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Coverage control for mobile sensor networks with unknown terrain roughness and nonuniform time-varying communication delays

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Abstract This paper investigates the coverage control problem on a circle with unknown terrain roughness and nonuniform time-varying communication delays. Adaptive coverage control laws are proposed for mobile sensors to collaboratively estimate the unknown roughness function using the basis function approximation approach. Moreover, contrary to existing studies, nonuniform time-varying communication delays are considered in this paper. Under the proposed adaptive coverage control laws, the sensor network can be driven to its optimal configuration minimizing the coverage cost function in the presence of nonuniform communication delays, and each sensor can learn the true roughness function. Finally, a simulation example is provided to show the effectiveness of the proposed control laws.

Keywords multi-agent systems, cooperative control, adaptive control, cost function, time-varying communication delays

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1 Introduction

In the past few decades, many studies on the coverage control problem that has a broad range of potential applications, including cooperative surveillance, patrolling, and environmental monitoring, have been reported [1–3]. In the coverage control problem, the networked mobile sensors are supposed to be placed in the mission field to drive themselves to the optimal configuration minimizing the coverage cost function. For instance, the integral of a weighted distance on the mission space was adopted to measure the coverage performance [4–8], and the joint probability for event detection by the sensor network was defined as another coverage cost function [9–12]. Moreover, coverage performance measured based on the response time from the sensor network to any point in the mission space was investigated [13–15].

Though numerous results have been reported for two-dimensional coverage problems, coverage control problem in a one-dimensional mission space has also gained extensive attention in the past decade. By formulating the one-dimensional deployment problem as a linear program, coverage control for unreliable mobile sensor networks (MSNs) was addressed in [16]. In [17], a sufficient and necessary condition to minimize the response time for heterogeneous sensors on a circle was derived, and this work was extended in [18, 19], where the limited communication ranges and bounded position measurement errors were considered, respectively. Moreover, the decomposition of the Hessian matrix derived from the locational cost function was used to derive a sufficient condition for achieving centroidal Voronoi tessellation configurations in a one-dimensional environment in [20].

In this work, we pay attention to the coverage control problem on a circle. The circular mission space can be extended to any closed curve using the parameterization method with a single variable, such as

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the curve arc length [21]. Thus, the coverage control problem on the circular mission space can be applied to many practical scenarios. For example, for the target-capturing and target-enclosing problems [22, 23], the deployment of mobile agents around the target on a circle for better coverage performance increases the possibility that the agents will either capture the target or defend against attackers when the target or attackers reach the circle. In [24], mobile sensors were deployed along the perimeter of a fire, and each sensor was responsible for the surveillance of a perimeter section. Coverage control laws can be adopted to rapidly detect the propagation of a fire. Moreover, on the ocean surface, mobile sensors need to move around a closed curve periodically for oceanographic sampling, and the coverage control laws can be adopted for the optimal data collection here [25].

In practice, the travel time of mobile sensors will be influenced not only by the sensors' movement capabilities but also by the roughness of the environment. In the sweep coverage problem, online partitioning algorithms were proposed to balance the workload of each agent while considering a rectangular region with unknown workload distribution [26, 27]. Through the use of the lifting Markov chains, fully distributed coverage control laws were developed for mobile sensors on a line with varying roughness [28]. Assuming that sensors can only access noisy measurements of the roughness function, a randomized protocol was presented to drive the sensor network to a near-optimal configuration on the line [29]. However, only homogeneous mobile sensors with identical movement capabilities were considered in [28, 29]. By contrast, coverage control for heterogeneous mobile sensors on a line with varying roughness was considered in [30], where the roughness function was known by all sensors at the beginning.

In practice, even a small time delay may damage the stability of MSNs with well-designed control laws. The constant time delay of the two-dimensional coverage control was considered in [31] by introducing a guaranteed multiplicatively weighted Voronoi diagram. Considering both constant delays and switching communication topology, the equidistant deployment on a line was investigated [32]. In [33], the mission space of a closed curve was considered for the coverage control problem, and uniform time-varying communication delays were investigated.

However, detailed roughness information is often hard to be collected using the sensors before the coverage task and the nonuniform communication delays will always occur between sensors, particularly when the networked mobile sensors are heterogeneous. Thus, the investigation of the coverage control problem subject to unknown terrain roughness and nonuniform time-varying communication delays is of practical significance. The contributions of this paper can be summarized as follows: First, the basis function approximation approach is adopted to design adaptive coverage control laws considering that the terrain roughness is unknown to the sensor network. The true roughness function will be estimated using the adaptive coverage control laws; meanwhile, the MSN will achieve the optimal configuration asymptotically. Moreover, the persistent excitation condition proposed in [5] is realized by selecting appropriate basis functions, so that the exact optimal configuration can be achieved. Second, compared with [30], communication delays and terrain roughness are considered in this work. Then, the non-Euclidean distance, which takes the integral of the interval of delayed position variables, will be considered. However, the non-Euclidean distance with delayed position variables cannot be directly decomposed into the non-Euclidean distance with current position variables and an integral on a period of time using the Newton-Leibniz theorem. This problem has not been seen in any existing study, which makes our techniques innovative. To solve this problem, the relationship between the integral of the delayed position variables and the integral on a period of time is derived in this paper so that the Lyapunov-Krasovskii functional can be applied to the convergence analysis. Third, unlike the previous results [33], where uniform time-varying communication delays are considered for the coverage control problem, nonuniform time-varying communication delays are allowed among MSNs in this work. When nonuniform communication delays are considered, a compact form of the closed-loop system often contains a sum of Laplacian matrices [34, 35], which will induce a condition expressed by linear matrix inequalities (LMI) to guarantee the convergence of the system. To avoid using large LMI in the convergence condition, the periodic property of circular communication topology and a new Lyapunov-Krasovskii functional without a Laplacian matrix are utilized to analyze the convergence of MSNs in this paper.

2 Problem formulation and preliminaries

Consider the mission space of a unit circle, and a group of n mobile sensors is constrained to move on it. The coordinate system is defined as the angle in counterclockwise from the positive direction of the horizontal axis. Then, the coordinate of any position p on the circle can be determined, and let p_i represent the angle position of sensor i, where $i \in \mathcal{I}_n = \{1, \ldots, n\}$. The set of all points on the circle is defined as $\mathcal{S}$, then one has $p \in \mathcal{S}$.

The influence of nonuniform terrain roughness on the sensors' movement capability is considered in [28, 29], where the same distance with a larger terrain roughness value will take the sensor more time to traverse. Let $\xi : [0,\ 2\pi] \to \mathbb{R}^+$ denote a continuous function representing terrain roughness on a certain position in the mission space, where $\mathbb{R}^+$ represents the set of positive real numbers. Make the following assumption, that is, $0 < \xi_{\min} \leqslant \xi(p) \leqslant \xi_{\max}$, which implies that ξ is bounded by two constants $\xi_{\min}$ and $\xi_{\max}$. In addition, the following equation can be obtained using the periodic property:

$$\xi(p) = \xi(p + 2k\pi), \tag{1}$$

where $k \in \mathbb{Z}$. It should be noted that the roughness function $\xi(\cdot)$ is unknown to the sensors at the beginning of the coverage task.

The mobile sensors' dynamics with nonuniform terrain roughness is described as follows:

$$\dot{p}_i = \frac{u_i}{\xi(p_i)}, \tag{2}$$

where p_i is sensor i's position and u_i is the corresponding control input. From (2), one has that the velocity of the sensor will be decreased by larger terrain roughness. Moreover, the mobile sensors' dynamics will become common single-integrator dynamics when $\xi(p_i) = 1$. According to the sensors' initial positions, they are sorted in a counterclockwise direction to simplify the subsequent analysis, as follows:

$$0 \leqslant p_1(0) < \cdots < p_i(0) < p_{i+1}(0) < \cdots < p_n(0) < 2\pi. \tag{3}$$

Noting the periodic property, one has $p_{n+1} = p_1 + 2\pi$. Thus, if sensor 1 is in the positive direction of sensor n, then the relative position of sensor 1 with respect to sensor n can be calculated using $p_{n+1} - p_n$ instead of $p_1 - p_n$, which is negative.

