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Command filter-based I&I adaptive control for MIMO uncertain systems with input saturation and disturbances

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Abstract This paper develops a control strategy based on immersion and invariance (I&I) adaptive methodology for a class of multi-input multi-output (MIMO) systems in the presence of parametric uncertainty, input saturation, and external disturbance. To avoid the analytic calculation in the backstepping process, a high-gain auxiliary system is constructed to compensate for the effect of command filter error. The first-order command filters are also employed in the construction procedure of the I&I adaptive law to simplify its design and remove the structural conditions on the regressors. A filter-based disturbance observer is developed to counteract the effect of the external disturbance produced by a partially known exogenous system. To overcome the input saturation nonlinearity, a smooth function is introduced to approximate the input saturation with an extended state and a bounding estimation law. Stringent analysis guarantees the stability of closed-loop system. Finally, simulated examples confirm the effectiveness of the suggested method.

Keywords adaptive control, command filtered backstepping, disturbance observer, immersion and invariance, MIMO systems

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1 Introduction

Adaptive control issues of single-input single-output (SISO) and multi-input multi-output (MIMO) nonlinear systems have attracted the attention of many researchers as a hot control field topic. Extensive research has been performed so far. The purpose of adaptive control is to control plants with unknown parameters, which is a reason for its rapid development and ongoing popularity [1].

To address shortcomings such as poor transient response and sensitivity in the presence of noises or disturbances in the classical adaptive approach, immersion and invariance (I&I) adaptive methodology provides an extra design of freedom to shape the estimation error manifold via domination to enhance the transient performance of the system [2,3]. However, this shaping strategy heavily depends on the solution of the partial differential equation (PDE), which is particularly challenging for multivariable systems. One way to remove this obstacle is the dynamic scaling method [4], in which a reduced-order observer, an output filter, and a dynamic scaling parameter are co-designed with the adaptive law. Another technique to avoid solving PDEs is to use low-pass filters as in [5,6], whereby filtering the state or error variables, the derivatives needed in the adaptive law are replaced by the constructed filter equations. Although the I&I adaptive method has been effectively applied in typical MIMO systems such as robot manipulator [6] and quadrotor [7,8], it is still an open issue to apply I&I estimators in higher-order plants and consider the above-mentioned problems at the same time. Therefore, to construct more feasible I&I adaptive laws for a general class of MIMO high-order nonlinear systems is one of the goals of this study.

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For higher-order plants, backstepping provides a systematic approach to design stabilizing controllers, yet its implementation becomes increasingly difficult, and increasing order of the system causes the "explosion of terms" problem. To address this, dynamic surface control is suggested in [9] for creating backstepping-based controllers where virtual controls are passed through first-order low-pass filters to eliminate the arising complexity. This method has been effectively applied in many classes of MIMO systems subject to unmeasured states, output constraints, and disturbances [10–12]. On the other hand, command-filtered backstepping method in [13,14] additionally constructs compensating signals to remove the effect of the unachieved portion between the virtual control and the filtered virtual control. This method is frequently used in conjunction with other techniques, such as adaptive neural network [15–17], and fuzzy logic system [18–20]. Therefore, the command-filtered backstepping technique is a plausible contender for our interest in the control problem of higher-order plants.

Physical restrictions always result in saturation constraints for control inputs or actuators, which can impair control performances and possibly lead to instability. One technique to compensate for input saturation is to introduce an auxiliary system [21, 22], where an auxiliary system is created to remove the effect of saturation between the error of the control input and saturated input, and this method has been applied in studies such as [16, 23–25] for various kinds of systems with saturated input. Another solution for this problem is to use a smooth function to approximate the saturation nonlinearity [17,19,20,26]. In [19,20], the smooth function and mean-value theorem are employed to deal with saturation limitations for adaptive fuzzy control of nonlinear systems. In [26], a well-defined smooth function is used to approximate the saturation, and the Nussbaum function is introduced to compensate for the nonlinear term arising from the input saturation. However, it has been reported that the usage of numerous kinds of Nussbaum functions shows poor transients and control shock [27, 28], which suggests that there is still room for further improvement of smooth function-based approach.

