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Distributed optimal consensus of multiagent systems with Markovian switching topologies: synchronous and asynchronous communications

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Abstract Considering limited system configurations and communication network bandwidths, this paper proposes two fully distributed event-triggered control schemes, namely synchronous and asynchronous communication, to solve the optimal consensus problem of linear multiagent systems (MASs) with Markovian switching topologies. The concept here is to design a novel composite dynamic event-triggered mechanism for each agent (using the information of all neighbors and one of them), based on which two fully distributed event-triggered protocols are designed to reach consensus and minimize a global convex team performance function. The effectiveness of the proposed control schemes is analyzed theoretically, that is, the optimal consensus problem can be realized, and the MAS does not exhibit Zeno behavior. Compared with existing optimal consensus results, the outstanding advantage of the proposed schemes is that a fully distributed asynchronous event-triggered communication scheme is designed, which fills the gap of optimal consensus results in this respect.

Keywords optimal consensus, dynamic event-triggered control, synchronous and asynchronous communications

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1 Introduction

In recent years, due to the application of multiagent systems (MASs) in robotics, remote control, economics, and microgrids [1], they have emerged as a hot research topic. MASs are systems comprising multiple single agents that complete a complex task through mutual communication. Thus far, there have been several impressive research findings related to the cooperative control of MASs (see [2–6] and references therein).

In addition, the research on the optimal control of MASs is of great significance in several applications, such as satellites, automatic ground vehicles, unmanned aerial vehicles, and medical control, as reported in [7, 8] and references therein. Further, several enlightening studies on the optimization problems of MASs have been reported, such as [9–16]. Particularly, the distributed optimal leader-follower consensus problems of MASs under directed graphs were considered in [7,9,10], where the linear quadratic regulator technique was adopted to design the performance index functions. In [11–13,17], optimizing performance was chosen to minimize a team performance function, where the function was the sum of local performance functions. In the application of MASs, a single agent is designed as an individual with certain sensing, computing, storage, and communication capabilities. Thus, event-triggered mechanisms, wherein the agent transmits the current information to all its neighbors when the triggering condition is satisfied,

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have been introduced in the optimal control problem in [14–16, 18]. In addition, the dynamic event-triggered mechanism was proposed to further reduce the triggering frequency, which has been rigorously demonstrated in [19].

Some impressive schemes reported in [16, 20] for addressing optimal consensus problems are no longer applicable for large-scale MASs because they depend on the global information of communication topology. In this regard, much pioneering effort has been focused on avoiding using global information (see [11, 13–15, 21–26] and references therein), where adaptive gains have been used to replace fixed gains. For example, the authors in [11] addressed the distributed optimal consensus problem of MASs through edge- and node-based adaptive design methods. However, continuous communication between neighbors was required for the controller design. Meanwhile, in [13, 15], fully distributed event-triggered optimal consensus protocols were designed without relying on global information or continuous communication between neighbors, where synchronous communication between neighbors was achieved. Further, edge-based event-triggered algorithms in [23, 24] were proposed to solve the leader-follower consensus problem of MASs to further save communication resources. In contrast, asynchronous event-triggered communication was implemented between neighbors. However, these proposed methods did not optimize the performance index and relied on fixed connectivity communication topology. The communication environment of MASs is often disturbed by random factors, resulting in switching communication topologies of the MAS (see [1, 27]).

Inspired by the above discussion, this paper considers distributed optimal consensus control for MASs with Markovian switching topologies and limited communication resources. Compared with existing studies, the main challenges in dealing with the optimal consensus problem are as follows: (1) The problem considered herein cannot be solved solely by combining the results of existing studies. In fact, because of the existence of Markovian random switching topologies, the typical properties and results corresponding to the fixed connectivity communication topology might not be applicable. Therefore, the stochastic theory was used for strict analysis. (2) The single adaptive edge event-triggered conditions in [23, 24] were theoretically insufficient for establishing optimal consensus conditions. (3) The single adaptive learning rates in [23, 24, 28, 29] were insufficient for constructing the event-triggered optimal consensus protocol.

The contributions of this paper are threefold:

(1) For large-scale MASs with Markovian switching topologies and limited communication resources, the novel asynchronous event-triggered communication scheme is proposed to solve the optimal consensus problem.

(2) To the best of our knowledge, this paper designs asynchronous event-triggered control schemes for the first time to deal with optimal consensus problems. Compared with the synchronous communication scheme, the asynchronous counterpart can further reduce communication congestion.

(3) The proposed method is more general, and the dynamic triggering mechanisms can further reduce communication frequency compared with the traditional mechanisms in [28]. Compared with previous methods [16, 20, 30], the method proposed herein does not rely on fixed connectivity communication topology and has scalability resulting from novel adaptive learning rates.

Notations. $\mathbb{R}^{m\times n}$ denotes an $m \times n$ dimensional-real matrix, and $\mathbb{R}^m$ denotes an m-dimensional column vector. $\mathbf{0}$ denotes a zero matrix with compatible dimensions. For $P \in \mathbb{R}^{n\times n}$, $P > 0$ denotes a positive definite matrix. T denotes the transposition of a matrix or vector. $\|\cdot\|_p$ denotes the p norm. For $\nu = (\nu_1, \dots, \nu_p)^{\mathrm{T}} \in R^p$, $\mathrm{sgn}(\nu) = (\mathrm{sgn}(\nu_1), \dots, \mathrm{sgn}(\nu_p))^{\mathrm{T}}$, where sgn denotes the signum function. $I[1, N]$ denotes the set $\{1, \dots, N\}$. $\mathbb{P}[\cdot]$ denotes the probability measure, and $\mathbb{E}[\cdot]$ denotes the mathematical expectation.

