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• **Supplementary File** •

Designing Novel Adaptive Dynamic Event-Triggered Protocols for Uncertain Multi-Agent Systems

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Appendix A Key symbols of this study

The main symbols of this study are listed in the following table.

Table A1 The main symbols of this study.

Symbol	Meaning	Dynamics
x_i	state of agent i	$\dot{x}_i = Ax_i + Bu_i$
$\hat{x}_i$	state of agent i at triggering time	$\hat{x}_i = x_i(t_k^i)$, $t \in [t_k^i, t_{k+1}^i)$
$\tilde{x}_i$	measurement error of state	$\tilde{x}_i = \hat{x}_i - x_i$
ϕ_i	consensus error	$\phi_i = \sum_{j=1}^N a_{ij}(x_i - x_j)$
$\hat{\phi}_i$	consensus error at triggering time	$\hat{\phi}_i = \sum_{j=1}^N a_{ij}(\hat{x}_i - \hat{x}_j)$
$\tilde{\phi}_i$	error between $\hat{\phi}_i$ and ϕ_i	$\tilde{\phi}_i = \hat{\phi}_i - \phi_i$
c_i	adaptive gain of agent i	$\dot{c}_i = \hat{\phi}_i^T \Gamma \hat{\phi}_i$, $c_i(0) \geqslant 1$
$\hat{c}_i$	adaptive gain at triggering time	$\hat{c}_i = c_i(t_k^i)$, $t \in [t_k^i, t_{k+1}^i)$
$\tilde{c}_i$	measurement error of adaptive gain	$\tilde{c}_i = \hat{c}_i - c_i$
$\bar{c}_i$	virtual adaptive gain of agent i	$\dot{\bar{c}}_i = \phi_i^T \Gamma \phi_i$, $\bar{c}_i(0) \geqslant 1$
$\check{c}_i$	error between c_i and $\bar{c}_i$	$\check{c}_i = c_i - \bar{c}_i$
d_i	adaptive gain of agent i	$\dot{d}_i = \lVert K\hat{\phi}_i \rVert$, $d_i(0) \geqslant 1$
$\hat{d}_i$	adaptive gain at triggering time	$\hat{d}_i = d_i(t_k^i)$, $t \in [t_k^i, t_{k+1}^i)$
$\tilde{d}_i$	measurement error of adaptive gain	$\tilde{d}_i = \hat{d}_i - d_i$
$\bar{d}_i$	virtual adaptive gain of agent i	$\dot{\bar{d}}_i = \lVert K\phi_i \rVert$, $\bar{d}_i(0) \geqslant 1$
$\check{d}_i$	error between d_i and $\bar{d}_i$	$\check{d}_i = d_i - \bar{d}_i$

Appendix B The proof of Theorem 1

This proof includes two parts, showing that the consensus can be achieved and Zeno behavior can be excluded, respectively.

Part 1: We first prove that consensus can be achieved. Letting $\tilde{\phi}_i = \sum_{j=1}^M a_{ij}(\tilde{x}_i - \tilde{x}_j)$, then $\tilde{\phi}_i = \hat{\phi}_i - \phi_i$. Denote $\bar{c}_i$ and $\bar{d}_i$ virtual adaptive gains, whose dynamics satisfy

$$\begin{aligned} \dot{\bar{c}}_i &= \phi_i^T \Gamma \phi_i, \\ \dot{\bar{d}}_i &= \lVert K\phi_i \rVert, \ i = 1, \cdots, M, \end{aligned} \tag{B1}$$

with $\bar{c}_i(0) \geqslant 1$ and $\bar{d}_i(0) \geqslant 1$. Further let $\check{c}_i = c_i - \bar{c}_i$ and $\check{d}_i = d_i - \bar{d}_i$. We have

$$\dot{x} = (I_N \otimes A)x + (\bar{C} \otimes BK)\phi + (C \otimes BK)\tilde{\phi} + (\check{C} \otimes BK)\phi + (\tilde{C} \otimes BK)\hat{\phi} + (\hat{D} \otimes B)G + (I_M \otimes B)F, \tag{B2}$$