Then, we consider the response time of a sensor to arrive at the point where an emergency happens. Let W_i denote the sensor i's maximum driving velocity, that is, $|u_i(t)| \leqslant W_i$, $\forall t \geqslant 0$. Further, when an emergency happens at $q \in \mathcal{S}$, sensor i will be assigned to move to q from its current position p_i. Thus, the following equation to compute the shortest response time T_i is derived:

$$T_i = \int_{\min(p_i,q)}^{\max(p_i,q)} \frac{1}{W_i/\xi(p)} \mathrm{d}p = \frac{\bar{d}_\xi(p_i, q)}{W_i}, \tag{4}$$

where $\bar{d}_\xi(p_i, q) = \int_{\min(p_i,q)}^{\max(p_i,q)} \xi(p)\mathrm{d}p$ is the non-Euclidean distance, which is adopted in [28]. Hence, higher terrain roughness will increase the moving time of sensor i for the same distance. Furthermore, Eq. (4) also shows that the sensor will take more time to traverse the same distance for larger terrain roughness.

The coverage cost function, which is similar to that in [17], is expressed as follows:

$$T(p_1, \ldots, p_n) = \max_{q \in \mathcal{S}} \min_{i \in \mathcal{I}_n} \frac{\bar{d}_\xi(p_i, q)}{W_i}, \tag{5}$$

where $\min_{i \in \mathcal{I}_n} \frac{\bar{d}_\xi(p_i,q)}{W_i}$ is the response time to q from the sensor network, and $i \in \mathcal{I}_n$ is the sensor with the minimum arrival time. After deciding the response time for q, the coverage cost function T is defined as the longest response time over all $q \in \mathcal{S}$. Thus, a smaller coverage cost function implies a shorter largest response time for the emergency occurs at any point on the circle.

Moreover, nonuniform time-varying communication delays are investigated in this work. In this coverage task, each sensor i exchanges its information with its neighbors, that is, sensors $i-1$ and $i+1$, indicating that the communication topology is fixed during the entire evolution process. Let $\tau_{i,j}(t)$ be the time delay in transmitting information from sensor j to i.

The objective of this work is to develop coverage control laws under unknown terrain roughness and nonuniform time-varying communication delays so that the MSN can be driven to its optimal positions minimizing equation (5). Two assumptions are needed for the subsequent design and analysis. One assumption is about the roughness function $\xi(p)$ [5], and the other is about the time-delays [36–42].

Assumption 1 (Matching conditions [5])**.** There exists an ideal parameter vector $a \in \mathbb{R}^m$ such that

$$\xi(p) = \mathcal{D}^{\mathrm{T}}(p)a, \tag{6}$$

where $\mathcal{D}^{\mathrm{T}}(p) = [D_1(p), D_2(p), \ldots, D_m(p)]$ is a vector known to each sensor and $D : \mathcal{S} \to \mathbb{R}_+$ is a bounded, smooth basis function, while $\xi(p)$ and a are unknown to the sensor network.

Many families of basis functions can be considered for $\mathcal{D}(p)$, such as Gaussians, wavelets, sigmoids, and splines [5], under the condition that the coordinates of the points in the set $\mathcal{S}$ are in the basis functions domains. Then, the roughness function approximation can then be computed as $\hat{\xi}_i(p,t) = \mathcal{D}^{\mathrm{T}}(p)\hat{a}_i(t)$, where $\hat{a}_i(t)$ is computed using adaptation laws in Section 3.

Assumption 2. The nonuniform time-varying communication delay $\tau_{i,j}(t)$ satisfies $0 < \tau_{i,j}(t) < \tau_{i,j}^m$, $\dot{\tau}_{i,j}(t) < d_{i,j}^m$ and $\ddot{\tau}_{i,j}(t)$ is bounded for all $t > 0$, $i, j \in \mathcal{I}_n$. Moreover, communication delays between sensors are symmetrical, that is, $\tau_{i,j}(t) = \tau_{j,i}(t)$.

Notably, $d_{i,j}^m$ is allowed to be unknown to the sensors, whereas $\tau_{i,j}^m$ should be known.

3 Adaptive coverage control laws

In this section, adaptive coverage control laws are developed to drive the MSN to their optimal positions considering both unknown terrain roughness and nonuniform time-varying communication delays.

Recalling that sensor i's maximum driving velocity is W_i, the proposed coverage control laws are expressed as follows:

$$u_i = W_i \mathrm{sat}(\bar{u}_i), \tag{7}$$

with

$$\bar{u}_i = \sigma_i \left(\frac{d_{\hat{\xi}_i}(p_i(t-\tau_{i,i+1}(t)), p_{i+1}(t-\tau_{i,i+1}(t)))}{W_i + W_{i+1}} - \frac{d_{\hat{\xi}_i}(p_{i-1}(t-\tau_{i,i-1}(t)), p_i(t-\tau_{i,i-1}(t)))}{W_{i-1} + W_i} \right), \tag{8}$$

where σ_i is a low control gain and $\mathrm{sat}(x) = \mathrm{sign}(x)\min\{1, |x|\}$. Moreover, given the periodic property, one has the following expression:

$$W_{n+i} \equiv W_n, u_{n+i} \equiv u_n. \tag{9}$$

The non-Euclidean distance $d_{\hat{\xi}_i}(a,b)$ is estimated using the following equation:

$$d_{\hat{\xi}_i}(a,b) = \int_a^b \hat{\xi}_i(p)\mathrm{d}p, \tag{10}$$

where

$$\hat{\xi}_i(p,t) = \mathcal{D}^{\mathrm{T}}(p)\hat{a}_i(t), \tag{11}$$

the vector $\hat{a}_i(t)$ is estimated by sensor i. Consequently, the parameter error is computed as follows:

$$\tilde{a}_i(t) = \hat{a}_i(t) - a. \tag{12}$$

In (8), when sensor i receives delayed positions from its neighbors and estimates the non-Euclidean distance, the difference of the weighted non-Euclidean distance is calculated as feedback, so that the optimal condition (16) can be achieved.

To estimate the unknown terrain roughness, the adaptive laws, which are inspired by [5], are designed as follows:

$$\dot{\hat{a}}_i = -\left(\frac{\int_{p_i(t-\tau_{i,i+1}(t))}^{p_{i+1}(t-\tau_{i,i+1}(t))} \mathcal{D}(p)\mathrm{d}p}{W_i + W_{i+1}} - \frac{\int_{p_{i-1}(t-\tau_{i,i-1}(t))}^{p_i(t-\tau_{i,i-1}(t))} \mathcal{D}(p)\mathrm{d}p}{W_{i-1} + W_i} \right) u_i - \gamma_i(\Omega_i \hat{a}_i - \omega_i), \tag{13}$$

where γ_i are adaptive gains. Ω_i and ω_i are defined as follows:

$$\Omega_i(t) = \int_0^t \mathcal{D}(p_i(\tau))\mathcal{D}^{\mathrm{T}}(p_i(\tau))\mathrm{d}\tau \tag{14}$$

and

$$\omega_i(t) = \int_0^t \mathcal{D}(p_i(\tau))\xi(p_i(\tau))\mathrm{d}\tau, \tag{15}$$

where $\xi(p_i(t))$ is the real terrain roughness at $p_i(t)$, which can be detected by sensor i using a laser stripe generator [29]. The first term of (13) is the compensation term, which is introduced to resolve the estimation errors, and the second term is a gradient descent term to minimize the time integration of $\tilde{\xi}_i(p_i)$, which is the error of the estimated terrain roughness.

Remark 1. In (15), the real terrain roughness at each sensor's current position needs to be obtained using the laser stripe generator. Notably, the coverage control law (8), which drives the sensors to their optimal configuration, is dependent on the estimated non-Euclidean distance (10) between neighbors. Thus, the terrain roughness at each sensor's current position needs to be measured as samples to estimate the real roughness $\xi(p)$ through the entire mission space $p \in [0, 2\pi]$ using (11) and (13). Then, the estimated non-Euclidean distance (10) in (8) can be calculated.

Remark 2. The mechanism of collision avoidance is not discussed in this paper. To enable the possible swap between sensors during the coverage task, assume that the sensors can be driven to side lanes of the circle by other controllers when they need to cross each other. Notably, the communication network topology is determined by the initial position of the sensors on the circle, and the topology is fixed during the entire coverage task.