2 Preliminaries

2.1 Algebraic graph basics

The communication graph $\mathcal{G}$ is modeled by $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$, where $\mathcal{V} = I[1, N]$ denotes the node set, $\mathcal{E} \subseteq I[1, N] \times I[1, N]$ denotes the edge set, $\mathcal{A} = [a_{ij}]_{N\times N}$ denotes the adjacency matrix. If $(j, i) \in \mathcal{E}$, $a_{ij} \geqslant 0$, otherwise, $a_{ij} = 0$. The set $\mathcal{N}_i$ denotes the neighbor set of node i with $\mathcal{N}_i = \{j|(j, i) \in \mathcal{E}\}$. Define the Laplace matrix $\mathcal{L} = [l_{ij}]_{N\times N}$, where $l_{ij} = -a_{ij}$ if $i \neq j$, and $l_{ii} = \sum_{j\in I[1,N]} a_{ij}$. The graph $\mathcal{G}$ is said to be undirected if $\forall$ $(i, j) \in \mathcal{E}$ is equal to $(j, i) \in \mathcal{E}$. If there exists a path between each pair of different nodes, a graph is connected.

In this paper, the communication graph among agents is modeled as a random switching graph $\mathcal{G}^{\sigma(t)} = (\mathcal{V}, \mathcal{E}^{\sigma(t)}, \mathcal{A}^{\sigma(t)})$, and they randomly switch among s different graphs $\mathcal{G}_1, \ldots, \mathcal{G}_s$. $\mathcal{G}^{\sigma(t)} = \mathcal{G}_i$ if and only if $\sigma(t) = i$. The switching signal $\sigma(t)$ obeys a Markovian process, which takes values in a finite state space $\mathcal{S} = \{1, \ldots, s\}$. Let $\Lambda = [\lambda_{mq}]_{s\times s}$ denote the transition rate matrix, where λ_{mq} satisfies

$$\mathbb{P}\{\sigma(t+\Delta t) = q|\sigma(t) = m\} = \begin{cases} 1 + \lambda_{mm}\Delta t + o(\Delta t), & q = m, \\ \lambda_{mq}\Delta t + o(\Delta t), & q \neq m, \end{cases}$$

with $\lambda_{mm} = -\sum_{q\neq m} \lambda_{mq}$, $\lambda_{mq} \geqslant 0$, and $o(\Delta t)$ denotes the higher order infinitesimal with $\lim_{\Delta t\to 0} \frac{o(\Delta t)}{\Delta t} = 0$.

Assumption 1. Each switching graph $\mathcal{G}^{\sigma(t)}$ is undirected, and the union graph $\mathcal{G}_u$ is connected.

Assumption 2. The Markovian process $\{\sigma(t), t \geqslant 0\}$ is ergodic.

Lemma 1 ([28])**.** If Assumptions 1 and 2 hold, $\mathbf{0}$ is an eigenvalue of the Laplace matrix $\mathcal{L}_u$ with 1_N as a corresponding right eigenvector and the other eigenvalues are positive. In addition, the smallest nonzero eigenvalue λ_2 of $\mathcal{L}_u$ satisfies $\lambda_2 = \min_{x\neq \mathbf{0}, 1_N^{\mathrm{T}} x = 0} \frac{x^{\mathrm{T}}\mathcal{L}_u x}{x^{\mathrm{T}} x}$.

Lemma 2 ([31])**.** For a given function $w(t)$, if it satisfies $\mathbb{E}[w(t)\mathbf{1}_{\{\sigma(t)=m\}}] \triangleq w_m(t)$, $m \in I[1, s]$, then, one has

$$\mathbb{E}\big[w(t)\mathrm{d}(\mathbf{1}_{\{\sigma(t)=m\}})\big] = \sum_{q\in I[1,s]} \lambda_{qm} w_q(t)\mathrm{d}t + o(\mathrm{d}t),$$

where $o(\mathrm{d}t)$ denotes the higher order infinitesimal with $\lim_{\mathrm{d}t\to 0} \frac{o(\mathrm{d}t)}{\mathrm{d}t} = 0$, $\mathbf{1}_{\{\sigma(t)=m\}} = \left\{\begin{smallmatrix} 1, & q = m, \\ 0, & q \neq m. \end{smallmatrix}\right.$

Remark 1. In the sense of classical functions, it is evident that the function $1_{\{\sigma(t)=m\}}$ is undifferentiable, since it is discontinuous at $q = m$ and is clearly undifferentiable. When generalized functions are introduced, the function $1_{\{\sigma(t)=m\}}$ is differentiable, where the function $\delta = \left\{\begin{smallmatrix} 0, & q = m, \\ \infty, & q \neq m \end{smallmatrix}\right.$ is its derivative.

2.2 System model

Consider a linear MAS composed of N agents. The mathematical model of the ith agent is

$$\dot{x}_i = Ax_i + Bu_i, \ i \in I[1, N], \tag{1}$$

where $x_i \in \mathbb{R}^n$ and $u_i \in \mathbb{R}^m$ denote the state and control input for agent i, respectively, $A \in \mathbb{R}^{n\times n}$ and $B \in \mathbb{R}^{n\times m}$ are the system matrices.

The purpose of this paper is to design the fully distributed event-triggered control strategy to realize the distributed optimal consensus problem. The definition of optimal consensus problem is given as follows.

