the actual application scenario	Gas pipeline monitoring system [12]
Large scale		Adding the wired connection	Industrial WSN [61, 84, 86]
			Air route network [67] China air traffic system [34] Grid system [35]
		Adding the long-range connection	East China power grid [36] Campus traffic [33] Topology planning [85] Emergency response networks/tactical wireless networks [58]
		Utilizing the information sharing of small-world network	Detection of the community structure of smart city network system [60]
		Optimizing the relay nodes	Delay tolerant network [59]
		Topology creation mechanism based on levelling	Network routing [87]
	Small-world networks	Introducing small world characteristics by in-network caching	Information center network [52]
		Introducing multi-dimensional switching capability	All-optical or hybrid optical/electric switching architecture [53]
		Reducing heterogeneity and enhancing the spreading speed	Network connection information dissemination [56]
		Multi-objective firefly algorithm	Job scheduling [62]
		Network deployment based on energy saving	Large-scale industrial WSN [22, 97]
		Swap connected edges to enhance robustness	Network attack [96]
	Scale-free networks	Finding the similarity between nodes to enhance the ability of community detection	Community detection [88]
		Optimizing neighbor relationship to get strongly connection	Wireless sensor networking [92]
		Using the method of the evolutionary game to improve the balance of big data partitioning	Big data balanced partitioning [93]
Reducing the APL and traffic intensity at bottleneck links		The end-to-end TCP [91]	

In Section 4, we will introduce a metric that can evaluate the performance of multiple network models.

4 System entropy theory

Sections 2 and 3 give a systematic overview of existing research on industrial networks based on different network models. However, due to the differences in optimization methods based on different network models, there is a lack of performance metrics that can cover various network models. Inspired by the method introduced in [98, 99], a novel measurement is proposed to evaluate the network topology and reflect the system performance of the network.

Compared to existing specific criteria like power consumption and delay, system entropy is more abstract and provides an overall measure of network performance from the network architecture perspective. In this regard, the system entropy can be used to guide network layout in the real world.

As a starting point, Renyi entropy with parameter α is introduced,

$$H_\alpha(\mathbf{L}^n) = \frac{1}{1-\alpha} \log_2(\text{tr}((\mathbf{L}^n)^\alpha)). \quad (14)$$

It can be seen that the Renyi entropy is a function of the trace of the normalized Laplacian matrix \mathbf{L}^n . For the undirected graph with n nodes, its $N \times N$ Laplacian matrix \mathbf{L}^n is symmetric and semi-definite. Therefore, the trace is equal to the sum of eigenvalues. Furthermore, the Renyi entropy can be seen as a function of the eigenvalue vector of the Laplacian matrix \mathbf{L}^n .

In this paper, the Renyi entropy with α equal to 0.5 is called system entropy, whose expression is given by

$$H_{0.5}(\mathbf{L}^n) = H_{0.5}(\mathbf{\Lambda}) = \frac{1}{1-\alpha} \log_2 \left(\sum_{l=1}^{N-1} \sqrt{\lambda_l} \right), \quad (15)$$

where $\mathbf{\Lambda} = [\lambda_1, \dots, \lambda_{N-1}]^T$ and λ_l denotes the l -th eigenvalue of \mathbf{L}^n . Because \mathbf{L}^n is semi-definite, its first eigenvalue λ_0 is equal to 0. For clearness, the system entropy of the target network is defined as $H_{0.5}(\mathbf{\Lambda})$.

In this paper, the Laplace matrix is considered to be unweighted. In Subsections 4.1–4.3, several properties of system entropy are derived, including concavity, invariance under permutation, and non-negativity.

4.1 Concave property

In this subsection, the concave property of the system entropy will be introduced. It is well known that the concave optimization theory is based on concave functions. If the system entropy function has the concave property, the system entropy function can be utilized to optimize the network. The concave property allows us to obtain the optimal values of the system entropy function by some mathematical tools such as the gradient descent method.

It should be noted that the concavity of the system entropy can be described by the following expression with $0 \leq \beta \leq 1$:

$$H_{0.5}(\beta\mathbf{\Lambda}_1 + (1-\beta)\mathbf{\Lambda}_2) \geq \beta H_{0.5}(\mathbf{\Lambda}_1) + (1-\beta)H_{0.5}(\mathbf{\Lambda}_2). \quad (16)$$

However, it is difficult to prove the concave property directly from (16), because it involves the operation of inequalities of multiple parameters and matrices. We can start with the system entropy function itself. Firstly, the function $\log(\cdot)$ is concave over the domain $(0, +\infty)$, because

$$\frac{\partial^2}{(\partial x)^2} \log(x) = \frac{-1}{x^2} < 0. \quad (17)$$

In addition, we define

$$f(\mathbf{\Lambda}) = \sum_{l=1}^{N-1} \sqrt{\lambda_l}. \quad (18)$$

It should be noted that $f(\mathbf{\Lambda})$ is a concave function of $\mathbf{\Lambda}$. Because \mathbf{L}_n is a semi-definite matrix and only its first eigenvalue λ_0 is equal to 0, $f(\mathbf{\Lambda})$ is positive. Since $f(\mathbf{\Lambda})$ is concave and positive, the concavity of $\log f(\mathbf{\Lambda})$ is achieved according to the rule of composition with monotone concave function. Therefore, the system entropy $H_{0.5}(\mathbf{\Lambda})$ is proven to be a concave function.