Disturbance attenuation problem continues to receive frequent attention recently due to its significance. Among existing methods, disturbance observer-based approaches have been created and applied to many practical systems such as quadcopters, robot manipulators, and hydraulic knee exoskeleton systems [29–34], to mention a few. Various kinds of disturbance observers can be used in different settings, namely nonlinear disturbance observer [31], sliding mode disturbance observer [32], and fixed-time disturbance observer [33], aiming to obtain asymptotic, finite-time, or practical tracking control results. Furthermore, the studies in [29,34,35] created efficient disturbance observers to estimate unknown external disturbances produced by an exogenous system whose model information is only partially known. This encourages us to build a disturbance observer that estimates unknown exosystem states and embed it in the controller along with adaptive laws.

Based on above discussions, this study creates an adaptive control strategy based on the I&I method with command filters for a class of MIMO uncertain nonlinear systems that are subject to control input saturation. To be more precise, to compensate for the parametric uncertainties, I&I adaptive laws are developed with command filters for the plant with satisfactory performance, and the challenge of solving PDEs is removed. As will be depicted in the backstepping procedure, the control design is based on the filtered dynamics using command filters, and the virtual control law to be used in the tracking error variable requires to be recovered from its filtered form, which varies from traditional command-filtered backstepping design with Lyapunov-based adaptive law [14–20]. In addition to compensating for parametric uncertainties, a filter-based disturbance observer is built to estimate external disturbances produced by exogenous systems. Motivated by [24], a high-gain auxiliary system is created to remove the effect of the errors of command-filtered virtual control law and the original virtual control law. Furthermore, a smooth function is introduced to approximate the abrupt saturation control with an extended state, and the bounding estimation approach is used to offset the effect of time-varying input gain. The main contributions of this study are listed as follows.

(1) The filter-based I&I adaptive method [5,6] in this study is co-designed with the backstepping method and is applied to MIMO systems with subsystems in the strict feedback form, and the structural conditions on the regressors needed in [3] are removed. Compared with the traditional adaptive backstepping technique, the performance of estimation errors and tracking errors can be efficiently enhanced due to the I&I adaptation mechanism. Furthermore, the controller is free from solving PDEs, which is not considered in I&I adaptation-based results [36, 37].

(2) A filter-based disturbance observer is created in this study to deal with the external disturbances caused by unknown exosystems with bounded modeling errors. Compared with the existing approach [29, 34], the disturbance observer in this study is developed in the filtered fashion to make the design more

integrated with the command filter-based I&I adaptive law.

(3) Compared with the smooth function-based approach [26] for input saturation using the Nussbaum function, in this paper, the bounding estimation approach [38, 39] is selected to deal with the effect of time-varying input gain for better transient performance.

The rest of the study is organized as follows. The plant model and problem formulation are depicted in Section 2. The adaptive control design is introduced in Section 3, as well as the stability analysis of the closed-loop system. Numerical simulation examples are provided in Section 4 to confirm the suggested control strategy, and finally, Section 5 draws a summary of this paper.

2 Problem formulation

Consider the following MIMO nonlinear system ($j = 1, \ldots, m$):

$$\begin{aligned}
\dot{x}_{j,1} &= x_{j,2} + \theta_{j,1}^{\mathrm{T}}\varphi_{j,1}(\bar{x}_{j,1}) + f_{j,1}(\bar{x}_{j,1}) + d_{j,1}(t),\\
&\vdots\\
\dot{x}_{j,n_j-1} &= x_{j,n_j} + \theta_{j,n_j-1}^{\mathrm{T}}\varphi_{j,n_j-1}(\bar{x}_{j,n_j-1}) + f_{j,n_j-1}(\bar{x}_{j,n_j-1}) + d_{j,n_j-1}(t),\\
\dot{x}_{j,n_j} &= u_j(v_j) + \theta_{j,n_j}^{\mathrm{T}}\varphi_{j,n_j}(x) + f_{j,n_j}(x) + d_{j,n_j}(t),
\end{aligned} \tag{1}$$