Definition 1 ([12,13])**.** Design the control strategy such that the states of all agents reach consensus at the optimal solution to the following convex optimization problem: $\min \sum_{i\in I[1,N]} f_i(w), \ w \in \mathbb{R}^n$, where $f_i(w) : \mathbb{R}^n \to \mathbb{R}$ is a local cost function known only to agent i.

The following assumptions ensure the solvability of the distributed optimal consensus problem.

Assumption 3. The system matrices satisfy that (A, B) is stabilizable.

Assumption 4. The local cost function $f_i(\cdot) : \mathbb{R}^n \to \mathbb{R}$ for agent i is differentiable, convex and satisfies

$$\frac{\partial f_i(w)}{\partial w} = \mathcal{R}\big[Aw + B\varphi_i(w)\big], \tag{2}$$

where $\mathcal{R} \in \mathbb{R}^{n\times n}$ is a negative definite matrix, $\varphi_i(w)$ is a sub gradient satisfying $\|\varphi_i(w)\|_2 \leqslant \varphi_m, \ \varphi_m > 0$, and $\|\varphi_i(w) - \varphi_j(w)\|_\infty \leqslant \varpi_{ij}, \ \varpi_{ij} > 0, \ w \in \mathbb{R}^n$.

Assumption 5. The optimal solution set of $f_i(x_i)$ for each local cost function is nonempty; i.e., there exists x_i^* satisfying that $f_i(x_i^*)$ is minimum.

Remark 2. Assumptions 3–5 are standard to deal with the distributed optimal consensus problem for MAS in (1). Assumption 4 implies that MAS in (1) can generate the descending direction of $f_i(x_i)$. Note that the system satisfying Assumption 4 is considered in [16]. The term $\varphi_i(x_i)$ plays an important role in reaching optimal consensus, which can be seen from optimization proof in Theorem 1.

Lemma 3 ([32])**.** Given a continuous differentiable convex function $F(w): \mathbb{R}^n \to \mathbb{R}$. Variable $w^* \in \mathbb{R}^n$ is called as a global minimum of $F(w)$ if and only if $\lim_{w\to w^*} \frac{\partial F(w)}{\partial w} = \mathbf{0}$.

3 Main results

In this section, synchronous and asynchronous communication schemes are given.

3.1 Synchronous communication scheme

Firstly, a fully distributed event-triggered control scheme for agent i by synchronous communication between neighbors is designed as follows:

$$u_i = F\sum_{j\in I[1,N]} a_{ij}^{\sigma(t)}\mu_{ij}(\tilde{x}_i - \tilde{x}_j) + \sum_{j\in I[1,N]} a_{ij}\iota_{ij}\mathrm{sgn}\Big[F(\tilde{x}_i - \tilde{x}_j)\Big] + \varphi_i(x_i), \tag{3a}$$

$$\dot{\mu}_{ij} = v_{ij}a_{ij}^{\sigma(t)}\|F(\tilde{x}_i - \tilde{x}_j)\|_2^2, \tag{3b}$$

$$\dot{\iota}_{ij} = o_{ij}a_{ij}^{\sigma(t)}\|F(\tilde{x}_i - \tilde{x}_j)\|_1, \tag{3c}$$

where $\tilde{x}_i$ is the open-loop estimate for x_i, μ_{ij}, and ι_{ij} are the adaptive gains with $\mu_{ij}(0) = \mu_{ji}(0)$ and $\iota_{ij}(0) = \iota_{ji}(0)$, F is the feedback gain matrix to be specified, v_{ij} and o_{ij} are the positive constants with $v_{ij} = v_{ji}$ and $o_{ij} = o_{ji}$, and $\varphi_i(\cdot)$ is a sub gradient given in (2).

Remark 3. The controller in (3a) consists of two parts. The first one is to realize the consensus, and the second one is to minimize the local cost function $f_i(x_i)$. Compared with [16, 30, 33], adaptive gains are introduced to avoid dependence on the global information of the topology graph. Compared with [11, 12, 34], the estimated state of agent j, $j \in \mathcal{N}_i$ is used instead of the real-time state, which can relieve the communication between neighbors.

Define the state estimate $\tilde{x}_i$ as

$$\begin{cases} \dot{\tilde{x}}_i = A\tilde{x}_i, & t \in [t_k^i, t_{k+1}^i), \\ \tilde{x}_i(t) = x_i(t_k^i), & t = t_k^i. \end{cases} \tag{4}$$

Define the measurement error e_i as

$$e_i = \tilde{x}_i - x_i,\ t \in [t_k^i, t_{k+1}^i). \tag{5}$$

Triggered instant t_k^i is determined by the following composite triggering mechanism:

$$t_{k+1}^i = \inf\big\{t > t_k^i | h_i(e_i, \pi_i) \geqslant 0 \vee g_i(e_i, \pi_i^1) \geqslant 0\big\}, \tag{6}$$

where $h_i(e_i, \pi_i)$ and $g_i(e_i, \pi_i^1)$ are the triggering functions. They are designed as

$$h_i(e_i, \pi_i) = \sum_{j\in I[1,N]} (1 + \delta\mu_{ij})a_{ij}^{\sigma(t)}\|Fe_i\|_2^2 - \frac{1}{4}\sum_{j\in I[1,N]} a_{ij}^{\sigma(t)}\|F(\tilde{x}_i - \tilde{x}_j)\|_2^2 - \gamma_1\pi_i, \tag{7a}$$