4.2 Invariance under permutation

Based on the fact that the Laplace matrix is symmetric, we further explain that the value of the system entropy remains unchanged with the permutation of the Laplace matrix.

Without loss of generality, we begin with the permutation of exchange of a pair of nodes. The exchange of a pair of nodes in the Laplace matrix requires the permutation matrix, marked as \mathbf{M}_2 (the subscript

Table 2 Comparison of Shannon entropy and system entropy

Property	System entropy	Shannon entropy
Convexity/concavity	$H_{0.5}(\beta \mathbf{L}_1^n + (1-\beta) \mathbf{L}_2^n) \geq \beta H_{0.5}(\mathbf{L}_1^n) + (1-\beta) H_{0.5}(\mathbf{L}_2^n)$	$H(\theta \mathbf{p}_1 + (1-\theta) \mathbf{p}_2) \geq \theta H(\mathbf{p}_1) + (1-\theta) H(\mathbf{p}_2)$
Invariance under permutation	$H_{0.5}(\mathbf{L}^n) = H_{0.5}(\mathbf{M}_2 \mathbf{L}^n \mathbf{M}_2)$	$H(p_1, p_2, \dots, p_n) = H(p_{j_1}, p_{j_2}, \dots, p_{j_n})$
Nonnegativity	$H_{0.5}(\mathbf{L}^n) \geq 0$	$H(\mathbf{p}) \geq 0$

‘2’ means swapping two elements), which is given by

$$\mathbf{M}_2 = \begin{bmatrix} 1 & & & & & \\ & \ddots & & & & \\ & & 0 \cdots 1 & & & \\ & & \vdots & \ddots & \vdots & \\ & & 1 \cdots 0 & & & \\ & & & & \ddots & \\ & & & & & 1 \end{bmatrix}. \quad (19)$$

The permutation operation of the Laplace matrix is as follows:

$$(\mathbf{L}_2^n)^{\frac{1}{2}} = \mathbf{M}_2 (\mathbf{L}^n)^{\frac{1}{2}} \mathbf{M}_2, \quad (20)$$

where \mathbf{L}_2^n is the Laplace matrix after the permutation operation. At the same time, we noticed that the matrix \mathbf{M}_2 is also an orthogonal matrix.

Therefore, the eigenvalues of \mathbf{L}^n remain the same with \mathbf{M}_2 operation. Arbitrary permutation can be achieved through multiple \mathbf{M}_2 operations. Since the permutation does not change the eigenvalues, the entropy of the system remains the same.

This property means that once the network topology is generated, its system entropy is determined.

4.3 Non-negativity

The non-negative property of system entropy indicates that system entropy also has the physical meaning of a general entropy value. It is also an important evidence of the rationality of the system.

To prove this property, we need to prove $\text{tr}((\mathbf{L}^n)^{\frac{1}{2}}) > 1$.

The maximum eigenvalue of the normalized Laplace matrix of a nonbinary graph can be described as [100]

$$\frac{N}{N-1} \leq \lambda_{N-1} < 2. \quad (21)$$

The result of taking the square root of anything greater than 1 is also greater than 1. So $1 < \sqrt{\lambda_{N-1}}$ remains established. Thus $\text{tr}((\mathbf{L}^n)^{\frac{1}{2}}) > 1$ and $H_{0.5}(\mathbf{L}^n) \geq 0$ are established. The nonnegative of the system entropy function is proven.

4.4 Summary of the above properties

It should be noted that the above three properties are also available for traditional Shannon entropy. In Shannon entropy, the concavity comes from the logarithmic function. Moreover, when exchanging the order of random variables, the value of Shannon entropy remains unchanged. Nonnegativity also holds for Shannon entropy.

To better compare Shannon entropy with system entropy, we compare the above properties of system entropy with Shannon entropy, which are summarized in Table 2.