where $\phi = [\phi_1^T, \cdots, \phi_M^T]^T$, $\tilde{\phi} = [\tilde{\phi}_1^T, \cdots, \tilde{\phi}_M^T]^T$, $\hat{\phi} = [\hat{\phi}_1^T, \cdots, \hat{\phi}_M^T]^T$, $\bar{C} = \mathrm{diag}(\bar{c}_1, \cdots, \bar{c}_M)$, $C = \mathrm{diag}(c_1, \cdots, c_M)$, $\tilde{C} = \mathrm{diag}(\tilde{c}_1, \cdots, \tilde{c}_M)$, $\check{C} = \mathrm{diag}(\check{c}_1, \cdots, \check{c}_M)$, $\hat{D} = \mathrm{diag}(\hat{d}_1, \cdots, \hat{d}_M)$, $G = [g^T(K\hat{\phi}_1), \cdots, g^T(K\hat{\phi}_M)]^T$, and $F = [f_1^T, \cdots, f_M^T]^T$. The closed-loop dynamics of ϕ is written as

$$\dot{\phi} = (I_M \otimes A)\phi + (\mathcal{L}\bar{C} \otimes BK)\phi + (\mathcal{L}C \otimes BK)\tilde{\phi} + (\mathcal{L}\check{C} \otimes BK)\phi + (\mathcal{L}\tilde{C} \otimes BK)\hat{\phi} + (\mathcal{L}\hat{D} \otimes B)G + (\mathcal{L} \otimes B)F. \tag{B3}$$

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Choose a Lyapunov function as

$$V_1 = x^T(\mathcal{L}\otimes Q)x + \sum_{i=1}^{M}\Big[\frac{1}{2}(\bar{c}_i-\alpha_c)^2 + \beta_c\bar{c}_iV_0 + \frac{1}{2}(\bar{d}_i-\alpha_d)^2 + \beta_d\bar{d}_iV_0\Big], \tag{B4}$$

where $\alpha_c, \alpha_d, \beta_c, \beta_d > 0$ are design constants, and $V_0 = \sum_{j=1}^{M}\frac{\varepsilon_j}{k_j}$. According to (4) and (5), we have $e^{-(k_i+\sigma_i\gamma_i)t}\varepsilon_i(0) \leqslant \varepsilon_i(t) \leqslant e^{-k_it}\varepsilon_i(0)$. Then, we have

$$\begin{aligned}\dot{V}_1 = {}& x^T[\mathcal{L}\otimes(QA+A^TQ)]x - 2\phi^T(\bar{C}\otimes\Gamma)\phi - 2\phi^T(C\otimes\Gamma)\tilde{\phi} - 2\phi^T(\check{C}\otimes\Gamma)\phi - 2\phi^T(\tilde{C}\otimes\Gamma)\hat{\phi} + 2\phi^T(\hat{D}\otimes QB)G\\ &+ 2\phi^T(I_N\otimes QB)F + \sum_{i=1}^{M}(\bar{c}_i-\alpha_c+\beta_cV_0)\phi_i^T\Gamma\phi_i + \sum_{i=1}^{M}(\bar{d}_i-\alpha_d+\beta_dV_0)\|K\phi_i\|\\ &- \sum_{i=1}^{M}(\beta_c\bar{c}_i+\beta_d\bar{d}_i)\sum_{j=1}^{M}\left(\varepsilon_j + \frac{\sigma_j}{k_j}\tilde{x}_j^T\Gamma\tilde{x}_j + \frac{\delta_j}{k_j}\sqrt{\tilde{x}_j^T\Gamma\tilde{x}_j}\right).\end{aligned} \tag{B5}$$

Next, we focus on analyzing (B5).

