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20736-node weighted max-cut problem solving by quadrature photonic spatial Ising machine

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The max-cut problem is one of the fundamental NP-hard problems in combinatorial optimization, which can describe the practical applications such as data clustering, machine scheduling, and image recognition. This problem can be formulated into an equivalent Ising model without local fields, given by $H = -\sum_{\langle l,k\rangle} J_{l,k}x_lx_k$ where $J_{l,k}$ is the interaction between spins and binary spin state $x_l \in \{1,-1\}$. And photonic Ising machines are designed to search for the ground state of the Ising model by either iterative sampling or directly evolving the ensemble energy regarding the established mapping of a particular combinatorial problem [1–4]. Among these solutions, spatial photonic Ising machines encoding the spins as a phase matrix in spatial light modulators (SLMs), translate the Ising model into another form as follows:

$$H = -\sum_{\langle l,k\rangle} \varepsilon_l\varepsilon_k x_lx_k, \tag{1}$$

in which the interaction coefficient $J_{l,k}$ is set by the amplitude modulation ε_l and ε_k [2]. This scheme is highly suitable for solving large-scale max-cut problems that exhibit fully connected interactions, owing to its superior connectivity and scalability [3]. However, several proposed Ising machines tend to focus on solving the benchmark unweighted max-cut problems, as they can be easily mapped onto the Ising model with $J_{l,k}$ taking values of $\{0,\pm1\}$ while the weighted ones are more practical and complex.

In this article, we successfully solve 20736-node weighted max-cut problems with the quadrature photonic spatial Ising machine (Q-SIM). To configure the weights, we perform intensity configuration based on Euler's formula by extending the quadrature phase configuration in previously proposed architecture [4]. Furthermore, we extend our experiments to instances of dense graphs and compare our results with those obtained through numerical simulations and other reviewed methods. Our experimental results show a 33% improvement in the maximum cut value over the simulation results and a 34% improvement over the standard Sahni-Gonzales (SG) algorithm. Moreover, our approach provides a substantial overall speed-up.

Architecture and principle. As shown in Figure 1(a), an extended coherent light source shines on the SLM screen. The phase mask of SLM is composed of four parts that encode both the interaction coefficients and the spin states, allowing for a spin with amplitude information to be represented by four distinct components: $e^{i(\phi_k-\alpha_k)}$, $e^{i(\theta_k-\beta_k)}$, $e^{i(\phi_k+\alpha_k)}$, $e^{i(\theta_k+\beta_k)}$. Here, $\phi_k \in \{0,\pi\}$ and $\theta_k \in \{\frac{\pi}{2},-\frac{\pi}{2}\}$ are two sets of mutually orthogonal phases to construct the Q-SIM, as demonstrated in our previous work [4]. These orthogonal phases allow for flexible configurations of low-rank matrices, overcoming the inherent restriction of rank $= 1$ imposed by the form of (1). In addition, some non-fully-connected Ising models with negative amplitudes are successfully configured with this architecture. On the other hand, the sum of $e^{i(\phi_k-\alpha_k)}$ and $e^{i(\phi_k+\alpha_k)}$ can form the amplitude of the spin, according to Euler's formula

$$\varepsilon_k x_k = \frac{1}{2}\left[e^{i(\phi_k-\alpha_k)} + e^{i(\phi_k+\alpha_k)}\right], \tag{2}$$

where $\varepsilon_k = \cos\alpha_k$. This approach simplifies the step of amplitude configuration while eliminating the limitation of non-negative amplitude. Then, after the two-dimensional Fourier transform by the lens, the central intensity detected by the CCD Camera is given in the form of

$$I(0,0) = (x^{\mathrm{T}}\varepsilon + y^{\mathrm{T}}\eta)\overline{(\varepsilon^{\mathrm{T}}x + \eta^{\mathrm{T}}y)}. \tag{3}$$

A specific derivation procedure regarding the principle of the quadrature photonic spatial Ising machine and the intensity configuration method based on Euler's formula is provided in Appendixes A and B.

Now the weighted max-cut instances can be mapped to the Ising model for the solution. When a cut is defined, the corresponding cut value can be written as $W = \frac{1}{2}\sum_{\langle l,k\rangle} w_{l,k}(1-x_lx_k)$, in which $w_{l,k}$ is the edge weight between the l-th node and the k-th node. The related Hamiltonian we use is $H = \sum_{\langle l,k\rangle} w_{l,k}x_lx_k$ and the weight can be expressed as $w_{l,k} = \cos\alpha_l\cos\alpha_k \pm \cos\beta_l\cos\beta_k$. Hence we can search for the maximum cut value by maximizing the central intensity during the experiment. Here, we calculate