$$\dot{\pi}_i = \gamma_2\left[\frac{1}{4}\sum_{j\in I[1,N]} a_{ij}^{\sigma(t)}\|F(\tilde{x}_i - \tilde{x}_j)\|_2^2 - \sum_{j\in I[1,N]} (1 + \delta\mu_{ij})a_{ij}^{\sigma(t)}\|Fe_i\|_2^2\right] - \gamma_3\pi_i, \tag{7b}$$

$$g_i(e_i, \pi_i^1) = \sum_{j\in I[1,N]} (1 + \delta^1\iota_{ij})a_{ij}^{\sigma(t)}\|Fe_i\|_1 - \delta_2\sum_{j\in I[1,N]} a_{ij}^{\sigma(t)}\|F(\tilde{x}_i - \tilde{x}_j)\|_1 - \gamma_1^1\pi_i^1, \tag{7c}$$

$$\dot{\pi}_i^1 = \gamma_2^1\left[\delta_2\sum_{j\in I[1,N]} a_{ij}^{\sigma(t)}\|F(\tilde{x}_i - \tilde{x}_j)\|_1 - \sum_{j\in I[1,N]} (1 + \delta^1\iota_{ij})a_{ij}^{\sigma(t)}\|Fe_i\|_1\right] - \gamma_3^1\pi_i^1, \tag{7d}$$

where $\delta, \delta^1, \delta_2, \gamma_1, \gamma_2, \gamma_3, \gamma_1^1, \gamma_2^1, \gamma_3^1$ are the positive constants with $\gamma_2 \in (0, 1)$, $\gamma_3 > (1-\gamma_2)\gamma_1$, $\delta^1 > \frac{1}{|\mathcal{N}_i|\varphi_m}$, and π_i and π_i^1 are the internal dynamic variables for agent i with $\pi_i(0) > 0$ and $\pi_i^1(0) > 0$.

Remark 4. When agent i triggers, i.e., $h_i(e_i,\pi_i) \geqslant 0$ or $g_i(e_i,\pi_i^1) \geqslant 0$ holds, reset $e_i = \mathbf{0}$. Then, agent i transmits current information to all its out neighbors. Therefore, the synchronous communication between neighbor agents is realized. Furthermore, the internal dynamic variables π_i and π_i^1 are introduced to expand the triggering interval compared with [28], which can be obtained from the simulation results.

Remark 5. The triggering mechanism in (6) is composed of complex triggering functions, and a single triggering function cannot achieve optimal consensus control. The triggering function in (7a) and (7b) is designed to deal with the first item in the controller (3a) and the triggering function in (7c) and (7d) is used to process the last two items in the controller (3a).

Define the consensus error as $\xi = [\xi_1^{\rm T},\dots,\xi_N^{\rm T}]^{\rm T}$ with $\xi_i = x_i - \frac{1}{N}\sum_{j\in I[1,N]} x_j$. Let $x = [x_1^{\rm T},\dots,x_N^{\rm T}]^{\rm T}$, $M = I_N - \frac{1}{N}\mathbf{1}\mathbf{1}^{\rm T}$. Then, $\xi = (M\otimes I_N)x$. It is not difficult to find that $\xi = \mathbf{0}$ if and only if $x_1 = x_2 = \cdots = x_N$. Therefore, the cooperative consensus problem of MAS in (1) is solved if $\xi \to \mathbf{0}$ as $t\to\infty$. Then, ξ is called as the consensus error. Furthermore, one can obtain that $M\mathcal{L} = \mathcal{L}M = M$.

The closed-loop system dynamics can be obtained by (1) and (3a):

$$\dot{x}_i = Ax_i + B\left[F\sum_{j\in I[1,N]} a_{ij}^{\sigma(t)}\mu_{ij}(\tilde{x}_i - \tilde{x}_j) + \sum_{j\in I[1,N]} a_{ij}^{\sigma(t)}\iota_{ij}\mathrm{sgn}[F(\tilde{x}_i - \tilde{x}_j)] + \varphi_i(x_i)\right]. \tag{8}$$

Then, the dynamics of consensus error ξ_i satisfies:

$$\begin{aligned}\dot{\xi}_i = {} & A\xi_i + BF\sum_{j\in I[1,N]} a_{ij}^{\sigma(t)}\mu_{ij}(\tilde{x}_i - \tilde{x}_j) + B\sum_{j\in I[1,N]} a_{ij}^{\sigma(t)}\iota_{ij}\mathrm{sgn}[F(\tilde{x}_i - \tilde{x}_j)] \\ & + B\left[\varphi_i(x_i) - \frac{1}{N}\sum_{j\in I[1,N]}\varphi_j(x_j)\right].\end{aligned} \tag{9}$$

Theorem 1. The distributed optimal consensus of MAS in (1) can be reached under Assumptions 1–5, if the fully distributed event-triggered control protocol is selected as shown in (3) and (5) with $F = -B^{\rm T}P$, where $P > 0$ is the solution of the following equality:

$$PA + A^{\rm T}P - PBB^{\rm T}P + lI_n = \mathbf{0}, \tag{10}$$

for l being the positive constant. Furthermore, there is no Zeno behavior.

Proof. See Appendix A for the detailed proof.

Remark 6. The proposed controller in (3) includes sign function, and the stability of error system in (9) will be understood in the sense of Filippov [35]. Because the sign function is locally essentially bounded and measurable, the system in (9) holds almost everywhere. Since it does not affect the stability analysis, almost everywhere is omitted in the analysis of this paper.