(i) We first deal with $-2\phi^T(C\otimes\Gamma)\tilde{\phi}$. By Young's inequality [1], we get

$$\begin{aligned}-2\phi^T(C\otimes\Gamma)\tilde{\phi} &\leqslant \frac{1}{2}\phi^T(C\otimes\Gamma)\phi + 2\tilde{\phi}^T(C\otimes\Gamma)\tilde{\phi}\\ &= \frac{1}{2}\phi^T(\bar{C}\otimes\Gamma)\phi + \frac{1}{2}\phi^T(\check{C}\otimes\Gamma)\phi + 2\tilde{\phi}^T(\bar{C}\otimes\Gamma)\tilde{\phi} + 2\tilde{\phi}^T(\check{C}\otimes\Gamma)\tilde{\phi}.\end{aligned}$$

Since

$$\begin{aligned}\check{c}_i(t) &= \int_0^t(\hat{\phi}_i^T(\tau)\Gamma\hat{\phi}_i(\tau) - \phi_i^T(\tau)\Gamma\phi_i(\tau))d\tau + \check{c}_i(0)\\ &= \int_0^t(\tilde{\phi}_i^T(\tau)\Gamma\tilde{\phi}_i(\tau) + 2\phi_i^T(\tau)\Gamma\tilde{\phi}_i(\tau))d\tau + \check{c}_i(0)\\ &\leqslant \int_0^t\phi_i^T(\tau)\Gamma\phi_i(\tau)d\tau + 2\int_0^t\tilde{\phi}_i^T(\tau)\Gamma\tilde{\phi}_i(\tau)d\tau + \check{c}_i(0)\\ &= \bar{c}_i(t) + 2\int_0^t\tilde{\phi}_i^T(\tau)\Gamma\tilde{\phi}_i(\tau)d\tau + \check{c}_i(0) - \bar{c}_i(0),\end{aligned} \tag{B6}$$

and

$$\begin{aligned}\tilde{\phi}_i^T\Gamma\tilde{\phi}_i &= \sum_{j=1}^{M}a_{ij}(\tilde{x}_i-\tilde{x}_j)^T\Gamma\sum_{j=1}^{M}a_{ij}(\tilde{x}_i-\tilde{x}_j)\\ &\leqslant 2l_{ii}^2\tilde{x}_i^T\Gamma\tilde{x}_i + 2\sum_{j=1}^{M}a_{ij}\tilde{x}_j^T\Gamma\sum_{j=1}^{M}a_{ij}\tilde{x}_j\\ &\leqslant 2l_{ii}^2\sum_{j=1}^{M}\tilde{x}_j^T\Gamma\tilde{x}_j,\end{aligned} \tag{B7}$$

by substituting (B7) into (B6), we get that

$$\begin{aligned}\check{c}_i &\leqslant \bar{c}_i - \bar{c}_i(0) + \check{c}_i(0) + 4l_{ii}^2\sum_{j=1}^{M}\int_0^t\tilde{x}_j^T(\tau)\Gamma\tilde{x}_j(\tau)d\tau\\ &\leqslant \bar{c}_i - \bar{c}_i(0) + \check{c}_i(0) + 4l_{ii}^2\sum_{j=1}^{M}\int_0^\infty\gamma_j\varepsilon_j(\tau)d\tau\\ &\leqslant \bar{c}_i - \bar{c}_i(0) + \check{c}_i(0) + 4l_{ii}^2\sum_{j=1}^{M}\frac{\gamma_j\varepsilon_j(0)}{k_j}.\end{aligned} \tag{B8}$$

Thus,

$$2\tilde{\phi}^T(\check{C}\otimes\Gamma)\tilde{\phi} = \sum_{i=1}^{M}2\check{c}_i\tilde{\phi}_i^T\Gamma\tilde{\phi}_i \leqslant \sum_{i=1}^{M}(2\bar{c}_i+\beta_1)\,\tilde{\phi}_i^T\Gamma\tilde{\phi}_i,$$

where $\beta_1 \geqslant \max\{-2\bar{c}_i(0) + 2\check{c}_i(0) + 8l_{ii}^2\sum_{j=1}^{M}\frac{\gamma_j\varepsilon_j(0)}{k_j}\}$ is a positive constant. Then, it is easy to derive

$$-2\phi^T(C\otimes\Gamma)\tilde{\phi} \leqslant \frac{1}{2}\phi^T(\bar{C}\otimes\Gamma)\phi + \frac{1}{2}\phi^T(\check{C}\otimes\Gamma)\phi + \sum_{i=1}^{M}(8\bar{c}_i+2\beta_1)\,l_{ii}^2\sum_{j=1}^{M}\tilde{x}_j^T\Gamma\tilde{x}_j. \tag{B9}$$

