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Designing novel adaptive dynamic event-triggered protocols for uncertain multi-agent systems

Bin CHENG[1,2] & Bin HE[1,2*]

[1]*College of Electronics and Information Engineering, Tongji University, Shanghai* 201804*, China;*
[2]*Frontiers Science Center for Intelligent Autonomous Systems, Shanghai* 201210*, China*

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Multi-agent systems (MASs) have garnered widespread attention and have been investigated by numerous researchers in recent years, primarily due to their broad applications in industry, transportation, agriculture, and astronomy. While studying the cooperative control of MASs, numerous researchers place emphasis on reducing the frequency of communication among neighboring agents. The authors of [1] studied the consensus control of linear MASs under limited communication bandwidth conditions and presented several static distributed event-triggered protocols. In [2], the authors devised dynamic event-based rules to compute communication time for the consensus control problem. Ref. [3] employed the event-based method to reduce the frequency of control updating, thereby lowering the risk of fatigue damage to mechanical equipment and a consequent shortening of its service life resulting from frequent control updating. Given that various models and systems feature uncertainty or disturbance, take its existence into consideration in MASs is crucial. Ref. [4] investigated event-based consensus disturbance rejection for MASs.

In the aforementioned studies, certain parameters of the designed algorithms depend on global information, such as agent number, matrix eigenvalues with respect to the topology, and uncertainty bound. However, irrespective of the controller or triggering rule, such global information is not easily computed or may even be unavailable to the local agent; therefore, these algorithms are not likely to be applicable to practical MASs. Consequently, a natural challenge that arises is the development of appropriate event-based protocols that can be used in uncertain MASs in a fully-distributed control manner. In earlier studies [5], we proposed adaptive event-triggered protocols to avoid continuous communications among neighboring agents. Nevertheless, the protocols outlined in [5] require continuous updating of the controller and only guarantee practical consensus; i.e., the consensus error tends to be bounded. Therefore, it is crucial to devise fully-distributed event-driven protocols for uncertain MASs with limited updating frequency and interaction.

This study addresses the distributed cooperative control of MASs in the presence of unknown uncertainties, with the constraints of control updating and the interaction among neighboring agents. Novel adaptive event-triggered protocols have been proposed, which comprise two-component controllers: a linear term and a sliding-mode nonlinear term, as well as dynamic event-triggering conditions.

Problem formulation. Consider M agents satisfying

$$\dot{x}_i = Ax_i + B[u_i + f_i(t, x_i)],\ i = 1, \ldots, M, \tag{1}$$

where $x_i \in \mathbb{R}^n$, $u_i \in \mathbb{R}^p$, and $f_i(t, x_i) \in \mathbb{R}^p$ represent the state, the control input, and the uncertainty of agent i, respectively, and (A, B) is stabilizable. Assume that $\exists \bar{f}_i > 0 \Rightarrow \|f_i(t, x_i)\| \leqslant \bar{f}_i, i = 1, \ldots, M$. In general, let $f_0 \triangleq \max\{\bar{f}_1, \ldots, \bar{f}_M\}$, which is an unknown positive constant. The agents are connected by an undirected graph.

The purpose of this study is to design fully-distributed event-triggered protocols such that a consensus can be realized; that is, $\lim_{t\to\infty} \|x_i - x_j\| = 0$, $i, j = 1, \ldots, M$, and Zeno behavior is not exhibited [5]; that is, continuous updating and interaction can be avoided.

Protocol design. Let $\hat{x}_j(t) = x_j(t_k^j), \forall t \in [t_k^j, t_{k+1}^j)$, where t_k^j represents the k-th triggering instant of agent j. Let $x = [x_1^{\mathrm{T}}, \ldots, x_M^{\mathrm{T}}]^{\mathrm{T}}$ and $\phi = [\phi_1^{\mathrm{T}}, \ldots, \phi_M^{\mathrm{T}}]^{\mathrm{T}}$, where $\phi_i = \sum_{j=1}^M a_{ij}(x_i - x_j)$. Consequently, $\phi = (\mathcal{L} \otimes I_n)x$. Then, the consensus is achieved if and only if ϕ is asymptotically stable; therefore, we refer to ϕ as the consensus error. Letting $\hat{\phi}_i = \sum_{j=1}^M a_{ij}(\hat{x}_i - \hat{x}_j)$, we devise an adaptive event-driven controller as