4 Convergence analysis

Considering nonuniform time-varying communication delays, we first analyze the convergence of the closed-loop system using the Lyapunov-Krasovskii functional. Then, we show that the MSN will learn the true roughness function $\xi(p)$ and the coverage cost function will be minimized asymptotically under the proposed coverage control laws, indicating that the networked mobile sensors will achieve the optimal configuration asymptotically.

Lemma 1 ([43])**.** Let V be an inner product space, and let $\boldsymbol{v}_1, \ldots, \boldsymbol{v}_n \in V$. The associated Gram matrix

$$\begin{pmatrix} \langle \boldsymbol{v}_1, \boldsymbol{v}_1\rangle & \langle \boldsymbol{v}_1, \boldsymbol{v}_2\rangle & \cdots & \langle \boldsymbol{v}_1, \boldsymbol{v}_n\rangle \\ \langle \boldsymbol{v}_2, \boldsymbol{v}_1\rangle & \langle \boldsymbol{v}_2, \boldsymbol{v}_2\rangle & \cdots & \langle \boldsymbol{v}_2, \boldsymbol{v}_n\rangle \\ \vdots & \vdots & \ddots & \vdots \\ \langle \boldsymbol{v}_n, \boldsymbol{v}_1\rangle & \langle \boldsymbol{v}_n, \boldsymbol{v}_2\rangle & \cdots & \langle \boldsymbol{v}_n, \boldsymbol{v}_n\rangle \end{pmatrix}$$

is positive definite if and only if $\boldsymbol{v}_1, \ldots, \boldsymbol{v}_n$ are linearly independent.

Lemma 2. The coverage cost function T defined by (5) is minimized to $T^* = \bar{d}_\xi(0, 2\pi)/(2\sum_{i=1}^n W_i)$ if and only if

$$\frac{\bar{d}_\xi(p_i, p_{i+1})}{W_i + W_{i+1}} = \frac{\bar{d}_\xi(p_j, p_{j+1})}{W_j + W_{j+1}}, \ \forall i, j \in \mathcal{I}_n. \tag{16}$$

Moreover, $T \leqslant \max_{i\in\mathcal{I}_n} \bar{d}_\xi(p_i, p_{i+1})/(W_i + W_{i+1})$.

Proof. Note that the response time is derived as (4). The proof is omitted here because it is similar to Theorem 2 in [17].

In the following, we propose three key lemmas in this work. First, the relationship between the integral of the time and the integral on the delayed position variables will be established in Lemma 3. Then, this relationship will be used to derive the result of Lemma 4.

Lemma 3. Letting

$$d_\xi(a, b) = \int_a^b \xi(p)\mathrm{d}p, \tag{17}$$

one has the following equation:

$$\int_{t-\tau_{i,j}(t)}^t \dot{d}_\xi(p_i(s), p_j(s))\mathrm{d}s = -\left(\int_{p_i(t-\tau_{i,j}(t))}^{p_i(t)} \xi(p)\mathrm{d}p + \int_{p_j(t)}^{p_j(t-\tau_{i,j}(t))} \xi(p)\mathrm{d}p\right), \tag{18}$$

where $0 < \tau_{i,j}(t) < \tau_{i,j}^m$.

Proof. The following equation can be obtained from the continuity of $\xi(q)$ and (17), that is,

$$\begin{aligned} d_\xi(p_i(t-\tau_{i,j}(t)), p_j(t-\tau_{i,j}(t))) &= \int_{p_i(t-\tau_{i,j}(t))}^{p_j(t-\tau_{i,j}(t))} \xi(p)\mathrm{d}p \\ &= \int_{p_i(t-\tau_{i,j}(t))}^{p_i(t)} \xi(p)\mathrm{d}p + \int_{p_i(t)}^{p_j(t)} \xi(p)\mathrm{d}p + \int_{p_j(t)}^{p_j(t-\tau_{i,j}(t))} \xi(p)\mathrm{d}p. \end{aligned} \tag{19}$$

Notably, $d_\xi(p_i(t-\tau_{i,j}(t)), p_j(t-\tau_{i,j}(t)))$ can also be expressed as the time integration as follows:

$$\begin{aligned} d_\xi(p_i(t-\tau_{i,j}(t)), p_j(t-\tau_{i,j}(t))) &= d_\xi(p_i(t), p_j(t)) - \int_{t-\tau_{i,j}(t)}^{t} \dot{d}_\xi(p_i(s), p_j(s))\mathrm{d}s \\ &= \int_{p_i(t)}^{p_j(t)} \xi(p)\mathrm{d}p - \int_{t-\tau_{i,j}(t)}^{t} \dot{d}_\xi(p_i(s), p_j(s))\mathrm{d}s. \end{aligned} \tag{20}$$

Thus, by comparing the results of (19) and (20), Eq. (18) can be obtained.

In Lemma 4, Eq. (22) will be derived using Lemma 3 and the periodic property of the circular communication topology. Lemma 4 is important in the subsequent convergence analysis. The derivative of a part of the Lyapunov-Krasovskii functional can be expressed as follows:

$$A = \sum_{i=1}^{n} \frac{d_\xi(p_i(t), p_{i+1}(t))}{W_i + W_{i+1}}(u_{i+1}(t) - u_i(t)), \tag{21}$$

which will appear in the proof of Lemma 5.

Lemma 4. When $u_i = W_i\bar{u}_i$, one has the following equation:

$$\begin{aligned} A = \sum_{i=1}^{n} &-\frac{1}{W_i\sigma_i}u_i^2(t) + \frac{\int_{t-\tau_{i,i+1}(t)}^{t} \dot{d}_\xi(p_i(s), p_{i+1}(s))\mathrm{d}s}{W_i + W_{i+1}}\dot{d}_\xi(p_i(t), p_{i+1}(t)) \\ &+ \left(\frac{\int_{p_i(t-\tau_{i,i+1}(t))}^{p_{i+1}(t-\tau_{i,i+1}(t))} \tilde{a}_i^{\mathrm{T}}(t)\mathcal{D}(p)\mathrm{d}p}{W_i + W_{i+1}} - \frac{\int_{p_{i-1}(t-\tau_{i-1,i}(t))}^{p_i(t-\tau_{i-1,i}(t))} \tilde{a}_i^{\mathrm{T}}(t)\mathcal{D}(p)\mathrm{d}p}{W_{i-1} + W_i}\right)u_i(t). \end{aligned} \tag{22}$$

Proof. Recalling (1), (9), and (17), one has the following equation:

$$\begin{aligned} \sum_{i=1}^{n} \frac{d_\xi(p_i(t), p_{i+1}(t))}{W_i + W_{i+1}}u_{i+1}(t) &= \sum_{i=1}^{n-1} \frac{d_\xi(p_i(t), p_{i+1}(t))}{W_i + W_{i+1}}u_{i+1}(t) + \frac{d_\xi(p_n(t), p_{n+1}(t))}{W_n + W_{n+1}}u_{n+1}(t) \\ &= \sum_{i=2}^{n} \frac{d_\xi(p_{i-1}(t), p_i(t))}{W_{i-1} + W_i}u_i(t) + \frac{d_\xi(p_0(t), p_1(t))}{W_0 + W_1}u_1(t) \\ &= \sum_{i=1}^{n} \frac{d_\xi(p_{i-1}(t), p_i(t))}{W_{i-1} + W_i}u_i(t). \end{aligned} \tag{23}$$

Then, one can also derive the following equation:

$$\begin{aligned} A &= \sum_{i=1}^{n} \frac{d_\xi(p_{i-1}(t), p_i(t))}{W_{i-1} + W_i}u_i(t) - \sum_{i=1}^{n} \frac{d_\xi(p_i(t), p_{i+1}(t))}{W_i + W_{i+1}}u_i(t) \\ &= \sum_{i=1}^{n} -\left(\frac{d_\xi(p_i(t), p_{i+1}(t))}{W_i + W_{i+1}} - \frac{d_\xi(p_{i-1}(t), p_i(t))}{W_{i-1} + W_i}\right)u_i(t). \end{aligned} \tag{24}$$