(ii) We then consider $-\frac{3}{2}\phi^T(\check{C}\otimes\Gamma)\phi$. Obviously,

$$\begin{aligned}-\check{c}_i &= -\check{c}_i(0) + \int_0^t(\phi_i^T(\tau)\Gamma\phi_i(\tau) - \hat{\phi}_i^T(\tau)\Gamma\hat{\phi}_i(\tau))d\tau\\ &\leqslant -\check{c}_i(0) + \frac{1}{3}\int_0^t\phi_i^T(\tau)\Gamma\phi_i(\tau)d\tau + 2\int_0^t\tilde{\phi}_i^T(\tau)\Gamma\tilde{\phi}_i(\tau)d\tau\\ &\leqslant -\check{c}_i(0) + \frac{1}{3}\bar{c}_i - \frac{1}{3}\bar{c}_i(0) + 4l_{ii}^2\sum_{j=1}^{M}\frac{\gamma_j\varepsilon_j(0)}{k_j}.\end{aligned}$$

It follows that

$$
\begin{aligned}
-\frac{3}{2}\phi^T(\check{C}\otimes\Gamma)\phi &= \sum_{i=1}^{M} -\frac{3}{2}\check{c}_i\phi_i^T\Gamma\phi_i \\
&\leqslant \sum_{i=1}^{M}\Big(\frac{1}{2}\bar{c}_i - \frac{3}{2}(\check{c}_i(0)+\frac{1}{3}\bar{c}_i(0)) + 6\sum_{i=1}^{M} l_{ii}^2\sum_{j=1}^{M}\frac{\gamma_i\varepsilon_j(0)}{k_j}\Big)\times\phi_i^T\Gamma\phi_i \\
&\leqslant \frac{1}{2}\phi^T(\bar{C}\otimes\Gamma)\phi + \phi^T(\beta_2 I_M\otimes\Gamma)\phi,
\end{aligned}
\tag{B10}
$$

where $\beta_2 \geqslant \max\{-\frac{3}{2}(\check{c}_i(0)+\frac{1}{3}\bar{c}_i(0)) + 6\sum_{i=1}^{M} l_{ii}^2\sum_{j=1}^{M}\frac{\gamma_i\varepsilon_j(0)}{k_j}\}$.

(iii) Now, we consider $-2\phi^T(\tilde{C}\otimes\Gamma)\hat{\phi}$. It follows from the triggering rule (3) that

$$
\begin{aligned}
-2\phi^T(\tilde{C}\otimes\Gamma)\hat{\phi} &= -2\sum_{i=1}^{M}\tilde{c}_i\phi^T\Gamma\hat{\phi}_i \\
&\leqslant \theta_1\sum_{i=1}^{M}\phi_i^T\Gamma\phi_i + \theta_1\sum_{i=1}^{M}\hat{\phi}_i^T\Gamma\hat{\phi}_i \\
&\leqslant 3\theta_1\sum_{i=1}^{M}\phi_i^T\Gamma\phi_i + 2\theta_1\sum_{i=1}^{M}\tilde{\phi}_i^T\Gamma\tilde{\phi}_i \\
&\leqslant 3\theta_1\sum_{i=1}^{M}\phi_i^T\Gamma\phi_i + 4\theta_1\sum_{i=1}^{M} l_{ii}^2\sum_{j=1}^{M}\tilde{x}_j^T\Gamma\tilde{x}_j.
\end{aligned}
\tag{B11}
$$