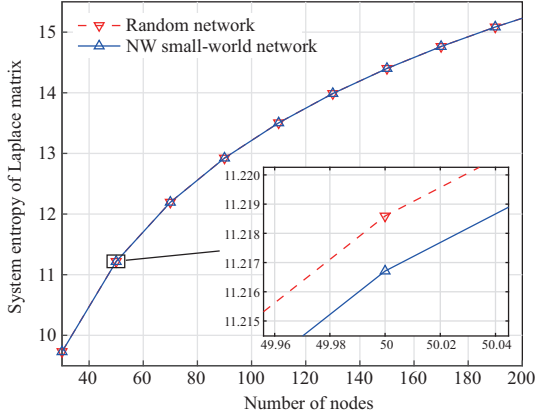


Figure 4 (Color online) System entropy varies with node size.

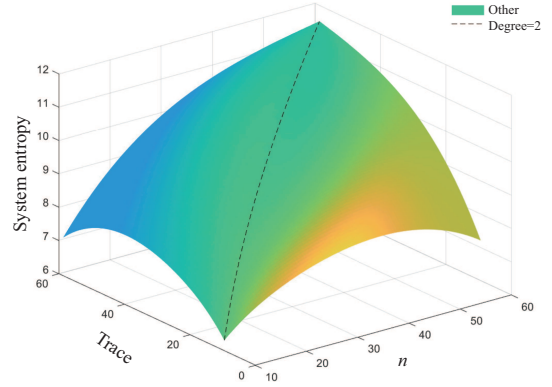


Figure 5 (Color online) The concavity of the system entropy function.

5 System entropy numerical results

This section exhibits numerical results of system entropy. In Subsection 5.1, the system entropy calculation results for different types of network topologies are shown. In Subsection 5.2, numerical results on the concave property of system entropy are given. In Subsection 5.3, the simulation result reflects the rationality of the application of system entropy to real systems. Subsection 5.4 presents the relationship between system entropy and APL.

5.1 System entropy of ER and NW networks

In this subsection, two kinds of networks are analyzed: ER random networks and Newman-Watts (NW) small-world networks. The following simulation strategies are adopted:

- (1) For the generation of an ER random network, the network scale and edge connection probability are preset.
- (2) For the NW small-world network, the average degree is specified and an initial regular network is generated. Then the structure of the regular network is modified by rewiring some edges without changing the number of edges in the network.

In our simulations, we need to ensure the consistency of the scale of the above two networks.

Based on the above strategy, we set the average node degree d_v equal to 8. Through simulations, we analyze the impact of the change in network size on system entropy. For an ER random network with n nodes, the connectivity probability of the network p_c is calculated based on the average degree d_v .

$$p_c = \frac{d_v}{n-1}. \quad (22)$$

Figure 4 shows the simulation results for varying network scales while keeping the average node degree at 8.

It can be seen that the performance of ER and NW networks reflected by system entropy is very close. However, upon closer examination of this slight discrepancy, we can find that the system entropy of the NW small-world network is smaller. This implies that the NW small-world network can organize a network with better comprehensive performance with the same number of edges and nodes.

5.2 Concave property of system entropy examples

In this subsection, the numerical results demonstrate the concavity of system entropy. To establish this, we obtain a large number of samples by generating a large number of Laplace matrices. These samples were then analyzed to discern specific patterns. The simulation results are shown in Figure 5, presenting the concavity of system entropy in terms of node scale and the sum of eigenvalues of the squared Laplace matrix.

Figure 5 illustrates that the entropy of the system exhibits concavity with the increase of both node size and the sum of eigenvalues. The dotted line in Figure 5 represents a network with a degree of 2. The

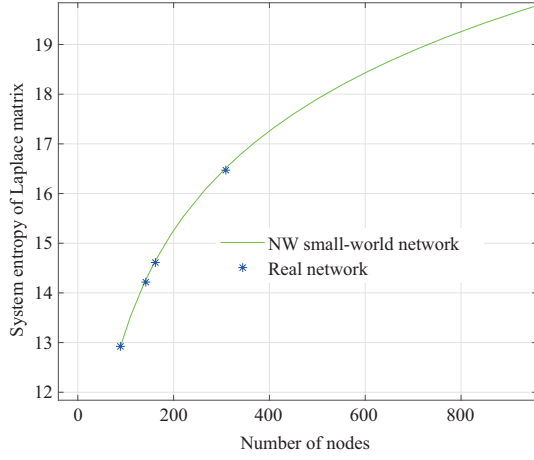


Figure 6 (Color online) System entropy of real and simulated NW small world networks.