It follows from (6), (11), and (12) that

$$\hat{\xi}_i(p,t) - \xi(p) = \mathcal{D}^{\mathrm{T}}(p)\tilde{a}_i(t). \tag{25}$$

Then, one has the following equation:

$$d_\xi(p_i(t), p_j(t)) = \int_{p_i(t)}^{p_j(t)} \xi(p)\mathrm{d}p = \int_{p_i(t)}^{p_j(t)} (\hat{\xi}_i(p,t) - \mathcal{D}^{\mathrm{T}}(p)\tilde{a}_i(t))\mathrm{d}p$$

$$=\int_{p_i(t)}^{p_j(t)}\hat{\xi}_i(p,t)\mathrm{d}p-\int_{p_i(t)}^{p_j(t)}\mathcal{D}^{\mathrm{T}}(p)\tilde{a}_i(t)\mathrm{d}p=d_{\hat{\xi}_i}(p_i(t),p_j(t))-\int_{p_i(t)}^{p_j(t)}\tilde{a}_i^{\mathrm{T}}(t)\mathcal{D}(p)\mathrm{d}p. \quad (26)$$

Consequently,

$$\begin{aligned}A=\sum_{i=1}^{n}-&\left(\frac{d_{\hat{\xi}_i}(p_i(t),p_{i+1}(t))}{W_i+W_{i+1}}-\frac{d_{\hat{\xi}_i}(p_{i-1}(t),p_i(t))}{W_{i-1}+W_i}\right)u_i(t)\\&+\left(\frac{\int_{p_i(t)}^{p_{i+1}(t)}\tilde{a}_i^{\mathrm{T}}(t)\mathcal{D}(p)\mathrm{d}p}{W_i+W_{i+1}}-\frac{\int_{p_{i-1}(t)}^{p_i(t)}\tilde{a}_i^{\mathrm{T}}(t)\mathcal{D}(p)\mathrm{d}p}{W_{i-1}+W_i}\right)u_i(t).\end{aligned} \quad (27)$$

Similar to (19), it follows from (10) that

$$\begin{aligned}d_{\hat{\xi}_i}(p_i(t-\tau_{i,j}(t)),p_j(t-\tau_{i,j}(t)))&=\int_{p_i(t-\tau_{i,j}(t))}^{p_j(t-\tau_{i,j}(t))}\hat{\xi}_i(p)\mathrm{d}p\\&=\int_{p_i(t-\tau_{i,j}(t))}^{p_i(t)}\hat{\xi}_i(p)\mathrm{d}p+\int_{p_i(t)}^{p_j(t)}\hat{\xi}_i(p)\mathrm{d}p+\int_{p_j(t)}^{p_j(t-\tau_{i,j}(t))}\hat{\xi}_i(p)\mathrm{d}p\\&=d_{\hat{\xi}_i}(p_i(t),p_j(t))+\int_{p_i(t-\tau_{i,j}(t))}^{p_i(t)}\hat{\xi}_i(p)\mathrm{d}p+\int_{p_j(t)}^{p_j(t-\tau_{i,j}(t))}\hat{\xi}_i(p)\mathrm{d}p\end{aligned} \quad (28)$$

and

$$\int_{p_i(t-\tau_{i,j}(t))}^{p_j(t-\tau_{i,j}(t))}\mathcal{D}(p)\mathrm{d}p=\int_{p_i(t-\tau_{i,j}(t))}^{p_i(t)}\mathcal{D}(p)\mathrm{d}p+\int_{p_i(t)}^{p_j(t)}\mathcal{D}(p)\mathrm{d}p+\int_{p_j(t)}^{p_j(t-\tau_{i,j}(t))}\mathcal{D}(p)\mathrm{d}p. \quad (29)$$

Then, it follows from (27) and (28) that

$$\begin{aligned}A=\sum_{i=1}^{n}-&\left(\frac{d_{\hat{\xi}_i}(p_i(t-\tau_{i,i+1}(t)),p_{i+1}(t-\tau_{i,i+1}(t)))}{W_i+W_{i+1}}-\frac{d_{\hat{\xi}_i}(p_{i-1}(t-\tau_{i-1,i}(t)),p_i(t-\tau_{i-1,i}(t)))}{W_{i-1}+W_i}\right)u_i(t)\\&+\left(\frac{\int_{p_i(t-\tau_{i,i+1}(t))}^{p_i(t)}\hat{\xi}_i(p)\mathrm{d}p+\int_{p_{i+1}(t)}^{p_{i+1}(t-\tau_{i,i+1}(t))}\hat{\xi}_i(p)\mathrm{d}p}{W_i+W_{i+1}}\right.\\&\left.-\frac{\int_{p_{i-1}(t-\tau_{i-1,i}(t))}^{p_{i-1}(t)}\hat{\xi}_i(p)\mathrm{d}p+\int_{p_i(t)}^{p_i(t-\tau_{i-1,i}(t))}\hat{\xi}_i(p)\mathrm{d}p}{W_{i-1}+W_i}\right)u_i(t)\\&+\left(\frac{\int_{p_i(t)}^{p_{i+1}(t)}\tilde{a}_i^{\mathrm{T}}(t)\mathcal{D}(p)\mathrm{d}p}{W_i+W_{i+1}}-\frac{\int_{p_{i-1}(t)}^{p_i(t)}\tilde{a}_i^{\mathrm{T}}(t)\mathcal{D}(p)\mathrm{d}p}{W_{i-1}+W_i}\right)u_i(t).\end{aligned} \quad (30)$$

When $u_i=W_i\bar{u}_i$, recalling (8), one has the following equation:

$$u_i=W_i\sigma_i\left(\frac{d_{\hat{\xi}_i}(p_i(t-\tau_{i,i+1}(t)),p_{i+1}(t-\tau_{i,i+1}(t)))}{W_i+W_{i+1}}-\frac{d_{\hat{\xi}_i}(p_{i-1}(t-\tau_{i,i-1}(t)),p_i(t-\tau_{i,i-1}(t)))}{W_{i-1}+W_i}\right). \quad (31)$$

Thus, it follows from (29), (30), and (31) that

$$\begin{aligned}A=\sum_{i=1}^{n}-&\frac{1}{W_i\sigma_i}u_i^2(t)+\left(\frac{\int_{p_i(t-\tau_{i,i+1}(t))}^{p_i(t)}\hat{\xi}_i(p)\mathrm{d}p+\int_{p_{i+1}(t)}^{p_{i+1}(t-\tau_{i,i+1}(t))}\hat{\xi}_i(p)\mathrm{d}p}{W_i+W_{i+1}}\right.\\&\left.-\frac{\int_{p_{i-1}(t-\tau_{i-1,i}(t))}^{p_{i-1}(t)}\hat{\xi}_i(p)\mathrm{d}p+\int_{p_i(t)}^{p_i(t-\tau_{i-1,i}(t))}\hat{\xi}_i(p)\mathrm{d}p}{W_{i-1}+W_i}\right)u_i(t)\\&-\left(\frac{\int_{p_i(t-\tau_{i,i+1}(t))}^{p_i(t)}\tilde{a}_i^{\mathrm{T}}(t)\mathcal{D}(p)\mathrm{d}p}{W_i+W_{i+1}}+\frac{\int_{p_{i+1}(t)}^{p_{i+1}(t-\tau_{i,i+1}(t))}\tilde{a}_i^{\mathrm{T}}(t)\mathcal{D}(p)\mathrm{d}p}{W_i+W_{i+1}}-\frac{\int_{p_{i-1}(t-\tau_{i-1,i}(t))}^{p_{i-1}(t)}\tilde{a}_i^{\mathrm{T}}(t)\mathcal{D}(p)\mathrm{d}p}{W_{i-1}+W_i}\right.\end{aligned}$$

$$
\begin{aligned}
&-\frac{\int_{p_i(t)}^{p_i(t-\tau_{i-1,i}(t))}\tilde{a}_i^{\mathrm{T}}(t)\mathcal{D}(p)\mathrm{d}p}{W_{i-1}+W_i}\Bigg)u_i(t)\\
&+\left(\frac{\int_{p_i(t-\tau_{i,i+1}(t))}^{p_{i+1}(t-\tau_{i,i+1}(t))}\tilde{a}_i^{\mathrm{T}}(t)\mathcal{D}(p)\mathrm{d}p}{W_i+W_{i+1}}-\frac{\int_{p_{i-1}(t-\tau_{i-1,i}(t))}^{p_i(t-\tau_{i-1,i}(t))}\tilde{a}_i^{\mathrm{T}}(t)\mathcal{D}(p)\mathrm{d}p}{W_{i-1}+W_i}\right)u_i(t).
\end{aligned}\tag{32}
$$