(iv) We can get that

$$
\begin{aligned}
2\phi^T(\hat{D}\otimes QB)G &= 2\sum_{i=1}^{M}\hat{d}_i\phi_i^T QBg(K\hat{\phi}_i) \\
&= 2\sum_{i=1}^{M}\hat{d}_i\phi_i^T QB\frac{-B^TQ\hat{\phi}_i}{\|B^TQ\hat{\phi}_i\|} \\
&= -2\sum_{i=1}^{M}\hat{d}_i\|B^TQ\hat{\phi}_i\| + 2\sum_{i=1}^{M}\hat{d}_i\tilde{\phi}_i^T QB\frac{B^TQ\hat{\phi}_i}{\|B^TQ\hat{\phi}_i\|} \\
&\leqslant -2\sum_{i=1}^{M}\hat{d}_i\|B^TQ\phi_i\| + 4\sum_{i=1}^{M}\hat{d}_i\|B^TQ\tilde{\phi}_i\|.
\end{aligned}
\tag{B12}
$$

Note that

$$
\begin{aligned}
-\check{d}_i &= -\int_0^t(\|K\hat{\phi}_i(\tau)\| - \|K\phi_i(\tau)\|)d\tau - \check{d}_i(0) \\
&\leqslant \int_0^t(\frac{1}{2}\|K\phi_i(\tau)\| - \frac{1}{2}\|K\hat{\phi}_i(\tau)\| + \frac{1}{2}\|K\tilde{\phi}_i(\tau)\|)d\tau - \check{d}_i(0) \\
&\leqslant \frac{1}{2}\bar{d}_i - \frac{1}{2}\bar{d}_i(0) - \check{d}_i(0) + \frac{1}{2}\int_0^t\|K\tilde{\phi}_i(\tau)\|d\tau,
\end{aligned}
\tag{B13}
$$

where

$$
\begin{aligned}
\frac{1}{2}\int_0^t\|K\tilde{\phi}_i(\tau)\|d\tau &\leqslant \frac{1}{2}\int_0^t\sqrt{2l_{ii}^2\sum_{j=1}^{M}\tilde{x}_j^T(\tau)\Gamma\tilde{x}_j(\tau)}d\tau \\
&\leqslant \frac{\sqrt{2}}{2}l_{ii}\sum_{j=1}^{M}\sqrt{\int_0^t\gamma_j\varepsilon_j(\tau)d\tau} \\
&\leqslant \frac{\sqrt{2}}{2}l_{ii}\sum_{j=1}^{M}\sqrt{\frac{\gamma_j\varepsilon_j(0)}{k_j}}.
\end{aligned}
\tag{B14}
$$

Substituting (B14) into (B13) yields

$$
-\check{d}_i \leqslant \frac{1}{2}\bar{d}_i - \frac{1}{2}\bar{d}_i(0) - \check{d}_i(0) + \frac{\sqrt{2}}{2}l_{ii}\sum_{j=1}^{M}\sqrt{\frac{\gamma_j\varepsilon_j(0)}{k_j}}.
\tag{B15}
$$

According to (B13), we then have

$$
\begin{aligned}
-2\sum_{i=1}^{M}\hat{d}_i\|B^TQ\phi_i\| &= -2\sum_{i=1}^{M}(\bar{d}_i+\tilde{d}_i+\check{d}_i)\|B^TQ\phi_i\| \\
&\leqslant -\sum_{i=1}^{M}\bar{d}_i\|B^TQ\phi_i\| + \sum_{i=1}^{M}\beta_3\|B^TQ\phi_i\|,
\end{aligned}
\tag{B16}
$$

where $\beta_3 \geqslant \max\{2\theta_3 - \bar{d}_i(0) - 2\check{d}_i(0) + \sqrt{2}l_{ii}\sum_{j=1}^{M}\sqrt{\frac{\gamma_j\varepsilon_j(0)}{k_j}}\}$.