$$\begin{aligned} u_i(t) &= \hat{c}_i(t)K\hat{\phi}_i(t) + \hat{d}_i(t)g(K\hat{\phi}_i(t)),\\ \dot{c}_i(t) &= \hat{\phi}_i^{\mathrm{T}}(t)\Gamma\hat{\phi}_i(t),\\ \dot{d}_i(t) &= \|K\hat{\phi}_i(t)\|,\ i = 1, \ldots, M, \end{aligned} \tag{2}$$

where $\hat{c}_i(t) = c_i(t_k^i)$, $\hat{d}_i(t) = d_i(t_k^i)$, $\forall t \in [t_k^i, t_{k+1}^i)$, c_i and d_i are adaptive gains satisfying $c_i(0) \geqslant 1$ and $d_i(0) \geqslant 1$, respectively; K and Γ are design matrices; and $g(s)$ is a nonlinear function for $s \in \mathbb{R}^p$ such that $g(s) = \begin{cases} \frac{s}{\|s\|}, & \text{if } \|s\| \neq 0,\\ 0, & \text{if } \|s\| = 0. \end{cases}$

Let $\tilde{x}_i = \hat{x}_i - x_i$, $\tilde{c}_i = \hat{c}_i - c_i$, and $\tilde{d}_i = \hat{d}_i - d_i$. We then design a triggering condition:

$$\begin{aligned} t_{k+1}^i \triangleq \inf \Big\{ & t > t_k^i \mid \tilde{x}_i^{\mathrm{T}}\Gamma\tilde{x}_i + \sqrt{\tilde{x}_i^{\mathrm{T}}\Gamma\tilde{x}_i} \geqslant \gamma_i\varepsilon_i \\ & \vee\ |\tilde{c}_i| \geqslant \theta_1 \mathrm{e}^{-\theta_2(t-t_k^i)}\ \vee\ |\tilde{d}_i| \geqslant \theta_3 \mathrm{e}^{-\theta_4(t-t_k^i)} \Big\}, \end{aligned} \tag{3}$$

* Corresponding author (email: hebin@tongji.edu.cn)

where $t_0^i \triangleq 0$, $\gamma_i > 0$, $i = 1, \ldots, M$, $\theta_i > 0$, $i = 1, \ldots, 4$, and $\varepsilon_i(t)$ satisfies

$$\dot{\varepsilon}_i = -k_i\varepsilon_i - \sigma_i\tilde{x}_i^{\mathrm{T}}\Gamma\tilde{x}_i - \delta_i\sqrt{\tilde{x}_i^{\mathrm{T}}\Gamma\tilde{x}_i},\ i = 1, \ldots, M \tag{4}$$

with $\varepsilon_i(0), k_i, \sigma_i, \delta_i > 0$. Under this triggering rule, agent i sends $x_i(t_k^i)$ to the neighbors only at time t_k^i, and $u_i(t)$ will not be updated until i or its neighbors are triggered.

Main results. The following results guarantee the convergence of agents and the exclusion of Zeno behavior.

Theorem 1. Consensus can be realized if $K = -B^{\mathrm{T}}Q$, and $\Gamma = QBB^{\mathrm{T}}Q$, where $P = Q^{-1}$ is a solution to the following linear matrix inequality (LMI): $AP + PA^{\mathrm{T}} - BB^{\mathrm{T}} < 0$. Furthermore, Zeno behavior does not exist.

Remark 1. Main symbols used in this study are categorized in Appendix A and the proof of Theorem 1 is presented in Appendix B. In the protocols (2)–(4), we introduce a linear term $\hat{c}_i(t)K\hat{\phi}_i(t)$ and a nonlinear term $\hat{d}_i(t)g(K\hat{\phi}_i(t))$ to achieve consensus and offset the effect of the uncertainties $f_i(t, x_i)$, respectively. To avoid relying on global information, we design two adaptive gains; that is, $c_i(t)$ and $d_i(t)$; furthermore, to avoid continuously updating the controller, we use values of the adaptive gains at event instants; that is, $\hat{c}_i(t)K\hat{\phi}_i(t)$ and $\hat{d}_i(t)g(K\hat{\phi}_i(t))$, rather than the time-varying values.