Then, let

$$
B_i=\left(\frac{\int_{p_i(t-\tau_{i,i+1}(t))}^{p_{i+1}(t-\tau_{i,i+1}(t))}\tilde{a}_i^{\mathrm{T}}(t)\mathcal{D}(p)\mathrm{d}p}{W_i+W_{i+1}}-\frac{\int_{p_{i-1}(t-\tau_{i-1,i}(t))}^{p_i(t-\tau_{i-1,i}(t))}\tilde{a}_i^{\mathrm{T}}(t)\mathcal{D}(p)\mathrm{d}p}{W_{i-1}+W_i}\right)u_i(t).\tag{33}
$$

Recalling (25), one has the following equation:

$$
\begin{aligned}
A=\sum_{i=1}^{n}-\frac{1}{W_i\sigma_i}u_i^2(t)+\Bigg(&\frac{\int_{p_i(t-\tau_{i,i+1}(t))}^{p_i(t)}\xi(p)\mathrm{d}p+\int_{p_{i+1}(t)}^{p_{i+1}(t-\tau_{i,i+1}(t))}\xi(p)\mathrm{d}p}{W_i+W_{i+1}}\\
&-\frac{\int_{p_{i-1}(t-\tau_{i-1,i}(t))}^{p_{i-1}(t)}\xi(p)\mathrm{d}p+\int_{p_i(t)}^{p_i(t-\tau_{i-1,i}(t))}\xi(p)\mathrm{d}p}{W_{i-1}+W_i}\Bigg)u_i(t)+B_i.
\end{aligned}\tag{34}
$$

Similar to (23), one can derive the following equation:

$$
\begin{aligned}
&\sum_{i=1}^{n}\frac{\int_{p_{i-1}(t-\tau_{i-1,i}(t))}^{p_{i-1}(t)}\xi(p)\mathrm{d}p+\int_{p_i(t)}^{p_i(t-\tau_{i-1,i}(t))}\xi(p)\mathrm{d}p}{W_{i-1}+W_i}u_i(t)\\
&\quad=\sum_{i=1}^{n}\frac{\int_{p_i(t-\tau_{i,i+1}(t))}^{p_i(t)}\xi(p)\mathrm{d}p+\int_{p_{i+1}(t)}^{p_{i+1}(t-\tau_{i,i+1}(t))}\xi(p)\mathrm{d}p}{W_i+W_{i+1}}u_{i+1}(t).
\end{aligned}\tag{35}
$$

Consequently,

$$
A=\sum_{i=1}^{n}-\frac{1}{W_i\sigma_i}u_i^2(t)+\frac{\int_{p_i(t-\tau_{i,i+1}(t))}^{p_i(t)}\xi(p)\mathrm{d}p+\int_{p_{i+1}(t)}^{p_{i+1}(t-\tau_{i,i+1}(t))}\xi(p)\mathrm{d}p}{W_i+W_{i+1}}(u_i(t)-u_{i+1}(t))+B_i.\tag{36}
$$

Noting the sensors' dynamics expressed in (2), it follows from (17) that

$$
\dot{d}_\xi(p_i(t),p_{i+1}(t))=u_{i+1}(t)-u_i(t).\tag{37}
$$

Recalling Lemma 3, it follows from (36) and (37) that

$$
A=\sum_{i=1}^{n}-\frac{1}{W_i\sigma_i}u_i^2(t)+\frac{\int_{t-\tau_{i,i+1}(t)}^{t}\dot{d}_\xi(p_i(s),p_{i+1}(s))\mathrm{d}s}{W_i+W_{i+1}}\dot{d}_\xi(p_i(t),p_{i+1}(t))+B_i.\tag{38}
$$

Thus, the proof is completed.

Then, convergence analysis will be conducted in Lemma 5, where the convergence condition expressed by LMI can be avoided using (22).

Lemma 5. When $u_i=W_i\bar{u}_i$, if the upper bounds on the communication delays and the upper bounds on their derivatives satisfy

$$
\tau_{i,i+1}^m<\frac{W_i+W_{i+1}}{4W_i\sigma_i}\ \text{and}\ d_{i,i+1}^m<1,\ \forall i\in\mathcal{I}_n,\tag{39}
$$

respectively, then

(1) $|p_{i+1}(t)-p_i(t)|$ is upper bounded for all $t\geqslant 0$ and $i\in\mathcal{I}_n$;

(2) $\lim_{t\to\infty}u_i(t)=0$ and $\lim_{t\to\infty}\hat{\xi}_i(p,t)=\xi(p)$ for all $i\in\mathcal{I}_n$.

Proof. Consider the Lyapunov-Krasovskii functional $V = V_1 + V_2$, where

$$V_1 = \frac{1}{2}\sum_{i=1}^{n} \frac{d_\xi^2(p_i(t), p_{i+1}(t))}{W_i + W_{i+1}} + \frac{1}{2}\sum_{i=1}^{n} \tilde{a}_i^{\mathrm{T}} \tilde{a}_i, \tag{40}$$

with $\tilde{a}_i(t) = \hat{a}_i(t) - a$ recalling (12) and

$$V_2 = \frac{1}{2}\sum_{i=1}^{n} \frac{1}{W_i + W_{i+1}} \int_{t-\tau_{i,i+1}(t)}^{t} (s - t + \tau_{i,i+1}^m) \dot{d}_\xi^2(p_i(s), p_{i+1}(s))\mathrm{d}s. \tag{41}$$

It follows from (17) that $d_\xi(p_i(t), p_j(t)) = \int_{p_i(t)}^{p_j(t)} \xi(p)\mathrm{d}p$. Thus, by recalling (21) and (37), the time derivative of V_1 along the mobile sensors' trajectories can be calculated as follows:

$$\dot{V}_1 = \sum_{i=1}^{n} \frac{d_\xi(p_i(t), p_{i+1}(t))}{W_i + W_{i+1}} (u_{i+1}(t) - u_i(t)) + \sum_{i=1}^{n} \tilde{a}_i^{\mathrm{T}} \dot{\hat{a}}_i = A + \sum_{i=1}^{n} \tilde{a}_i^{\mathrm{T}} \dot{\hat{a}}_i. \tag{42}$$

Then, recalling the adaptive control laws expressed in (13), $\tau_{i,j}(t) = \tau_{j,i}(t)$ in Assumption 2, and Lemma 4, one has the following equation:

$$\begin{aligned}\dot{V}_1 =& A + \sum_{i=1}^{n} - \left(\frac{\int_{p_i(t-\tau_{i,i+1}(t))}^{p_{i+1}(t-\tau_{i,i+1}(t))} \tilde{a}_i^{\mathrm{T}}(t)\mathcal{D}(p)\mathrm{d}p}{W_i + W_{i+1}} - \frac{\int_{p_{i-1}(t-\tau_{i,i-1}(t))}^{p_i(t-\tau_{i,i-1}(t))} \tilde{a}_i^{\mathrm{T}}(t)\mathcal{D}(p)\mathrm{d}p}{W_{i-1} + W_i} \right) u_i(t) - \tilde{a}_i^{\mathrm{T}} \gamma_i(\Omega_i \hat{a}_i - \omega_i) \\ =& \sum_{i=1}^{n} \left[-\frac{1}{W_i\sigma_i} u_i^2(t) + \frac{\int_{t-\tau_{i,i+1}(t)}^{t} \dot{d}_\xi(p_i(s), p_{i+1}(s))\mathrm{d}s}{W_i + W_{i+1}} \dot{d}_\xi(p_i(t), p_{i+1}(t)) - \tilde{a}_i^{\mathrm{T}} \gamma_i(\Omega_i \hat{a}_i - \omega_i) \right].\end{aligned} \tag{43}$$