Also note that

$$\begin{aligned}\check{d}_i &= \int_0^t (\|K\hat{\phi}_i(\tau)\| - \|K\phi_i(\tau)\|)d\tau + \check{d}_i(0) \\ &\leqslant \sqrt{2}l_{ii}\sum_{j=1}^{M}\sqrt{\frac{\gamma_j\varepsilon_j(0)}{k_j}} + \check{d}_i(0).\end{aligned} \tag{B17}$$

According to (B17), we have

$$\begin{aligned}4\sum_{i=1}^{M}\hat{d}_i\|B^TQ\tilde{\phi}_i\| &\leqslant 4\sum_{i=1}^{M}(\bar{d}_i + \tilde{d}_i + \check{d}_i)\|B^TQ\tilde{\phi}_i\| \\ &\leqslant 4(1+\theta_3+\check{d}_i(0)+\sqrt{2}l_{ii}\sum_{j=1}^{M}\sqrt{\frac{\gamma_j\varepsilon_j(0)}{k_j}})\sum_{i=1}^{M}\bar{d}_i\|B^TQ\tilde{\phi}_i\| \\ &\leqslant \beta_4\sum_{i=1}^{M}\bar{d}_i l_{ii}\sum_{j=1}^{M}\sqrt{\tilde{x}_j\Gamma\tilde{x}_j},\end{aligned} \tag{B18}$$

where $\beta_4 \geqslant \max\{4(1+\theta_3+\check{d}_i(0)+\sqrt{2}l_{ii}\sum_{j=1}^{M}\sqrt{\frac{\gamma_j\varepsilon_j(0)}{k_j}})\}$.

Substituting (B16) and (B18) into (B12) yields

$$2\phi^T(\hat{d}\otimes QB)G \leqslant -\sum_{i=1}^{M}\bar{d}_i(t)\|B^TQ\phi_i\| + \sum_{i=1}^{M}\beta_3\|B^TQ\phi_i\| + \sum_{i=1}^{M}\beta_4\bar{d}_i l_{ii}\sum_{j=1}^{M}\sqrt{\tilde{x}_j\Gamma\tilde{x}_j}. \tag{B19}$$

(v) We can also obtain that

$$\begin{aligned}2\phi^T(I_M\otimes QB)F &\leqslant 2\sum_{i=1}^{M}\|B^TQ\phi_i\|\cdot\|f_i\| \\ &\leqslant 2f_0\sum_{i=1}^{M}\|B^TQ\phi_i\|.\end{aligned} \tag{B20}$$

According to the above analysis, substituting (B9), (B10), (B11), (B19), and (B20) into (B5) yields

$$\begin{aligned}\dot{V}_1 \leqslant\ & x^T[\mathcal{L}\otimes(QA+A^TQ)]x + \phi^T[(\beta_2+3\theta_1)I_N\otimes\Gamma]\phi + \sum_{i=1}^{M}((-\alpha_c+\beta_cV_0)\phi_i^T\Gamma\phi_i + (-\alpha_d+\beta_dV_0)\|K\phi_i\|) \\ &+ (\beta_3+2f_0)\sum_{i=1}^{M}\|B^TQ\phi_i\| + \beta_4\sum_{i=1}^{M}\bar{d}_i l_{ii}\sum_{j=1}^{M}\sqrt{\tilde{x}_j\Gamma\tilde{x}_j} + \sum_{i=1}^{M}(8\bar{c}_i+2\beta_1+4\theta_1)l_{ii}^2\sum_{j=1}^{M}\tilde{x}_j^T\Gamma\tilde{x}_j \\ &- \sum_{i=1}^{M}(\beta_c\bar{c}_i+\beta_d\bar{d}_i)\sum_{j=1}^{M}(\varepsilon_j + \frac{\sigma_j}{k_j}\tilde{x}_j^T\Gamma\tilde{x}_j + \frac{\delta_j}{k_j}\sqrt{\tilde{x}_j^T\Gamma\tilde{x}_j}) \\ =\ & x^T[\mathcal{L}\otimes(QA+A^TQ)]x + \phi^T[(-\alpha_c+\beta_cV_0+\beta_2+3\theta_1)I_N\otimes\Gamma]\phi + \sum_{i=1}^{M}((-\alpha_d+\beta_dV_0+\beta_3+2f_0)\|K\phi_i\|) \\ &+ \sum_{i=1}^{M}\sum_{j=1}^{M}(\beta_4\bar{d}_i l_{ii} - \beta_d\bar{d}_i\frac{\delta_j}{k_j})\sqrt{\tilde{x}_j\Gamma\tilde{x}_j} + \sum_{i=1}^{M}\sum_{j=1}^{M}((8\bar{c}_i+2\beta_1+4\theta_1)l_{ii}^2 - \beta_c\bar{c}_i\frac{\sigma_j}{k_j})\tilde{x}_j^T\Gamma\tilde{x}_j.\end{aligned} \tag{B21}$$