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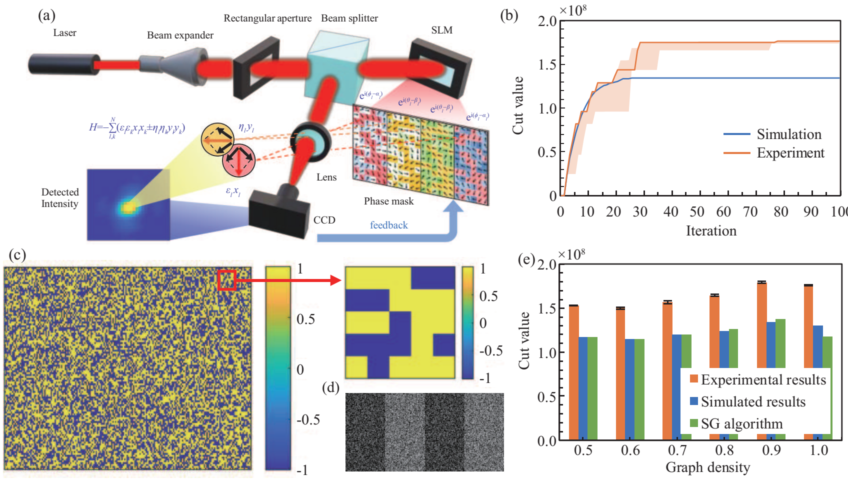

Figure 1 (Color online) (a) Schematic and principle of Q-SIM with phase-to-amplitude configuration. (b) Experimental (orange line) and simulated (blue line) evolution of the cut value. The light orange region represents the interval of the result distribution from the ten experiments, with the mean indicated by the orange line. (c) Final experimental spin states in (b). (d) Final experimental phase mask in (b). (e) Experimental results for max-cut problems with graph densities of $[0.5, 1.0]$.

the Euclidean distance $\|I_T - I\|_2$ between the initial detected intensity I and target intensity I_T as a cost function of the simulated annealing (SA) algorithm, thus generating a new phase mask to refresh SLM screen. This procedure is continuously cycled to govern the Hamiltonian evolution until the system stabilizes to the ground state. Details about the experimental setup and the operational procedures based on the SA algorithm are demonstrated in Appendixes C and D.

Experimental performance and numerical simulations. Given that the exact solvers generally fail with 1000 nodes, we need to perform a reference calculation on the conventional electrical computing platform. To tackle larger instances, we opt to utilize a classical greedy heuristic algorithm known as the SG algorithm [1]. On average, it requires 11 h to execute the SG algorithm on CPUs (Intel i9-13900K, 5.8 GHz) for addressing the max-cut problems of 20736 nodes. In the case of an all-to-all max-cut problem, the SG method generates a maximum cut value of 1.178×10^8, while our method produces a higher cut value of 1.759×10^8 with a 122 times speedup, as shown in Figure 1(b). Additionally, we conduct simulations to emulate the functioning of the photonic Ising machine. Notably, our findings reveal that the simulation results are also inferior to the experimental results.

Finally, we extrapolate our experiments and simulations for the max-cut problem with graph densities of $[0.5, 1.0]$ compared with the SG algorithm. The results are shown in Figure 1(e), which statistically demonstrate that our scheme offers compelling advantages for handling large-scale max-cut problem that outweighs electronic computers, in comparison with both simulations and the SG algorithm. The experimental maximum cut values exceed the SG algorithm by an average of 34% and achieve a maximum of 49% with the graph density of 1.0, which precisely captures the inherent advantage of fully connected systems. Additionally, the experimental results routinely outperform the simulated results by roughly 33%. These will be illustrated in the computational results given in Appendix E. We speculate that the detection susceptible to noise may cause some perturbation in experiments, making it easier to jump out of the local energy minima. This intrinsic property fits better with the SA algorithm and thus improves the machine performance. In fact, a related work reported that noise-enhanced photonic Ising machines can be used to solve large-scale combinatorial optimization problems [5].

Discussion and conclusion. We conduct a comparative analysis of our system performance, as elaborated in Appendix F, in solving max-cut problems against other Ising machines and identify the following advantages. (1) Efficient resolution of large-scale max-cut problems. (2) Flexible mapping capabilities for (non)fully connected max-cut problems, allowing for arbitrary amplitude assignments. (3) An uncomplicated and cost-effective experimental setup.

In summary, we performed extensive experiments on the Q-SIM with the phase-to-amplitude configuration applied to the instance of the max-cut problem. This system effectively addresses weighted problems with dense graphs of up to 20736 nodes, resulting in over 30% enhancements compared to classical solvers. Consequently, our proposal showcases superior optimization performance and rapid computational speed within the optical computing paradigm, making it a highly competitive solution for addressing large-scale NP problems.

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Supporting information Appendixes A–F. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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