Note also that

$$\begin{aligned}&\int_{t-\tau_{i,i+1}(t)}^{t} \dot{d}_\xi(p_i(s), p_{i+1}(s))\mathrm{d}s \dot{d}_\xi(p_i(t), p_{i+1}(t)) \\ &= \int_{t-\tau_{i,i+1}(t)}^{t} \dot{d}_\xi(p_i(s), p_{i+1}(s))\dot{d}_\xi(p_i(t), p_{i+1}(t))\mathrm{d}s \\ &\leqslant \frac{1}{2}\int_{t-\tau_{i,i+1}(t)}^{t} \dot{d}_\xi^2(p_i(s), p_{i+1}(s))\mathrm{d}s + \frac{1}{2}\tau_{i,i+1}^m \dot{d}_\xi^2(p_i(t), p_{i+1}(t)).\end{aligned} \tag{44}$$

Thus, one can derive the following equation:

$$\begin{aligned}\dot{V}_1 \leqslant \sum_{i=1}^{n} \Bigg[&- \frac{1}{W_i\sigma_i} u_i^2(t) + \frac{1}{2}\frac{\int_{t-\tau_{i,i+1}(t)}^{t} \dot{d}_\xi^2(p_i(s), p_{i+1}(s))\mathrm{d}s}{W_i + W_{i+1}} \\ &+ \frac{1}{2}\frac{\tau_{i,i+1}^m \dot{d}_\xi^2(p_i(t), p_{i+1}(t))}{W_i + W_{i+1}} - \tilde{a}_i^{\mathrm{T}} \gamma_i(\Omega_i \hat{a}_i - \omega_i) \Bigg].\end{aligned} \tag{45}$$

It follows from (41) that

$$V_2 = \frac{1}{2}\sum_{i=1}^{n} \frac{1}{W_i + W_{i+1}} \left[\int_{t-\tau_{i,i+1}(t)}^{t} s\dot{d}_\xi^2(p_i(s), p_{i+1}(s))\mathrm{d}s - (t - \tau_{i,i+1}^m) \int_{t-\tau_{i,i+1}(t)}^{t} \dot{d}_\xi^2(p_i(s), p_{i+1}(s))\mathrm{d}s \right].$$

Then, the time derivative of V_2 is calculated as follows:

$$\begin{aligned}\dot{V}_2 = \frac{1}{2}\sum_{i=1}^{n} \frac{1}{W_i + W_{i+1}} \Big\{ &t\dot{d}_\xi^2(p_i(t), p_{i+1}(t)) - (1 - \dot{\tau}_{i,i+1}(t))(t - \tau_{i,i+1}(t))\dot{d}_\xi^2(p_i(t - \tau_{i,i+1}(t)), p_{i+1}(t \\ &- \tau_{i,i+1}(t))) - \int_{t-\tau_{i,i+1}(t)}^{t} \dot{d}_\xi^2(p_i(s), p_{i+1}(s))\mathrm{d}s - (t - \tau_{i,i+1}^m)[\dot{d}_\xi^2(p_i(t), p_{i+1}(t))\end{aligned}$$

$$
\left.-(1-\dot{\tau}_{i,i+1}(t))\dot{d}_\xi^2(p_i(t-\tau_{i,i+1}(t)),p_{i+1}(t-\tau_{i,i+1}(t)))]\right\}
$$

$$
=\frac{1}{2}\sum_{i=1}^{n}\frac{1}{W_i+W_{i+1}}\Bigg[\tau_{i,i+1}^m\dot{d}_\xi^2(p_i(t),p_{i+1}(t))-\int_{t-\tau_{i,i+1}(t)}^{t}\dot{d}_\xi^2(p_i(s),p_{i+1}(s))\mathrm{d}s
$$

$$
+(1-\dot{\tau}_{i.i+1}(t))(\tau_{i,i+1}(t)-\tau_{i,i+1}^m)\dot{d}_\xi^2(p_i(t-\tau_{i,i+1}(t)),p_{i+1}(t-\tau_{i,i+1}(t)))\Bigg]. \tag{46}
$$

Recalling $0<\tau_{i,j}(t)<\tau_{i,j}^m$, $\dot{\tau}_{i,j}(t)<d_{i,j}^m$ in Assumption 2 and the condition $d_{i,i+1}^m<1$ expressed in condition (39), one has the following equation:

$$
\dot{V}_2\leqslant\frac{1}{2}\sum_{i=1}^{n}\frac{1}{W_i+W_{i+1}}\left(\tau_{i,i+1}^m\dot{d}_\xi^2(p_i(t),p_{i+1}(t))-\int_{t-\tau_{i,i+1}(t)}^{t}\dot{d}_\xi^2(p_i(s),p_{i+1}(s))\mathrm{d}s\right). \tag{47}
$$

Consequently, $\dot{V}$ satisfies

$$
\dot{V}=\dot{V}_1+\dot{V}_2\leqslant\sum_{i=1}^{n}\left[-\frac{1}{W_i\sigma_i}u_i^2(t)+\frac{\tau_{i,i+1}^m\dot{d}_\xi^2(p_i(t),p_{i+1}(t))}{W_i+W_{i+1}}-\tilde{a}_i^{\mathrm{T}}\gamma_i(\Omega_i\hat{a}_i-\omega_i)\right]. \tag{48}
$$

Moreover, similar to (23), it follows from (37) that

$$
\begin{aligned}
\sum_{i=1}^{n}\dot{d}_\xi^2(p_i(t),p_{i+1}(t))&=\sum_{i=1}^{n}(u_{i+1}(t)-u_i(t))^2=\sum_{i=1}^{n}(u_{i+1}^2(t)-2u_{i+1}(t)u_i(t)+u_i^2(t))\\
&\leqslant\sum_{i=1}^{n}(2u_{i+1}^2(t)+2u_i^2(t))=2\sum_{i=1}^{n}u_{i+1}^2(t)+2\sum_{i=1}^{n}u_i^2(t)=4\sum_{i=1}^{n}u_i^2(t).
\end{aligned} \tag{49}
$$

Then, recalling (14), (15), and (25), one has the following equation:

$$
\begin{aligned}
\dot{V}&\leqslant\sum_{i=1}^{n}\left[\left(-\frac{1}{W_i\sigma_i}+\frac{4\tau_{i,i+1}^m}{W_i+W_{i+1}}\right)u_i^2(t)-\tilde{a}_i^{\mathrm{T}}(t)\gamma_i\int_0^t\mathcal{D}(p_i(\tau))\mathcal{D}^{\mathrm{T}}(p_i(\tau))\tilde{a}_i(t)\mathrm{d}\tau\right]\\
&=\sum_{i=1}^{n}\left[\left(-\frac{1}{W_i\sigma_i}+\frac{4\tau_{i,i+1}^m}{W_i+W_{i+1}}\right)u_i^2(t)-\gamma_i\int_0^t(\mathcal{D}^{\mathrm{T}}(p_i(\tau))\tilde{a}_i(t))^2\mathrm{d}\tau\right].
\end{aligned} \tag{50}
$$

From $\tau_{i,i+1}^m<\frac{W_i+W_{i+1}}{4W_i\sigma_i}$ expressed in condition (39), one has $-\frac{1}{W_i\sigma_i}+\frac{4\tau_{i,i+1}^m}{W_i+W_{i+1}}<0$. Thus, one can conclude that $\dot{V}\leqslant0$. It follows from $\dot{V}\leqslant0$ that V is upper bounded. Note also that $V\geqslant0$. Therefore, the Lyapunov-Krasovskii functional $V(t)$ is upper bounded for all $t\geqslant0$. Then, $d_\xi(p_i(t),p_{i+1}(t))=\int_{p_i(t)}^{p_{i+1}(t)}\xi(q)\mathrm{d}q$ is always upper bounded for all $i\in\mathcal{I}_n$, which indicates that $|p_{i+1}(t)-p_i(t)|$ is upper bounded for all $t\geqslant0$ and $i\in\mathcal{I}_n$.