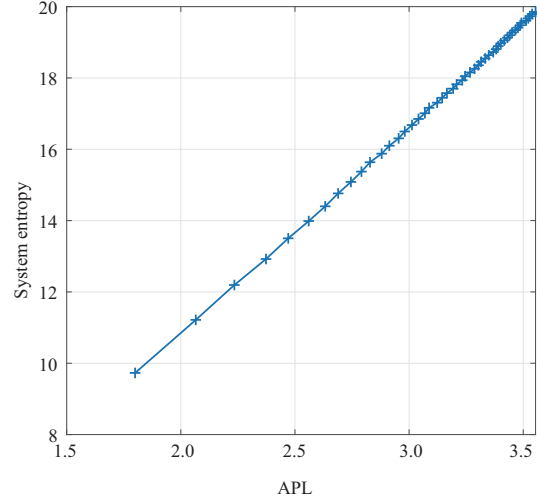


Figure 7 (Color online) APL vs. system entropy of NW small world network.

“ $n > \text{trace}$ ” side represents a situation where the network has fewer edges or lower connectivity, while the “ $n < \text{trace}$ ” side corresponds to a network with rich connections. It is evident that every degree section displays concavity. Therefore, the experiments in this subsection provide numerical evidence for the concavity of the system entropy function.

5.3 System entropy of real networks

To prove the validity of using the system entropy function to evaluate industrial networks, this subsection will give the value of the system entropy for a real-world network example.

The analyzed network in the example was generated from E-mail data from a large European research institution. The open-source dataset has been anonymized for the specific data in the network, leaving only the topology of the network visible. The average degree of the network is about 16. At the same time, the network is divided into four main groups, the number of nodes are 309, 162, 89, and 142, respectively. These four groups are all small-world networks. To intuitively express the relationship between the system entropy of these four actual networks and the system entropy curve obtained by computer simulation, we plotted the four discrete points alongside the corresponding curve for observation.

Figure 6 shows the simulation result under the above conditions. The data points on the curve are obtained by averaging the system entropy values from 100 small-world simulation networks of corresponding sizes. It can be seen from the simulation results that the system entropy of the four real networks is distributed near the curve of degree 16. Although the system entropy of the real network is slightly different from the average simulation results, the fitting effect is good. The results show that the system entropy of the small-world network closely matches that of the simulated small-world network.

5.4 The relationship between APL and system entropy

The average path length (APL) is a commonly used network performance measurement. In this subsection, we focus on the relationship between the average path length and the system entropy.

Because APL can describe the number of nodes required for the average information transmission, it is an important indicator of the information transmission speed within a network. Given that system entropy reflects the complexity of the network, we anticipate that the smaller the average path length, the lower the system entropy. Based on the above analysis, the simulation results are generated to reflect the relationship between average path length and system entropy. The NW small-world network is utilized. In Figure 7, APL is plotted on the horizontal axis, while the system entropy of the network is plotted on the vertical axis.

It can be seen that the system entropy function of the network is proportional to APL. The linear proportional relationship between APL and system entropy in this experiment shows that system entropy can realize the evaluation of network power consumption in the sense of measuring the forwarding times

and data transmission distance of the network, which further proves that system entropy has great guiding significance for measuring network performance.

6 Conclusion and perspective

This paper explores network models applied in industrial networks from a network scale perspective. It provides a summary of key definitions and metrics of graph theory related to network models and focuses on three network models: random networks, small-world networks, and scale-free networks. Based on the above models, this paper continues to discuss the network in the existing industrial applications and establishes the mapping relationship between the industrial application and the network model. Finally, a novel network performance metric called system entropy is proposed, and its mathematical properties are analyzed. System entropy is capable of covering a variety of network models and enabling a consistent performance comparison across different models. These findings highlight the potential for further expansion and research of system entropy as a useful tool in network performance evaluation.