3.2 Asynchronous communication scheme

In this subsection, a fully distributed event-triggered protocol for agent i by asynchronous communication between neighbors is designed as follows:

$$u_i = F\sum_{j\in I[1,N]} a_{ij}^{\sigma(t)} d_{ij}(\tilde{x}_{ij} - \tilde{x}_{ji}) + \sum_{j\in I[1,N]} a_{ij}^{\sigma(t)} c_{ij}\mathrm{sgn}[F(\tilde{x}_{ij} - \tilde{x}_{ji})] + \varphi_i(x_i), \tag{11a}$$

$$\dot{d}_{ij} = m_{ij}a_{ij}^{\sigma(t)}\|F(\tilde{x}_{ij} - \tilde{x}_{ji})\|_2^2, \tag{11b}$$

$$\dot{c}_{ij} = \tau_{ij}a_{ij}^{\sigma(t)}\|F(\tilde{x}_{ij} - \tilde{x}_{ji})\|_1, \tag{11c}$$

where $\tilde{x}_{ij} = \mathrm{e}^{A(t-t_k^{ij})}x_i(t_k^{ij}),\ t\in[t_k^{ij}, t_{k+1}^{ij})$ is the open-loop estimate for x_i, d_{ij}, and c_{ij} are the adaptive gains with $d_{ij}(0) = d_{ji}(0)$ and $c_{ij}(0) = c_{ji}(0)$, F is the feedback gain matrix to be specified, m_{ij}, and τ_{ij} are the positive constants with $m_{ij} = m_{ji}$ and $\tau_{ij} = \tau_{ji}$, and $\varphi_i(x_i)$ is given in (2).

Remark 7. The difference from the controller in (3) and the controller in (11) is reflected in the state estimation. The state estimation in the controller (3) is based on the triggering instant t_k^i, which depends on itself and all its neighbor information. The state estimation in the controller (11) is based on the triggering instant t_k^{ij}, which depends on the information of the nodes constituting the edge.

Define the measurement error e_{ij} as

$$e_{ij} = \tilde{x}_{ij} - x_i,\ t \in [t_k^{ij}, t_{k+1}^{ij}). \tag{12}$$

Triggering instant t_k^{ij} is determined by the following triggering mechanism:

$$t_{k+1}^{ij} = \inf\{t > t_k^{ij} | h_{ij}(e_{ij}, \pi_{ij}) \geqslant 0 \vee g_{ij}(e_{ij}, \pi_{ij}^1) \geqslant 0\}, \tag{13}$$

where $h_{ij}(e_{ij}, \pi_{ij}),\ g_{ij}(e_{ij}, \pi_{ij}^1)$ are the triggering functions. They are designed as

$$h_{ij}(e_{ij}, \pi_{ij}) = (1 + \sigma d_{ij})\|Fe_{ij}\|_2^2 - \frac{1}{4}\|F(\tilde{x}_{ij} - \tilde{x}_{ji})\|_2^2 - \bar{\gamma}_1 \pi_{ij}, \tag{14a}$$

$$\dot{\pi}_{ij} = \bar{\gamma}_2 \left[\frac{1}{4}\|F(\tilde{x}_{ij} - \tilde{x}_{ji})\|_2^2 - (1 + \sigma d_{ij})\|Fe_{ij}\|_2^2\right] - \bar{\gamma}_3 \pi_{ij}, \tag{14b}$$

$$g_{ij}(e_{ij}, \pi_{ij}^1) = (1 + \sigma^1 c_{ij})\|Fe_{ij}\|_1 - \sigma_2 \|F(\tilde{x}_{ij} - \tilde{x}_{ji})\|_1 - \bar{\gamma}_1^1 \pi_{ij}^1, \tag{14c}$$

$$\dot{\pi}_{ij}^1 = \bar{\gamma}_2^1 \left[\sigma_2 \|F(\tilde{x}_{ij} - \tilde{x}_{ji})\|_1 - (1 + \sigma^1 c_{ij})\|Fe_{ij}\|_1\right] - \bar{\gamma}_3^1 \pi_{ij}^1, \tag{14d}$$

where $\sigma, \sigma^1, \sigma_2, \bar{\gamma}_1, \bar{\gamma}_2, \bar{\gamma}_3, \bar{\gamma}_1^1, \bar{\gamma}_2^1, \bar{\gamma}_3^1$ are the positive constants with $\bar{\gamma}_2 \in (0,1)$, $\bar{\gamma}_3 > (1-\bar{\gamma}_2)\bar{\gamma}_1$, $\sigma^1 > \frac{1}{2\varphi_m}$, and $\bar{\pi}_{ij}$ and $\bar{\pi}_{ij}^1$ are the internal dynamic variables for agent i with $\pi_{ij}(0) > 0$ and $\pi_{ij}^1(0) > 0$.

Remark 8. When the edge (j, i) triggers, i.e., $h_{ij}(e_{ij}, \pi_{ij}) \geqslant 0$ or $g_{ij}(e_{ij}, \pi_{ij}^1) \geqslant 0$ holds, reset $e_{ij} = \mathbf{0}$. Then, agent j transmits current information to agent i instead of all its out neighbors. Therefore, the asynchronous communication between neighbor agents is realized. Moreover, the triggering function in (14) is easy to construct than the triggering function in (7), which only depends on the information of agents i and j.