By choosing $\beta_c \geqslant 2(4+\beta_1+2\theta_1)\max\{l_{ii}^2\}\max\{\frac{k_i}{\sigma_i}\}$, $\beta_d \geqslant \beta_4\max\{l_{ii}\}\max\{\frac{k_i}{\delta_i}\}$, $\alpha_c = \alpha_1+\beta_cV_0(0)+\beta_2+3\theta_1$, and $\alpha_d = \beta_dV_0(0)+\beta_3+2f_0$, we get from (B21) that

$$\begin{aligned}\dot{V}_1 &\leqslant x^T[\mathcal{L}\otimes(QA+A^TQ)]x - \phi^T(\alpha_1I_M\otimes\Gamma)\phi \\ &\leqslant x^T[\mathcal{L}\otimes(QA+A^TQ-\Gamma)]x \\ &\leqslant \frac{1}{\lambda_M(\mathcal{L})}\phi^T[I_M\otimes(QA+A^TQ-\Gamma)]\phi \\ &\leqslant 0,\end{aligned} \tag{B22}$$

where we have used Lemma 1 [1)] and chosen $\alpha_1 \geqslant \frac{1}{\lambda_2(\mathcal{L})}$, $\lambda_2(\mathcal{L})$ and $\lambda_M(\mathcal{L})$ represent the smallest and largest nonzero eigenvalues of $\mathcal{L}$, respectively.

Thus, $V_1(t)$ is bounded, and so are ϕ_i, $\bar{c}_i$, and $\bar{d}_i$. And V_1 has a finite limit as $t\to\infty$, denoted by V_1^∞. It then follows from (B22) that

$$\int_0^\infty \phi^T[I_M\otimes(-(QA+A^TQ-\Gamma))]\phi dt \leqslant \lambda_M(\mathcal{L})(V_1(0)-V_1^\infty).$$

Besides, by (B8) and (B17) we have $c_i = \check{c}_i + \bar{c}_i \leqslant 2\bar{c}_i - \bar{c}_i(0) + c_i(0) + 4l_{ii}^2\sum_{j=1}^{M}\frac{\gamma_j\varepsilon_j(0)}{k_j}$ and $d_i = \check{d}_i + \bar{d}_i \leqslant \bar{d}_i + \check{d}_i(0) + \sqrt{2}l_{ii}\sum_{j=1}^{M}\sqrt{\frac{\gamma_j\varepsilon_j(0)}{k_j}}$, implying that c_i and d_i are bounded. Then, $\check{c}_i = c_i - \bar{c}_i$ is also bounded. According to the triggering rule,

1) **Lemma 1.** [2] The smallest nonzero eigenvalue $\lambda_2(\mathcal{L})$ of an undirected connected graph $\mathcal{G}$ satisfies $\lambda_2(\mathcal{L}) = \min_{x\neq 0,\mathbf{1}^Tx=0}(x^T\mathcal{L}x/x^Tx)$.