Remark 2. In contrast to [2], which developed dynamic event-triggered protocols for multiple single integrators, the dynamic adaptive event-based algorithm provided in this study is applicable to general linear networks. Moreover, the current study is substantially different from [5], which designs event-based protocols to achieve a consensus of MASs with the existence of matched uncertainties with a limited communication frequency. The task here is more complex because we aim to design a fully-distributed protocol to avoid continuous interaction among neighbors and continuous controller updates. However, the method used here is essentially different from the one in [5]. In particular, to determine the optimal timing for communication, Ref. [5] designed static triggering conditions for agents, which include two state-based thresholds depending on the local network topology and a time-based threshold. Here, we design a novel dynamic event-triggering condition without such state-based thresholds, thereby enabling the decoupling of the triggering condition (3) and (4) from the topology. Compared with the case in [5], each agent can easily compute the triggering condition (3) and (4) and determine event instants.

Remark 3. The designed event-driven protocol guarantees that ϕ is asymptotically stable for MASs with matching uncertainties. In certain cases involving practical engineering, non-matching uncertainties may exist. Particularly, the system dynamics is described by the following equation:

$$\dot{x}_i = Ax_i + B[u_i + f_i(t, x_i)] + w_i,\ i = 1, \ldots, M, \tag{5}$$

where $w_i \in \mathbb{R}^n$ is a non-matching uncertainty of agent i satisfying $\|w_i\| \leqslant v_i$, and v_i is a positive constant. In this case, we use the σ-modification technology for (2) as

$$\begin{aligned} u_i &= \hat{c}_iK\hat{\phi}_i + \hat{d}_ig(K\hat{\phi}_i), \\ \dot{c}_i &= -\varsigma\hat{c}_i + \hat{\phi}_i^{\mathrm{T}}\Gamma\hat{\phi}_i, \\ \dot{d}_i &= -\upsilon\hat{c}_i + \|K\hat{\phi}_i\|, \end{aligned} \tag{6}$$

where $\varsigma, \upsilon > 0$. Triggering events are determined by the triggering condition (3). In contrast to Theorem 1, ϕ only converges to a residual set here rather than zero.

Simulation. Consider 10 agents satisfying (1), where $A = \left[\begin{smallmatrix} 0 & 1 \\ 0 & 0 \end{smallmatrix}\right]$, $B = \left[\begin{smallmatrix} 0 \\ 1 \end{smallmatrix}\right]$, $i = 1, \ldots, 10$, $f_i(t, x_i) = i/10\sin(t)$, $i = 1, \ldots, 3$, $f_i(t, x_i) = i\mathrm{e}^{-t}$, $i = 4, \ldots, 6$, $f_i(t, x_i) = i/(i + t)$, $i = 7, 8$, $f_i(t, x_i) = \sin(\|x_i(t)\|)/i$, $i = 9, 10$. The parameters of (2)–(4) are chosen as $\theta_i = 1$, $i = 1, \ldots, 4$, $\gamma_i = 2$, $k_i = 0.2$, $\sigma_i = 0.3$, $\delta_i = 0.3$, $i = 1, \ldots, 10$. Solving the LMI gives $P = \left[\begin{smallmatrix} 0.8660 & -0.5000 \\ -0.5000 & 0.8660 \end{smallmatrix}\right]$. Then, we further obtain $K = [\, -1.0000\ \ -1.7321 \,]$ and $\Gamma = \left[\begin{smallmatrix} 1.0000 & 1.7321 \\ 1.7321 & 3.0000 \end{smallmatrix}\right]$. The simulation results are shown in Figure 1.

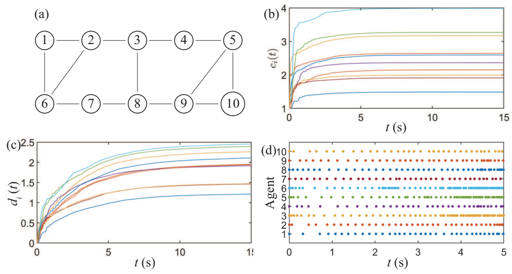

Figure 1 (Color online) Simulation results. (a) Undirected graph; (b) gain c_i; (c) gain d_i; (d) triggering instants.

Conclusion. This study addressed the cooperative control problem in the presence of uncertainties with constrained interaction and updating. We designed adaptive dynamic event-driven algorithms, guaranteeing consensus achievement and no Zeno behavior. The presented protocols are fully distributed because they do not depend on any global information associated with topologies or uncertainties. Future studies should further investigate this problem under general directed graphs.

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Supporting information Appendixes A and B. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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