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References

- 1 Rawat D B. Industrial Internet of Things: Cybermanufacturing Systems. Cham: Springer, 2017
- 2 Thangaramya K, Logambigai R, SaiRamesh L, et al. An energy efficient clustering approach using spectral graph theory in wireless sensor networks. In: Proceedings of the 2nd International Conference on Recent Trends and Challenges in Computational Models (ICRTCCM), 2017. 126–129
- 3 Gross J L, Yellen J, Anderson M. Graph Theory and Its Applications. New York: Chapman and Hall/CRC, 2018
- 4 Sengoku M, Tamura H, Mase K, et al. A routing problem on ad-hoc networks and graph theory. In: Proceedings of the International Conference on Communication Technology Proceedings, 2000. 2: 1710–1713
- 5 Zhang D G, Tang Y M, Cui Y Y, et al. Novel reliable routing method for engineering of internet of vehicles based on graph theory. *Eng Computations*, 2018, 36: 226–247
- 6 Kannadhasan S, KarthiKeyan G, Sethupathi V. A graph theory based energy efficient clustering techniques in wireless sensor networks. In: Proceedings of the IEEE Conference on Information & Communication Technologies, 2013. 151–155
- 7 Bollobás B. Modern Graph Theory. Berlin: Springer Science & Business Media, 2013
- 8 Bondy J A, Murty U S R. Graph Theory with Applications. London: Macmillan London, 1976
- 9 West D B. Introduction to Graph Theory. Upper Saddle River: Prentice Hall, 2001
- 10 van Steen M. Graph Theory and Complex Networks. New York: Springer, 2010
- 11 Yoon G, Kwon D H, Kwon S C, et al. Ring topology-based redundancy ethernet for industrial network. In: Proceedings of the SICE-ICASE International Joint Conference, 2006. 1404–1407
- 12 Lee M Y, Azman A S, Subramaniam S K, et al. Performance analysis of linear topology wireless sensor network in oil and gas industry. *IOP Conf Ser-Mater Sci Eng*, 2020, 765: 012070
- 13 Kanatas A G. Coordinates distributions in finite uniformly random networks. *IEEE Access*, 2022, 10: 49005–49014
- 14 Erdős P, Rényi A. On random graphs I. *Publ Math Debrecen*, 1959, 6: 290–297
- 15 Erdős P, Rényi A. On the evolution of random graphs. *Publ Math Inst Hung Acad Sci*, 1960, 5: 17–60
- 16 Pandey O J, Hegde R M. Low-latency and energy-balanced data transmission over cognitive small world WSN. *IEEE Trans Veh Technol*, 2018, 67: 7719–7733
- 17 Qiu T, Liu X Z, Li K Q, et al. Community-aware data propagation with small world feature for Internet of Vehicles. *IEEE Commun Mag*, 2018, 56: 86–91
- 18 Qiu T, Li B L, Qu W Y, et al. TOSG: a topology optimization scheme with global small world for industrial heterogeneous Internet of Things. *IEEE Trans Ind Inf*, 2018, 15: 3174–3184
- 19 Watts D J, Strogatz S H. Collective dynamics of “small-world” networks. *Nature*, 1998, 393: 440–442
- 20 Qiu T, Lu Z L, Li K Q, et al. An adaptive robustness evolution algorithm with self-competition for scale-free Internet of Things. In: Proceedings of IEEE Conference on Computer Communications, 2020. 2106–2115
- 21 Ding Y M, Li X H, Tian Y C, et al. Generating scale-free topology for wireless neighborhood area networks in smart grid. *IEEE Trans Smart Grid*, 2018, 10: 4245–4252
- 22 Peng H X, Si S Z, Awad M K, et al. Toward energy-efficient and robust large-scale WSNs: a scale-free network approach. *IEEE J Sel Areas Commun*, 2016, 34: 4035–4047
- 23 Qiu T, Liu J, Si W S, et al. Robustness optimization scheme with multi-population co-evolution for scale-free wireless sensor networks. *IEEE ACM Trans Networking*, 2019, 27: 1028–1042
- 24 Barabási A L, Albert R. Emergence of scaling in random networks. *Science*, 1999, 286: 509–512
- 25 Fronczak A, Fronczak P, Holyst J A. Average path length in random networks. *Phys Rev E*, 2004, 70: 056110
- 26 Vega-Redondo F. Complex Social Networks. Cambridge: Cambridge University Press, 2007