where $x = [x_1^{\mathrm{T}}, \ldots, x_m^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{R}^{n_1+\cdots+n_m}$ with $x_j = [x_{j,1}, \ldots, x_{j,n_j}]^{\mathrm{T}} \in \mathbb{R}^{n_j}$ and $\bar{x}_{j,i_j} = [x_{j,1}, \ldots, x_{j,i_j}]^{\mathrm{T}} \in \mathbb{R}^{n_j}, i_j = 1, \ldots, n_j$ are the state vectors, $\varphi_{j,i_j}(\cdot) \in \mathbb{R}^{q_{j,i_j}}$ and $f_{j,i_j}(\cdot) \in \mathbb{R}, i_j = 1, \ldots, n_j$ are known smooth functions, $\theta_{j,i_j} \in \mathbb{R}^{q_{j,i_j}}$ is the unknown constant parameter vector, and $d_{j,i_j}(t)$ is the external disturbance generated by the following exosystem:

$$\begin{aligned}
&\dot{\eta}_{j,i_j} = W_{j,i_j}\eta_{j,i_j} + \varpi_{j,i_j}(t),\\
&d_{j,i_j}(t) = C_{j,i_j}\eta_{j,i_j}, \quad i_j = 1, \ldots, n_j,\ j = 1, \ldots, m,
\end{aligned} \tag{2}$$

where $\eta_{j,i_j} \in \mathbb{R}^{r_j}$ is the state, $W_{j,i_j} \in \mathbb{R}^{r_j\times r_j}$ and $C_{j,i_j} \in \mathbb{R}^{1\times r_j}$ are the known system matrices with (W_{j,i_j}, C_{j,i_j}) observable, and $\varpi_{j,i_j} \in \mathbb{R}^{r_j}$ describes the modeling error of the exosystem. In (1), $u_j(v_j(t)) \in \mathbb{R}$ is the control input for the jth subsystem with saturation type nonlinearity described by

$$u_j(v_j(t)) = \mathrm{sat}(v_j(t)) = \begin{cases} \mathrm{sign}(v_j(t))u_{j,M}, & |v_j(t)| \geqslant u_{j,M},\\ v_j(t), & |v_j(t)| < u_{j,M}, \end{cases} \tag{3}$$

with $u_{j,M}$ being the known saturated bound of $u_j(v_j)(t)$.

To handle input saturation nonlinearity, as in [26] we introduce the following smooth function to approximate it:

$$g_j(v_j) = u_{j,M}\tanh(v_j/u_{j,M}), \tag{4}$$

and by using (3), the last dynamics of (1) is rewritten with the auxiliary signal v_j,

$$\begin{aligned}
&\dot{x}_{j,n_j} = g_j(v_j) + \theta_{j,n_j}^{\mathrm{T}}\varphi_{j,n_j}(x) + f_{j,n_j}(x) + d_{j,n_j}(t) + d_{j,s}(v_j),\\
&\dot{v}_j = -h_j v_j + w_j,
\end{aligned} \tag{5}$$

where $h_j > 0$ is a positive constant to be designed, $d_{j,s}(v_j) = u_j(v_j) - g_j(v_j)$ is bounded, and w_j is the control signal to be designed.

The following assumptions are made in this paper.

Assumption 1. The reference for the jth subsystem $y_{j,r}(t)$ and its first-order derivative $\dot{y}_{j,r}(t)$ are smooth, available, and bounded.

Assumption 2. The exosystem states $\eta_{j,i_j}(t)$ are bounded and the modeling error $\varpi_{j,i_j}(t)$ is bounded.

Remark 1. Assumption 1 is made thanks to the command filter backstepping technique which removes the requirement in traditional backstepping where the reference and its first n_jth order derivatives need to be known and bounded. Assumption 2 is reasonable since in practice the energy of external disturbance is finite, and similar requirements can be found in [29, 35].