Next, the boundedness of $\ddot{V}$ will be proven. From (43) and (46), one can find that the boundedness of $\ddot{V}$ depends on the boundedness of the following variables: u_i, $\dot{u}_i$, $\tau_{i,i+1}(t)$, $\dot{\tau}_{i,i+1}(t)$, $\dot{d}_\xi(p_i(t),p_{i+1}(t))$, $\dot{d}_\xi(p_i(t-\tau_{i,i+1}(t)),p_{i+1}(t-\tau_{i,i+1}(t)))$, $\ddot{d}_\xi(p_i(t),p_{i+1}(t))$, $\tilde{a}_i$, $\dot{\tilde{a}}_i$, $\Omega_i\hat{a}_i-\omega_i$, $\dot{\Omega}_i(t)$, $\dot{\omega}(t)$, $\xi(p_i(t))$, $\dot{\xi}(p_i(t))$, $\ddot{\tau}_{i,i+1}(t)$, and $\ddot{d}_\xi(p_i(t-\tau_{i,i+1}(t)),p_{i+1}(t-\tau_{i,i+1}(t)))$. Moreover, recalling (31), one can obtain that the boundedness of $\dot{u}_i$ depends on the boundedness of $d_{\hat{\xi}_i}(p_i(t-\tau_{i,i+1}(t)),p_{i+1}(t-\tau_{i,i+1}(t)))$ and $\dot{d}_{\hat{\xi}_i}(p_i(t-\tau_{i,i+1}(t)),p_{i+1}(t-\tau_{i,i+1}(t)))$. It can be found from (7) and Assumption 2 that u_i, $\tau_{i,i+1}(t)$, $\dot{\tau}_{i,i+1}(t)$, and $\ddot{\tau}_{i,i+1}(t)$ are bounded. Furthermore, noting that $K_i(q)$ is a bounded basis function in Assumption 1, one can obtain that $\xi(p_i(t))$, $\dot{\xi}(p_i(t))$, $\dot{\Omega}_i(t)$, and $\dot{\omega}(t)$ are bounded. Then, the boundedness of the remaining variables will be analyzed as follows. It follows from $\dot{V}\leqslant0$ that V is upper bounded. Then, one has $d_\xi(p_i(t),p_{i+1}(t))$, $\tilde{a}_i$, and $\dot{d}_\xi(p_i(t),p_{i+1}(t))$ are bounded. Consequently, $d_\xi(p_i(t-\tau_{i,i+1}(t)),p_{i+1}(t-\tau_{i,i+1}(t)))$ and $\dot{d}_\xi(p_i(t-\tau_{i,i+1}(t)),p_{i+1}(t-\tau_{i,i+1}(t)))$ are bounded. Recalling the definition of $\tilde{a}_i$ and (25), one can obtain that $\hat{a}_i$ and $\hat{\xi}_i$ are both bounded, which further leads to the finding that $d_{\hat{\xi}_i}(p_i(t-\tau_{i,i+1}(t)),p_{i+1}(t-\tau_{i,i+1}(t)))$ and $\dot{d}_{\hat{\xi}_i}(p_i(t-\tau_{i,i+1}(t)),p_{i+1}(t-\tau_{i,i+1}(t)))$ are bounded from (26). Then,

$\ddot{d}_\xi(p_i(t), p_{i+1}(t))$ and $\ddot{d}_\xi(p_i(t-\tau_{i,i+1}(t)), p_{i+1}(t-\tau_{i,i+1}(t)))$ are also bounded. Noting the form of the adaptive law expressed in (13), $\dot{\hat{a}}_i$, $\ddot{\hat{a}}_i$, and $\Omega_i\hat{a}_i - \omega_i$ are found to be bounded, and thus $\dot{\tilde{a}}_i$ is also bounded. Therefore, it can be concluded that all the above variables are bounded, implying that $\ddot{V}$ is bounded. Then, $\lim_{t\to\infty}\dot{V}=0$ is obtained using Barbalat's lemma.

Recalling (50), one can derive that $\lim_{t\to\infty} u_i(t) = 0$ and $\lim_{t\to\infty}\int_0^t(\mathcal{D}^{\mathrm{T}}(p_i(\tau))\tilde{a}_i(t))^2\mathrm{d}\tau = 0$. Finally, from (25), one has $\lim_{t\to\infty}\int_0^t(\mathcal{D}^{\mathrm{T}}(q_i(\tau))\tilde{a}_i(t))^2\mathrm{d}\tau = \lim_{t\to\infty}\int_0^t(\hat{\xi}_i(p,t)-\xi(p))^2\mathrm{d}\tau = 0$. Thus, $\lim_{t\to\infty}\hat{\xi}_i(p,t)=\xi(p)$ holds for all $i\in\mathcal{I}_n$.

Finally, we show that the coverage cost function will be minimized asymptotically under the proposed control law expressed in (7), indicating that the networked mobile sensors will achieve the optimal configuration asymptotically.

Theorem 1. Choose the basis function vector $\mathcal{D}$ from a set of functions that are linearly independent of each other in the interval $[0, 2\pi]$. Assuming that Eq. (39) holds for all sensor i, $T(q_1,\ldots,q_n)$ defined by (5) is minimized asymptotically using the adaptive coverage control law expressed in (7) with properly selected low gain σ_i.

Proof. According to Lemma 1, the Gram matrix is positive definite if and only if the corresponding vectors are linearly independent. Then, $\Omega_i(t) > 0$ will always hold with the inner product $\langle f, g\rangle = \int_0^t f(\tau)g(\tau)\mathrm{d}\tau$ if and only if the basis functions selected in $\mathcal{D}$ are linearly independent of each other in $[0, 2\pi]$.

Considering the second term on the right-hand side of inequality expressed in (50), one has $\int_0^t(\mathcal{D}^{\mathrm{T}}(p_i(\tau))\tilde{a}_i(t))^2\mathrm{d}\tau = \tilde{a}_i^{\mathrm{T}}(t)(\int_0^t\mathcal{D}(p_i(\tau))\mathcal{D}^{\mathrm{T}}(p_i(\tau))\mathrm{d}\tau)\tilde{a}_i(t) = \tilde{a}_i^{\mathrm{T}}(t)\Omega_i(t)\tilde{a}_i(t)$. Given that $\lim_{t\to\infty}\dot{V} = 0$ and $\Omega_i(t)$ is positive definite, one has $\lim_{t\to\infty}\tilde{a}_i(t) = 0$, $\forall i\in\mathcal{I}_n$, which indicates that each sensor's estimated roughness function converges to the true roughness function over the entire mission space, that is, $\lim_{t\to\infty}\hat{\xi}_i(p,t) = \xi(p)$, $\forall p\in[0,2\pi]$. Then, from (10), (17), and Assumption 2, one has $\lim_{t\to\infty} d_{\hat{\xi}_i}(p_i(t-\tau_{i,i+1}(t)), p_{i+1}(t-\tau_{i,i+1}(t))) = \lim_{t\to\infty} d_\xi(p_i(t), p_{i+1}(t))$.

Noting that q_i is continuous and all the nonuniform time-varying delays are bounded, $|p_{i+1}(t-\tau_{i,i+1}(0)) - p_i(t-\tau_{i,i+1}(0))|$ is upper bounded by some constant, which is obtained from the initial condition expressed in (3). Then, one can find that the first term of $\hat{a}_i(t)$ is also bounded at $t=0$. From (14) and (15), it follows that the term $\Omega_i\hat{a}_i - \omega_i$ in (13) contains the time integration of the estimation error $\hat{a}_i - a_i$. Thus, one can conclude that $\hat{a}_i(t)$ is bounded if $|p_{i+1}(t-\tau_{i,i+1}(t)) - p_i(t-\tau_{i,i+1}(t))|$ is bounded, $\forall i\in\mathcal{I}_n$. Moreover, $d_{\hat{\xi}_i}(p_i(t-\tau_{i,i+1}(t)), p_{i+1}(t-\tau_{i,i+1}(t))) = \int_{p_i(t-\tau_{i,i+1}(t))}^{p_{i+1}(t-\tau_{i,i+1}(t))}\hat{\xi}_i(p)\,\mathrm{d}p$ is bounded if $|p_{i+1}(t-\tau_{i,i+1}(t)) - p_i(t-\tau_{i,i+1}(t))|$ is bounded, $\forall i\in\mathcal{I}_n$. Thus, if σ_i is small enough, then $u_i(0) = W_i\bar{u}_i(0)$ holds recalling (7)–(10). It follows from the first result of Lemma 5 that when $u_i(t) = W_i\bar{u}_i(t)$, $|p_{i+1}(t) - p_i(t)|$ is bounded for all $t\geqslant 0$ and $i\in\mathcal{I}_n$. Consequently, $u_i(t) = W_i\bar{u}_i(t)$ can always be satisfied using the proposed coverage control laws (7) with properly chosen low gains σ_i.