- 27 Onnela J P, Saramäki J, Hyvönen J, et al. Structure and tie strengths in mobile communication networks. In: Proceedings of the National Academy of Sciences of the United States of America, 2007. 7332–7336
- 28 Choromański K, Matuszak M, Miekisz J. Scale-free graph with preferential attachment and evolving internal vertex structure. *J Stat Phys*, 2013, 151: 1175–1183
- 29 Zhou Y Q, Liu L, Wang L, et al. Service-aware 6G: an intelligent and open network based on the convergence of communication, computing and caching. *Digital Commun Networks*, 2020, 6: 253–260
- 30 Zhou Y Q, Tian L, Liu L, et al. Fog computing enabled future mobile communication networks: a convergence of communication and computing. *IEEE Commun Mag*, 2019, 57: 20–27
- 31 Cui Q M, Hu X X, Ni W, et al. Vehicular mobility patterns and their applications to Internet-of-Vehicles: a comprehensive survey. *Sci China Inf Sci*, 2022, 65: 211301
- 32 Zhu G X, Lyu Z H, Jiao X, et al. Pushing AI to wireless network edge: an overview on integrated sensing, communication, and computation towards 6G. *Sci China Inf Sci*, 2023, 66: 130301
- 33 Liu M F, Xiong S W. The properties of campus road traffic networks. In: Proceedings of the International Conference on Information System and Artificial Intelligence (ISAI), 2016. 603–607
- 34 Wang H H, Wen R Y. Analysis of air traffic network of China. In: Proceedings of the 24th Chinese Control and Decision Conference (CCDC), 2012. 2400–2403
- 35 Ding J, Bai X M, Zhao W, et al. The improvement of the small-world network model and its application research in bulk power system. In: Proceedings of the International Conference on Power System Technology, 2006. 1–5
- 36 Ding M, Han P P. Reliability assessment to large-scale power grid based on small-world topological model. In: Proceedings of the International Conference on Power System Technology, 2006. 1–5
- 37 Meng Z W, Lu Z, Song J. Comparison analysis of the small-world topological model of Chinese and American power grids. *Autom Electric Power Syst*, 2004, 28: 21–24
- 38 Mishra R R, Moharana L. Analysis of different grid types used for sensor deployment in wireless sensor network. In: Proceedings of the Communication, Control and Intelligent Systems (CCIS), 2015. 91–95
- 39 Dorogovtsev S N, Mendes J F F. *Evolution of Networks: From Biological Nets to the Internet and WWW*. New York: OUP Oxford, 2013
- 40 Coutinho R W L, Boukerche A. Transfer learning for disruptive 5G-enabled industrial Internet of things. In: Proceedings of the IEEE Transactions on Industrial Informatics, 2022. 4000–4007
- 41 Ghosh A A, Ratasuk R, Rost P, et al. *5G-Enabled Industrial IoT Networks*. Norwood: Artech, 2022
- 42 Fanucchi D, Staehle B, Knorr R. Network formation for industrial IoT: evaluation, limits and recommendations. In: Proceedings of the IEEE 23rd International Conference on Emerging Technologies and Factory Automation (ETFA), 2018. 227–234
- 43 Kada B, Alzubairi A, Tameem A Y. Industrial communication networks and the future of industrial automation. In: Proceedings of the Industrial & Systems Engineering Conference (ISEC), 2019. 1–5
- 44 Carugi M, Lou Z. Future industrial networks: requirements, challenges, research and standardization needs. In: Proceedings of the ITU Kaleidoscope: Connecting Physical and Virtual Worlds (ITU K), 2021. 1–9
- 45 Wang R X, Beshley H, Yan L Y, et al. Industrial 5G private network: architectures, resource management, challenges, and future directions. In: Proceedings of the IEEE 16th International Conference on Advanced Trends in Radioelectronics, Telecommunications and Computer Engineering (TCSET), 2022. 780–784
- 46 Sharma R, Villanyi B. Insights from a high-performance open-communication framework for advanced industrial networks. In: Proceedings of the IEEE 21st International Symposium on Computational Intelligence and Informatics (CINTI), 2021. 253–258
- 47 Fernandes M M, Bigheti J A, Pontarolli R P, et al. Industrial automation as a service: a new application to industry 4.0. *IEEE Latin Am Trans*, 2021, 19: 2046–2053
- 48 He X, He M S, Han Z J. A survey of network topology of data center. In: Proceedings of the IEEE 4th International Conference on Big Data Security on Cloud (BigDataSecurity), IEEE International Conference on High Performance and Smart Computing (HPSC) and IEEE International Conference on Intelligent Data and Security (IDS), 2018. 39–41
- 49 Trajkovic L. Complex networks. In: Proceedings of the IEEE 19th International Conference on Cognitive Informatics & Cognitive Computing, 2020
- 50 Toscano E, Lucia L B. A multichannel approach to avoid beacon collisions in IEEE 802.15.4 cluster-tree industrial networks. In: Proceedings of the IEEE Conference on Emerging Technologies Factory Automation, 2009
- 51 Li F, Zhang Q, Zhang W. Graph partitioning strategy for the topology design of industrial network. *IET Commun*, 2007, 1: 1104–1110
- 52 Xu X G, Zhang T K, Feng C Y. Study on small world characteristics of in-network caching in information-centric networks. In: Proceedings of the IEEE 85th Vehicular Technology Conference, 2017. 1–5
- 53 Zhang D X, Guo H X, Wu J, et al. A deterministic small-world topology based optical switching network architecture for data centers. In: Proceedings of the European Conference on Optical Communication (ECOC), 2014. 1–3
- 54 Slota G M, Madduri K, Rajamanickam S. PuLP: scalable multi-objective multi-constraint partitioning for small-world networks. In: Proceedings of the IEEE International Conference on Big Data (Big Data), 2014. 481–490
- 55 Zheng G Z, Liu Q M. A survey on the topology of wireless sensor networks based on small world network model. In: Proceedings of the 2nd International Conference on Future Computer and Communication, 2010