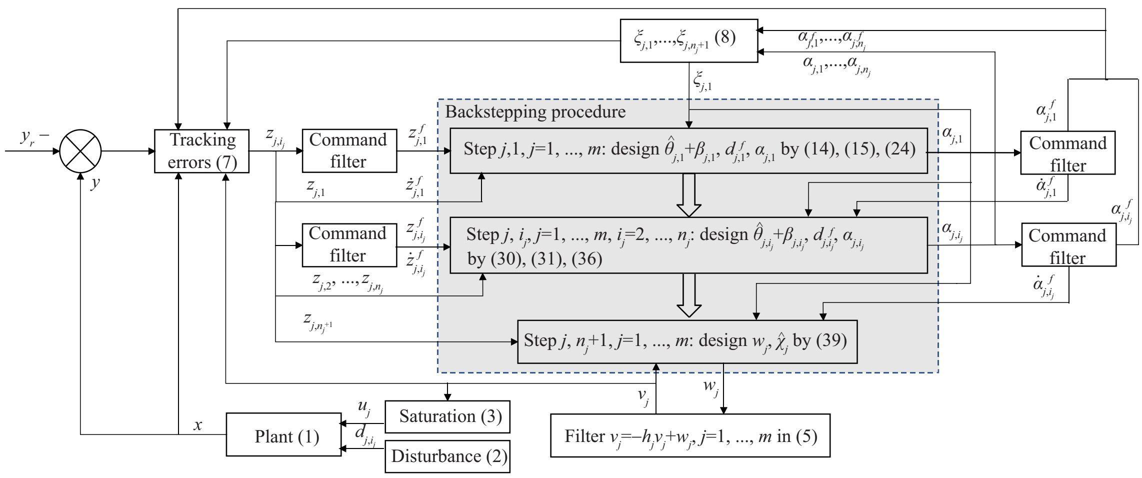

Figure 1 (Color online) The block diagram of the control scheme.

Remark 2. Compared with SISO systems, the MIMO system in the form of (1) exhibits high coupling characteristics and can represent a large class of practical systems, which is more meaningful to be investigated. Moreover, in this paper the factors of external disturbance and input saturation nonlinearity are also considered, which make the design task more challenging.

In this paper the following first-order command filter is defined:

$$\tau \dot{z}^f + z^f = z, \quad z^f(0) = z(0), \tag{6}$$

where $z \in \mathbb{R}$ is the input, the superscript $(\cdot)^f$ denotes the filtered variable, and $\tau > 0$ is the filter constant. The filter (6) will be employed to generate filtered versions of tracking error variables, regressors, and virtual controls, and the outputs z^f as well as $\dot{z}^f$ will be used in the control design.

The control objective of this paper is to design an appropriate adaptive controller for the uncertain and disturbed plant (1) subject to input saturation to track the given references $y_{j,r}(t), j = 1, \dots, m$ as precisely as possible.

3 Control design

The block diagram of the control scheme is shown in Figure 1. The tracking errors will be determined first which contain the states, filtered virtual controls, and auxiliary states. In every step except for the final step, we apply the command filter to filter both sides of the error equation; then the virtual control law, the corresponding adaptive law, and disturbance observer will be designed. The virtual control laws are passed through command filters and an auxiliary system is assigned to remove the effect of command filtered error of the virtual controls. Finally, the designed control law in the last step is passed through the filter in (5) to derive the signal v_j.