The closed-loop system dynamics can be obtained by (1) and (11a):

$$\dot{x}_i = Ax_i + B\left[F \sum_{j \in I[1,N]} a_{ij}^{\sigma(t)} d_{ij}(\tilde{x}_{ij} - \tilde{x}_{ji}) + \sum_{j \in I[1,N]} a_{ij}^{\sigma(t)} c_{ij} \mathrm{sgn}[F(\tilde{x}_{ij} - \tilde{x}_{ji})] + \varphi_i(x_i)\right]. \tag{15}$$

Then, the dynamics of consensus error ξ_i satisfies:

$$\begin{aligned} \dot{\xi}_i = {} & A\xi_i + BF \sum_{j \in I[1,N]} a_{ij}^{\sigma(t)} d_{ij}(\tilde{x}_{ij} - \tilde{x}_{ji}) + B \sum_{j \in I[1,N]} a_{ij}^{\sigma(t)} c_{ij} \mathrm{sgn}[F(\tilde{x}_{ij} - \tilde{x}_{ji})] \\ & + B\left[\varphi_i(x_i) - \frac{1}{N} \sum_{j \in I[1,N]} \varphi_j(x_j)\right]. \end{aligned} \tag{16}$$

Theorem 2. The distributed optimal consensus of MAS in (1) can be reached under Assumptions 1–5, if the fully distributed event-triggered control protocol is selected as shown in (11) and (13) with $F = -B^{\mathrm{T}}P$, where $P > 0$ is the solution of (10). Furthermore, there is no Zeno behavior.

Proof. See Appendix B for the detailed proof.

Remark 9. Asynchronous communication refers to "one-to-one" transmission of information, while synchronous communication refers to "one to many" transmission of information. From synchronous and asynchronous communication viewpoints, respectively, two kinds of fully distributed event-triggered control schemes are designed to solve optimal consensus problem. Asynchronous communication can further alleviate the burden of communication compared with synchronous communications. The cost of asynchronous communication is the high demand for hardware equipment, which requires installing multiple event detectors on each agent, while synchronous communication only requires installing one event detector on each agent. Therefore, the users can select an appropriate control scheme according to the configuration of the agent and the communication bandwidth.

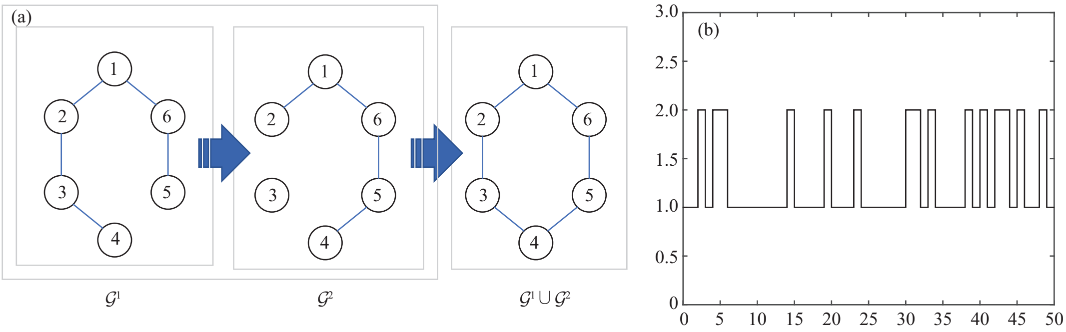

Figure 1 (Color online) (a) Switching topologies; (b) switching signal with two modes.

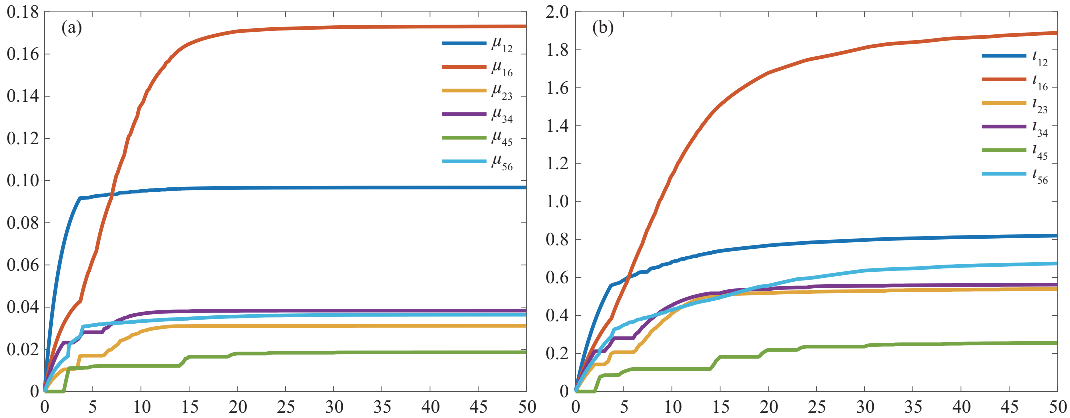

Figure 2 (Color online) Simulation results of Theorem 1. (a) Adaptive gain μ_{ij} in (3b); (b) adaptive gain ι_{ij} in (3c).

Remark 10. Note that the distributed optimization problem does not indicate that the sum of the optimal values is obtained by local optimization without communication networks. Communication networks act on an important role in the calculation process. In short, local optimization methods without communication networks cannot be used to solve distributed optimization problems, but the proposed distributed optimization schemes in this paper are useful.

4 Example

In this section, a numerical example is given to verify the validity of the theoretical results obtained in this paper. The switching topologies among 6 agents are shown in Figure 1(a), where initial stations are randomly selected.