- 56 Ou C G, Jin X L, Wang Y Z, et al. The impact of heterogeneous spreading abilities of network ties on information spreading. In: Proceedings of the IEEE International Conference on Computer and Information Technology; Ubiquitous Computing and Communications; Dependable, Autonomic and Secure Computing; Pervasive Intelligence and Computing, 2015. 454–460
- 57 Karsai M, Kivelä M, Pan R K, et al. Small but slow world: how network topology and burstiness slow down spreading. *Phys Rev E*, 2011, 83: 025102
- 58 Gaur N, Chakraborty A, Manoj B S. Delay optimized small-world networks. *IEEE Commun Lett*, 2014, 18: 1939–1942
- 59 Wei K M, Guo S, Zeng D Z, et al. Exploiting small world properties for message forwarding in delay tolerant networks. *IEEE Trans Comput*, 2015, 64: 2809–2818
- 60 Liu F Y, Xie G. A fast algorithm for community detection of network systems in smart city. *IEEE Access*, 2019, 7: 51856–51865
- 61 Wu J Y, Shao X Y, Zhu H P. Relay node deployment based small world effect in hierarchical industrial wireless sensor networks. In: Proceedings of the IEEE International Conference on Green Computing and Communications and IEEE Internet of Things and IEEE Cyber, Physical and Social Computing, 2013. 1066–1071
- 62 Arsuaga-Rios M, Vega-Rodriguez M A. Multiobjective small-world optimization for energy saving in grid environments. *Comput J*, 2015, 58: 432–447
- 63 Elmrini A, Amrani A G. Wireless sensors network for traffic surveillance and management in smart cities. In: Proceedings of the IEEE 5th International Congress on Information Science and Technology (CiSt), 2018. 562–566
- 64 Fathurrahman M F, Sutarto H Y, Semajski I. Urban network traffic analysis, data imputation, and flow prediction based on probabilistic PCA model of traffic volume data. In: Proceedings of the 8th International Conference on Advanced Informatics: Concepts, Theory and Applications (ICAICTA), 2021. 1–6
- 65 Qi L, Luan W J, Huang Y S. Traffic incident message dissemination for reducing traffic congestion. In: Proceedings of the IEEE 15th International Conference on Networking, Sensing and Control (ICNSC), 2018. 1–6
- 66 Zhang G L, Jia H F, Yang L L, et al. Research on a model of node and path selection for traffic network congestion evacuation based on complex network theory. *IEEE Access*, 2020, 8: 7506–7517
- 67 Sawai H. Reorganizing a new generation airline network based on an ant-colony optimization-inspired small-world network. In: Proceedings of the IEEE Congress on Evolutionary Computation, 2012. 1–8
- 68 Sidorov A I, Khanzhina O A, Tavarov S S, et al. Ensuring the efficiency of distribution networks C. Dushanbe and Republic of Tajikistan. In: Proceedings of the International Multi-Conference on Industrial Engineering and Modern Technologies, 2019. 1–4
- 69 Qiu J, Xia S X, Zhang J X, et al. Research on key technologies of communication for large-scale stability control system in modern power grid. In: Proceedings of the 12th IEEE PES Asia-Pacific Power and Energy Engineering Conference, 2020. 1–5
- 70 Chen L M, Dong X Z, Kuang X Y, et al. Towards ubiquitous power distribution communication: multi-service access and QoS guarantees for IoT applications in smart grid. In: Proceedings of the IEEE Innovative Smart Grid Technologies-Asia, 2019. 894–898
- 71 Gong L L, Zhang Y Z, Yang M H, et al. A review of reliability, vulnerability and resilience analysis of smart grid based on complex network. In: Proceedings of the IEEE 21st International Conference on Software Quality, Reliability and Security Companion, 2021. 117–126
- 72 Tuttokmai O, Kaygusuz A. Smart grids and industry 4.0. In: Proceedings of the International Conference on Artificial Intelligence and Data Processing (IDAP), 2018. 1–6
- 73 Bushuev V, Solovyev D, Shilova L. Management of functioning and development of complex integrated power supply system of smart city. In: Proceedings of the International Multi-Conference on Industrial Engineering and Modern Technologies, 2020. 1–6
- 74 Watts D J, Strogatz S H. Collective dynamics of “small-world” networks. *Nature*, 1998, 393: 440–442
- 75 Gladkikh T D. Dependability assessment of western Siberia electrical grids. In: Proceedings of the International Multi-Conference on Industrial Engineering and Modern Technologies, 2018. 1–5
- 76 Singh M K, Amin S I, Imam S A, et al. A survey of wireless sensor network and its types. In: Proceedings of the International Conference on Advances in Computing, Communication Control and Networking, 2018. 326–330
- 77 Srivastava S, Singh M, Gupta S. Wireless sensor network: a survey. In: Proceedings of the International Conference on Automation and Computational Engineering (ICACE), 2018. 159–163
- 78 Dash L, Khuntia M. Energy efficient techniques for 5G mobile networks in WSN: a survey. In: Proceedings of the International Conference on Computer Science, Engineering and Applications (ICCSEA), 2020. 1–5
- 79 Polavarapu S C, Panda S K. A survey on industrial applications using MEMS and WSN. In: Proceedings of the 4th International Conference on I-SMAC (IoT in Social, Mobile, Analytics and Cloud), 2020. 982–986
- 80 Biradar M, Mathapathi B. Secure, reliable and energy efficient routing in WSN: a systematic literature survey. In: Proceedings of the International Conference on Advances in Electrical, Computing, Communication and Sustainable Technologies (ICAECT), 2021. 1–13
- 81 Manikandan G, Sakthi U. A comprehensive survey on various key management schemes in WSN. In: Proceedings of the 2nd International Conference on I-SMAC (IoT in Social, Mobile, Analytics and Cloud), 2018. 378–383
- 82 Yaqoob A, Ashraf M A, Ferooz F, et al. WSN operating systems for Internet of Things (IoT): a survey. In: Proceedings of the International Conference on Innovative Computing (ICIC), 2019. 1–7