$\tilde{x}_i^T\Gamma\tilde{x}_i$ is bounded, implying that $(I_M \otimes BK)\tilde{x}$ and $(I_M \otimes BK)\hat{\phi}$ are bounded. Since $\hat{d}_i = d_i + \tilde{d}_i$ is bounded, we have $(\mathcal{L}\hat{d} \otimes B)G$ is bounded. Also noting that $(\mathcal{L} \otimes B)F$ is bounded, we get that

$$\dot{\phi} = (I_M \otimes A)\phi + (\mathcal{L}\bar{C} \otimes BK)\phi + (\mathcal{L}C \otimes BK)\tilde{\phi} + (\mathcal{L}\check{C} \otimes BK)\phi + (\mathcal{L}\tilde{C} \otimes BK)\hat{\phi} + (\mathcal{L}\hat{D} \otimes B)G + (\mathcal{L} \otimes B)F,$$

is bounded. Following Barbalat's lemma [3], $\phi^T[I_M \otimes (-(QA + A^TQ - \Gamma))]\phi \to 0$ as $t \to \infty$. Thus, $\phi \to 0$ as $t \to \infty$, i.e., consensus can be achieved.

Part 2: In the following, a contradiction argument is provided to exclude the Zeno behavior. In general, suppose agent i exists Zeno behavior, i.e., $\exists T < +\infty \Rightarrow \lim_{k\to\infty} t_k^i = T$. Then, for $\forall\Delta > 0$, $\exists k_0 \in \mathcal{Z} \Rightarrow$ for $k \geqslant k_0$, $t_k^i \in [T - \Delta, T)$.

Since consensus can be achieved, we have x_i, $\hat{\phi}_i$, $\hat{c}_i$, and $\hat{d}_i$ are all bounded for $t \in [0, T)$. Since $\tilde{x}_i(t_{k_0}^i) = 0$, $\tilde{c}_i(t_{k_0}^i) = 0$, and $\tilde{d}_i(t_{k_0}^i) = 0$, we have

$$\begin{aligned}\tilde{x}_i^T\Gamma\tilde{x}_i + \sqrt{\tilde{x}_i^T\Gamma\tilde{x}_i} &\leqslant \|\Gamma\|\rho_1^2(t - t_{k_0}^i)^2 + \sqrt{\|\Gamma\|}\rho_1(t - t_{k_0}^i),\\ |\tilde{c}_i| &\leqslant \rho_2(t - t_{k_0}^i),\\ |\tilde{d}_i| &\leqslant \rho_3(t - t_{k_0}^i),\end{aligned} \tag{B23}$$

where ρ_1, ρ_2, and ρ_3 denote the upper bounds of $\|Ax_i + Bf_i + BK\hat{c}_i\hat{\phi}_i\|$, $\hat{\phi}_i^T\Gamma\hat{\phi}_i$, and $\|K\hat{\phi}_i\|$ for $t \in [0, T]$, respectively.

Denote $\Delta \triangleq \min\{\Delta_1, \Delta_2, \Delta_3\}$, where $\Delta_1 = \frac{\sqrt{1+4\gamma_i e^{-(k_i+\sigma_i\gamma_i)T}}-1}{2\sqrt{\|\Gamma\|}\rho_1}$, $\Delta_2 = \frac{\theta_1 e^{-\theta_2 T}}{\rho_2}$, and $\Delta_3 = \frac{\theta_3 e^{-\theta_4 T}}{\rho_3}$. Under the triggering rule (3), next event of i after $t_{k_0}^i$ will happen first when

$$\tilde{x}_i^T\Gamma\tilde{x}_i + \sqrt{\tilde{x}_i^T\Gamma\tilde{x}_i} \geqslant \gamma_i\varepsilon_i \geqslant \gamma_i e^{-(k_i+\sigma_i\gamma_i)T} \ \vee\ |\tilde{c}_i| \geqslant \theta_1 e^{-\theta_2(t-t_k^i)} \geqslant \theta_1 e^{-\theta_2 T} \ \vee\ |\tilde{d}_i| \geqslant \theta_3 e^{-\theta_4(t-t_k^i)} \geqslant \theta_3 e^{-\theta_4 T}. \tag{B24}$$

It follows that $t_{k_0+1}^i - t_{k_0}^i \geqslant \Delta$. We have $t_{k_0+1}^i \geqslant T$, which does not consist with $t_{k_0+1}^i < T$. Thus, there is no Zeno behavior.

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