The dynamics of agent i are shown in (1), where the system matrices are $A = [-1\ 2; 1\ \ -3]$ and $B = [0; 1]$. The local cost function $f_i(x_i)$ is selected as

$$f_i(x_i) = \begin{bmatrix} x_{i1} \\ x_{i2} \\ \varphi_i \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} 1 & -2 & 0 \\ -2 & 6 & -2 \\ 0 & -2 & 3 \end{bmatrix} \begin{bmatrix} x_{i1} \\ x_{i2} \\ \varphi_i \end{bmatrix},$$

where $\varphi_i = \frac{i}{2}$, $i \in I[1,6]$. Moreover, the matrix $\mathcal{R}$ in (2) is selected as $\mathcal{R} = [-1\ 0; 0\ \ -\frac{1}{2}]$. The network topologies are governed by Markovian switching signal $\{\sigma(t), t \geqslant 0\}$ with two modes, as shown in Figure 1(b).

The initial states are selected as $x_1 = [2.9227; 1.7125]$, $x_2 = [-0.9969; -0.5535]$, $x_3 = [0.5155; 0.3307]$, $x_4 = [2.1500; 2.4590]$, $x_5 = [-0.2131; -2.6632]$, $x_6 = [0.0646; 0.4362]$. The initial internal dynamic variables $\pi_i(0)$, $\pi_i^1(0)$, $\pi_{ij}(0)$, $\pi_{ij}^1(0)$ are selected as 1. The solution P of (10) satisfies $P = [0.065\ 0.0412; 0.0412\ 0.0356]$.

Figure 2 shows the dynamic responses of the adaptive gains μ_{ij} and ι_{ij} under Theorem 1. It can be seen that the adaptive gains converge to some finite values. Figure 3(a) shows that three-dimensional

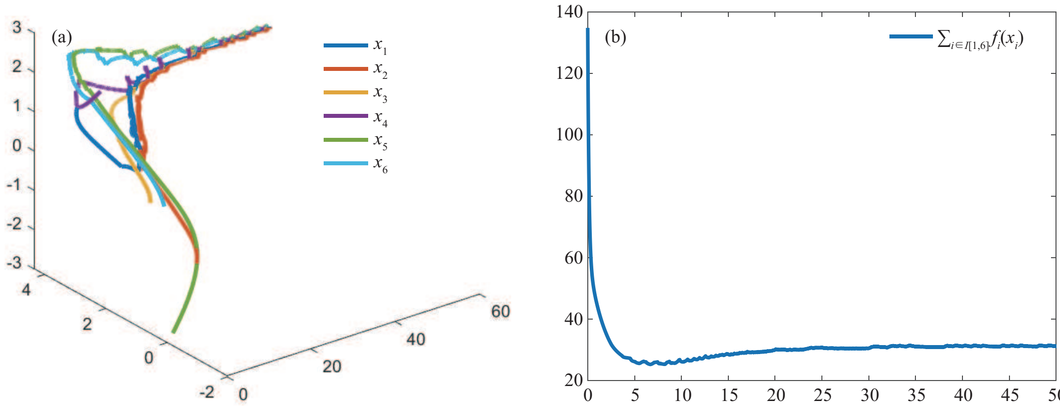

Figure 3 (Color online) Simulation results of Theorem 1. (a) Dynamic responses of agents' states; (b) dynamic response of team cost function $\sum_{i\in I[1,N]} f_i(x_i)$.

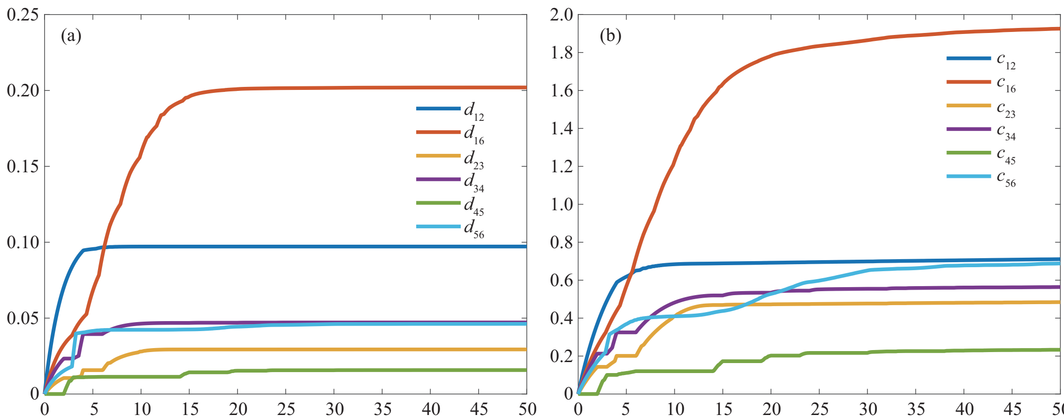

Figure 4 (Color online) Simulation results of Theorem 2. (a) Adaptive gain d_{ij} in (11b); (b) adaptive gain c_{ij} in (11c).