First, define the error variables for the jth subsystem,

$$\begin{aligned} z_{j,1} &= x_{j,1} - y_{j,r} - \xi_{j,1}, \\ z_{j,i_j} &= x_{j,i_j} - \alpha^f_{j,i_j-1} - \xi_{j,i_j}, \quad i_j = 2, \dots, n_j, \\ z_{j,n_j+1} &= g_j(v_j) - \alpha^f_{j,n_j} - \xi_{j,n_j+1}, \end{aligned} \tag{7}$$

where $\alpha_{j,1}, \dots, \alpha_{j,n_j}$ are the virtual control law to be determined in each step of the control design, α^f_{j,i_j} is the filtered variable of α_{j,i_j} passed through the command filter (6), and $\xi_{j,1}, \dots, \xi_{j,n_j+1}$ are the auxiliary system states to remove the effect of the error in the command-filtering backstepping process, designed as

$$
\begin{aligned}
&\dot{\xi}_{j,1} = -\epsilon_j c_{j,1}\xi_{j,1} + \xi_{j,2} + \alpha^f_{j,1} - \alpha_{j,1},\\
&\vdots\\
&\dot{\xi}_{j,n_j-1} = -\epsilon_j^{n_j-1} c_{j,n_j}\xi_{j,1} + \xi_{j,n_j} + \alpha^f_{j,n_j-1} - \alpha_{j,n_j-1},\\
&\dot{\xi}_{j,n_j} = -\epsilon_j^{n_j} c_{j,n_j}\xi_{j,1} + \xi_{j,n_j+1} + \alpha^f_{j,n_j} - \alpha_{j,n_j},\\
&\dot{\xi}_{j,n_j+1} = -\epsilon_j^{n_j+1} c_{j,n_j+1}\xi_{j,1},
\end{aligned} \tag{8}
$$

with $c_{j,1},\ldots,c_{j,n_j+1}$ Hurwitz coefficients and $\epsilon_j > 1$ the high-gain parameter. The design steps are illustrated below in detail.

Step $j,1$. First, the derivative of $z_{j,1}$ is calculated as

$$
\begin{aligned}
\dot{z}_{j,1} &= \dot{x}_{j,1} - \dot{y}_{j,r} - \dot{\xi}_{j,1}\\
&= z_{j,2} + \alpha^f_{j,1} + \xi_{j,2} + \theta^{\mathrm{T}}_{j,1}\varphi_{j,1} + f_{j,1} + d_{j,1} - \dot{y}_{j,r} - \dot{\xi}_{j,1}\\
&= z_{j,2} + \theta^{\mathrm{T}}_{j,1}\varphi_{j,1} + \bar{\alpha}_{j,1} + d_{j,1},
\end{aligned} \tag{9}
$$

where the equivalent virtual control $\bar{\alpha}_{j,1}$ is given by

$$
\bar{\alpha}_{j,1} = \alpha_{j,1} + \epsilon_j c_{j,1}\xi_{j,1} + f_{j,1} - \dot{y}_{j,r}. \tag{10}
$$

Now using the filter (6) to filter both sides of (9) as

$$
\dot{z}^f_{j,1} = z^f_{j,2} + \theta^{\mathrm{T}}_{j,1}\varphi^f_{j,1} + d^f_{j,1} + \bar{\alpha}^f_{j,1}, \tag{11}
$$

where the exponential decaying term due to filter error is omitted and $d^f_{j,1}$ to be estimated is generated by the following filtered exosystem:

$$
\begin{aligned}
\dot{\eta}^f_{j,1} &= W_{j,1}\eta^f_{j,1} + \varpi^f_{j,1},\\
d^f_{j,1} &= C_{j,1}\eta^f_{j,1}.
\end{aligned} \tag{12}
$$

Now we design the filtered equivalent virtual control $\bar{\alpha}^f_{j,1}$ as

$$
\bar{\alpha}^f_{j,1} = -k_{j,1}z^f_{j,1} - (\hat{\theta}_{j,1} + \beta_{j,1})^{\mathrm{T}}\varphi^f_{j,1} - \hat{d}^f_{j,1}, \tag{13}
$$

where $k_{j,1} > 0$ is a positive control parameter, $(\hat{\theta}_{j,1} + \beta_{j,1})$ is the I&I estimate for $\theta_{j,1}$ updated by