Thus, recalling $\lim_{t\to\infty}u_i(t) = 0$ in the second result of Lemma 5, one has

$$\lim_{t\to\infty}\frac{d_{\hat{\xi}_i}(p_i(t-\tau_{i,i+1}(t)), p_{i+1}(t-\tau_{i,i+1}(t)))}{W_i+W_{i+1}} = \lim_{t\to\infty}\frac{d_{\hat{\xi}_i}(p_{i-1}(t-\tau_{i,i-1}(t)), p_i(t-\tau_{i,i-1}(t)))}{W_{i-1}+W_i} \tag{51}$$

and

$$\lim_{t\to\infty}\frac{d_\xi(p_i(t), p_{i+1}(t))}{W_i+W_{i+1}} = \lim_{t\to\infty}\frac{d_\xi(p_{i-1}(t), p_i(t))}{W_{i-1}+W_i}.$$

Therefore, noting $\bar{d}_\xi(a,b) = |d_\xi(a,b)|$, one has $\frac{\bar{d}_\xi(p_i(t),p_{i+1}(t))}{W_i+W_{i+1}} = \frac{\bar{d}_\xi(p_{i-1}(t),p_i(t))}{W_{i-1}+W_i}$, $i\in\mathcal{I}_n$, as time proceeds to infinity. Then, it follows from Lemma 2 that $T(q_1,\ldots,q_n)$ will be minimized asymptotically.

Remark 3. The basis functions in $\mathcal{D}$ can be selected from trigonometric functions, exponential functions, Gabor systems, and wavelet systems, so that they are linearly independent of each other [44]. For example, the basis function vector can be selected from the following set:

$$\{\cos(\lambda_k x) + c_k\}\cup\{\sin(\mu_l x) + d_l\}, \tag{52}$$

where $k\in\{1,\ldots,\alpha\}$, $l\in\{1,\ldots,\beta\}$, $\alpha,\beta\in\mathbb{N}^+$, $\lambda_k,\mu_l\in\mathbb{R}/\{0\}$, $c_k, d_l\in\mathbb{R}$, $|\lambda_l|\neq|\lambda_j|$, and $|\mu_l|\neq|\mu_j|$ when $l\neq j$. It can be obtained from the proof of Theorem 1 in [44] that the functions selected in the set (52) are linearly independent of each other on any interval.

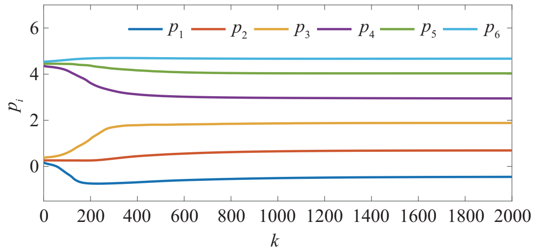

Figure 1 (Color online) Sensors' positions.

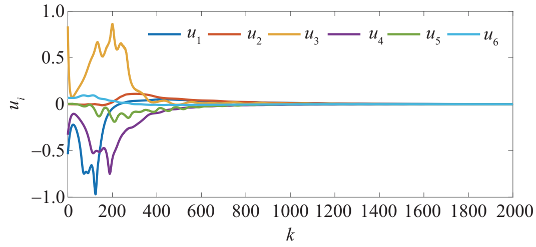

Figure 2 (Color online) Sensors' control inputs.

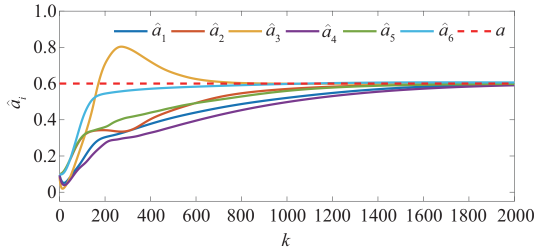

Figure 3 (Color online) Sensors' estimated parameters.

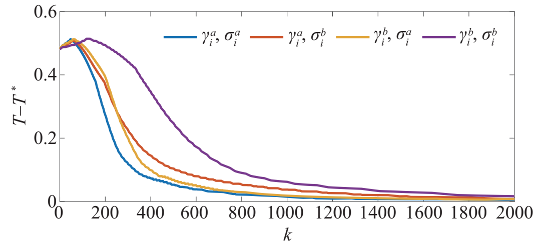

Figure 4 (Color online) Function $T - T^*$ for parameters γ_i and σ_i with different combinations of values.

Remark 4. In [45], the Gaussians are selected as the basis functions so that the unknown weighting function can be approximated. However, every center of the Gaussian function needs to be visited before the "cover" mode. In this study, we can remove this requirement by selecting the basis functions that are linearly independent of each other on the interval $[0, 2\pi]$, as shown in Theorem 1.

5 Simulation studies

This section presents a simulation example. In this case, six sensors with the maximum driving velocities W_i of $[7.2, 1.5, 6.0, 2.5, 4.1, 0.9]$ are considered. In [5], the density function of the environment can be parameterized using nine Gaussians that are known to each robot. Following the similar settings, the roughness function $\xi(p)$ is parameterized as a linear combination of three trigonometric functions, which is consistent with Assumption 1. In particular, choose $\mathcal{D}$ as $\mathcal{D}(p) = [\sin(3p)+2,\ \sin(15p)+2,\ \sin(30p)+2]$. The linear combination parameters a are set as $[0.6, 0.3, 0.1]$, which needs to be estimated using the sensor network. The adaptive rates γ_i are selected as $[0.23, 0.20, 0.22, 0.20, 0.21, 0.22]$. The control gains σ_i in (7) are selected as $[1.31, 1.31, 1.11, 1.12, 1.37, 1.31]$. The selection of γ_i and σ_i will not cause the divergence if (39) is satisfied. The time-delays in the MSN are $\tau_{i,i+1}(t) = 0.1\sin(0.02it) + 0.1$ which satisfy (39). Initialize all estimated parameters $\hat{a}_i$ to 0.1. At the beginning of the coverage task, mobile sensors are randomly deployed on a unit circle. The results obtained using the adaptive coverage control laws expressed in (7) and (13) are shown in Figures 1–4.

The history of the sensors' positions and control input of each sensor is shown in Figures 1 and 2, respectively. The control input fluctuates because of the nonuniform terrain roughness and time-varying delays recalling (8). Figure 3 shows the history of the first component of the estimated parameters $\hat{a}_i$. One can find that the true value of the first component of $\hat{a}_i$ is asymptotically achieved. It should be noted that though other components of $\hat{a}_i$ are not given, they also achieve their true values asymptotically. Finally, the function $T - T^*$ for parameters γ_i and σ_i with different combinations of values is shown in Figure 4, where $\gamma_i^a = \gamma_i$ and $\sigma_i^a = \sigma_i$ denote the previous settings, $\gamma_i^b = 0.5\gamma_i^a$ and $\sigma_i^b = 0.5\sigma_i^a$, respectively. One can find that the coverage cost function T with different combinations of values combination converges to its minimum value T^*, which is computed by $T^* = \bar{d}_\xi(0, 2\pi)/(2\sum_{i=1}^n W_i)$ given in Lemma 2. Moreover, either smaller γ_i or smaller σ_i will decrease the convergence speed of the coverage task.

6 Conclusion

The coverage control problem for MSNs with unknown terrain roughness and nonuniform time-varying communication delays has been investigated in this paper. Adaptive coverage control laws have been developed for mobile sensors. By adopting Barbalat's lemma and the matrix theory, we show that the MSN can be driven to their optimal configuration asymptotically under the proposed control laws. In the future, it will be interesting to extend the current work to the case of mobile sensors with limited communication ranges.

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