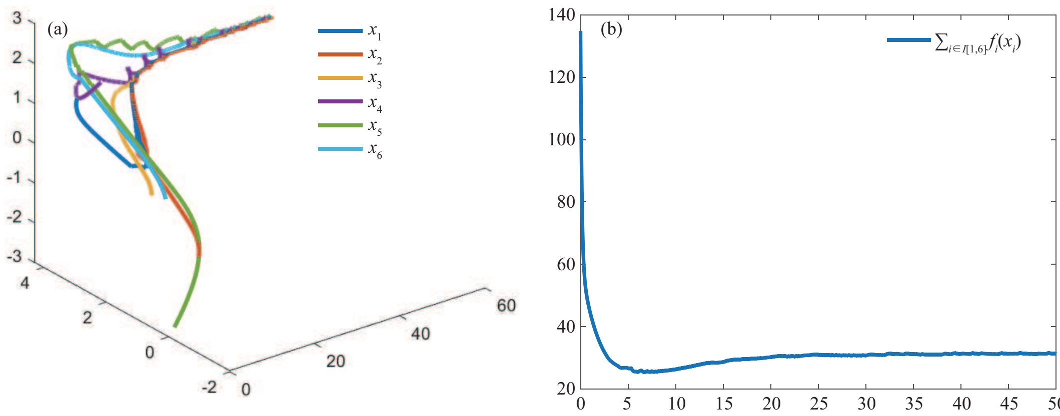

Figure 5 (Color online) Simulation results of Theorem 2. (a) Dynamic responses of agents' states; (b) dynamic response of team cost function $\sum_{i\in I[1,N]} f_i(x_i)$.

trajectories of six agents, which indicate that the consensus is reached. From Figure 3(b), the team cost function finally converges to 31.49. Therefore, it follows from Figure 3 that the optimal consensus problem of MAS in (1) is achieved. Next, by using the fully distributed event-triggered control scheme in (11) with the same parameters and initial conditions, the simulation results are shown in Figures 4 and 5. In Figure 4, one can obtain that adaptive gains converge to finite values. It can be seen from Figure 5 that the distributed optimal consensus problem is solved with the team cost function converging to 31.49 as $x_i \to x*$. Figure 6 depicts the triggering instants under two event-triggered mechanisms, which implies that Zeno behavior does not occur under the proposed fully distributed event-triggered

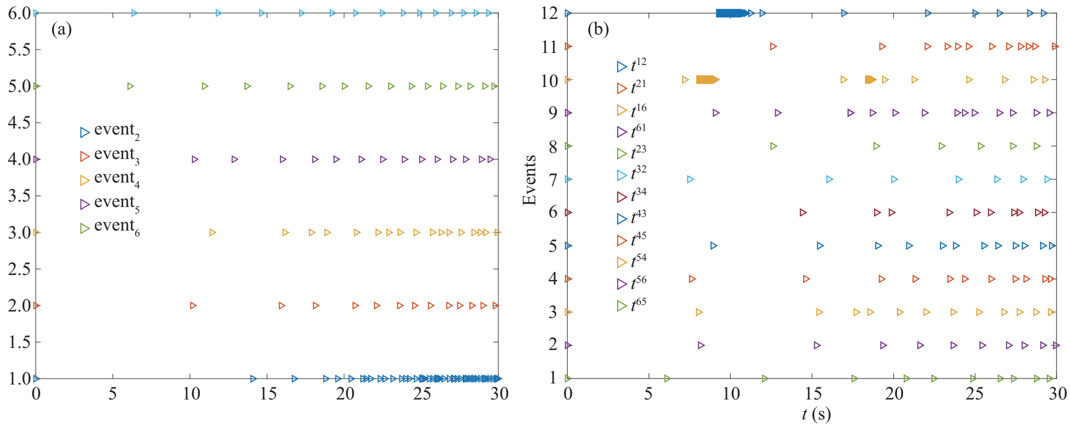

Figure 6 (Color online) (a) Triggering instants under Theorem 1; (b) triggering instants under Theorem 2.

Table 1 The triggering numbers under different mechanisms

Triggering mechanism	Agent 1	Agent 2	Agent 3	Agent 4	Agent 5	Agent 6
DETM in (6)	53	125	79	126	170	85
ETM in [28]	62	277	340	226	467	120

control protocols. When the triggering condition of agent i is satisfied, it sends information to all its out neighbors simultaneously under the fully distributed event-triggered control protocol in (3). When the triggering condition of edge (j, i) is satisfied, agent j only sends current information to agent i under the fully distributed event-triggered control protocol in (11). It can be seen from Figure 6(b) that agent i will not send information to its out neighbors at the same time. For example, triggering instants t^{12} are not equal to the triggering instants t^{16}, which means that the time when agent 1 sends information to agent 2 is independent of the time when agent 1 sends information to agent 6. Therefore, the asynchronous event-triggered communication scheme can further reduce the demand for network communication bandwidth compared with the synchronous event-triggered communication scheme, the defect is that more event-triggered detectors need to be designed and the requirements for intelligent hardware equipment are high. In conclusion, the users can choose two schemes according to the agent configuration to realize the optimal consensus problem.

In order to further highlight the advantages of designing a dynamic event-triggered mechanism in (6), Table 1 records the frequency of event occurrences under different triggering mechanisms. From Table 1, it is evident that the proposed dynamic event-triggered mechanism is more resource efficient for the MASs compared with triggering mechanisms in [28].

5 Conclusion

Herein, the distributed optimal consensus problem of MASs with Markovian switching topologies was investigated using fully distributed event-triggered control schemes, where synchronous and asynchronous communications are achieved. While minimizing the team cost function, the states of the MASs successfully reached a consensus. Moreover, theoretical analysis and simulation results verified the effectiveness of the proposed methods. In some cases, cooperative tasks must be completed within a fixed/prescribed time to meet certain system requirements (see [36]). Therefore, our future research may consider the fixed-time optimal consensus problem of MASs.

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Supporting information Appendixes A and B. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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