- 83 Swetha K, Lahari V, Manikrisha G V V, et al. A survey on placement of sensor nodes in deployment of wireless sensor networks. In: Proceedings of the International Conference on Intelligent Sustainable Systems (ICISS), 2019. 132–139
- 84 Araujo R P, Souza F S H D, Ueyama J, et al. On the analysis of Newman watts and Kleinberg small world models in wireless sensor networks. In: Proceedings of the IEEE 14th International Symposium on Network Computing and Applications, 2015. 17–21
- 85 Tadayon N, Zonouz A E, Aïssa S, et al. Cost-effective design and evaluation of wireless sensor networks using topology-planning methods in small-world context. *IET Wirel Sens Syst*, 2014, 4: 43–53
- 86 Ye X C, Xu L, Lin L W. Small-world model based topology optimization in wireless sensor network. In: Proceedings of the International Symposium on Information Science and Engineering, 2008. 102–106
- 87 Kumar M, Pattanaik K K, Yadav B, et al. Optimization of wireless sensor networks inspired by small world phenomenon. In: Proceedings of the IEEE 10th International Conference on Industrial and Information Systems (ICIIS), 2015. 66–70
- 88 Liu X Y, Pan Q, An Y Y, et al. Community detection algorithm based on the scale free property of networks. In: Proceedings of the IEEE International Conference on Computer and Information Technology; Ubiquitous Computing and Communications; Dependable, Autonomic and Secure Computing; Pervasive Intelligence and Computing, 2015. 2270–2275
- 89 Barabási A L, Bonabeau E. Scale-free networks. *Sci Am*, 2003, 288: 60–69
- 90 Barabási A L. The architecture of complexity. *IEEE Control Syst Mag*, 2007, 27: 33–42
- 91 Sakumoto Y, Ohsaki H. On the impact of scale-free structure on end-to-end TCP performance. In: Proceedings of the IEEE 39th Annual Computer Software and Applications Conference, 2015. 652–653
- 92 Wang L L, Dang J X, Jin Y, et al. Scale-free topology for large-scale wireless sensor networks. In: Proceedings of the 3rd IEEE/IFIP International Conference in Central Asia on Internet, 2007. 1–5
- 93 Li N, Dan W, Yu W W. Research on scale-free network user-side big data balanced partition strategy. In: Proceedings of the International Conference on Computer Network, Electronic and Automation (ICCNEA), 2019. 93–98
- 94 Yu A Q, Wang N, Wu N. Scale-free networks: characteristics of the time-variant robustness and vulnerability. *IEEE Syst J*, 2021, 15: 4082–4092
- 95 Herrmann H J, Schneider C M, Moreira A A, et al. Onion-like network topology enhances robustness against malicious attacks. *J Stat Mech*, 2011, 2011: P01027
- 96 Usman M, Javaid N, Khalid A, et al. Robustness optimization of scale-free IoT networks. In: Proceedings of the International Wireless Communications and Mobile Computing (IWCMC), 2020. 2240–2244
- 97 Jian Y H, Liu E W, Yue W, et al. Scale-free model for wireless sensor networks. In: Proceedings of the Wireless Communications and Networking Conference (WCNC), 2013
- 98 Brandes U. *Network Analysis: Methodological Foundations*. Berlin: Springer Science & Business Media, 2005
- 99 Cook D J, Holder L B. *Mining Graph Data*. Hoboken: John Wiley & Sons, 2006
- 100 Bs M, Chakraborty A, Singh R. *Complex Networks: A Networking and Signal Processing Perspective*. Boston: Prentice Hall, 2018