$$
\begin{aligned}
\dot{\hat{\theta}}_{j,1} &= -\gamma_{j,1}\varphi^f_{j,1}(-k_{j,1}z^f_{j,1} + z^f_{j,2}) - \gamma_{j,1}\dot{\varphi}^f_{j,1}z^f_{j,1} - \gamma_{j,1}\sigma_{j,1}(\hat{\theta}_{j,1} + \beta_{j,1}),\\
\beta_{j,1} &= \gamma_{j,1}\varphi^f_{j,1}z^f_{j,1},
\end{aligned} \tag{14}
$$

with $\gamma_{j,1}, \sigma_{j,1} > 0$ being positive design parameters with appropriate dimensions, and $\hat{d}^f_{j,1}$ is the disturbance observer developed as

$$
\begin{aligned}
\dot{\hat{\eta}}^f_{j,1} &= W_{j,1}(\hat{\eta}^f_{j,1} + \Gamma_{j,1}z^f_{j,1}) - \Gamma_{j,1}(-k_{j,1}z^f_{j,1} + z^f_{j,2}),\\
\hat{d}^f_{j,1} &= C_{j,1}(\hat{\eta}^f_{j,1} + \Gamma_{j,1}z^f_{j,1}),
\end{aligned} \tag{15}
$$

with $\Gamma_{j,1}$ a suitable parameter vector to be designed.

Now substituting (13) into (11) yields

$$
\dot{z}^f_{j,1} = -k_{j,1}z^f_{j,1} + z^f_{j,2} - \zeta^{\mathrm{T}}_{j,1}\varphi^f_{j,1} + C_{j,1}\tilde{\eta}^f_{j,1}, \tag{16}
$$

where $\zeta_{j,1} = \hat{\theta}_{j,1} - \theta_{j,1} + \beta_{j,1}$ is the off-the-manifold co-ordinate and $\tilde{\eta}^f_{j,1} = \eta^f_{j,1} - \hat{\eta}^f_{j,1} - \Gamma_{j,1}z^f_{j,1}$ is the observer error. The time-derivative of $\zeta_{j,1}$ can be calculated by invoking (14) and (16) as

$$
\begin{aligned}
\dot{\zeta}_{j,1} &= \dot{\hat{\theta}}_{j,1} + \gamma_{j,1}\varphi^f_{j,1}(-k_{i,1}z^f_{j,1} + z^f_{j,2} - \zeta^{\mathrm{T}}_{j,1}\varphi^f_{j,1} + C_{j,1}\tilde{\eta}^f_{j,1}) + \gamma_{j,1}\dot{\varphi}^f_{j,1}z^f_{j,1}\\
&= -\gamma_{j,1}\varphi^f_{j,1}\varphi^{f\mathrm{T}}_{j,1}\zeta_{j,1} + \gamma_{j,1}\varphi^f_{j,1}C_{j,1}\tilde{\eta}^f_{j,1} - \gamma_{j,1}\sigma_{j,1}(\hat{\theta}_{j,1} + \beta_{j,1}).
\end{aligned} \tag{17}
$$

Moreover, the time-derivative of $\tilde{\eta}^f_{j,1}$ satisfies

$$\begin{aligned}\dot{\tilde{\eta}}^f_{j,1} &= W_{j,1}\tilde{\eta}^f_{j,1} + \varpi^f_{j,1} - \Gamma_{j,1}(-\zeta^{\mathrm{T}}_{j,1}\varphi^f_{j,1} + C_{j,1}\tilde{\eta}^f_{j,1}) \\ &= (W_{j,1} - \Gamma_{j,1}C_{j,1})\tilde{\eta}^f_{j,1} + \varpi^f_{j,1} + \Gamma_{j,1}\zeta^{\mathrm{T}}_{j,1}\varphi^f_{j,1},\end{aligned} \tag{18}$$

where the matrix $W_{j,1} - \Gamma_{j,1}C_{j,1}$ can be made Hurwitz by selecting appropriate $\Gamma_{j,1}$ since $(W_{j,1}, C_{j,1})$ is observable. Now choosing the Lyapunov function,

$$V_{j,1} = \frac{1}{2}z^{f2}_{j,1} + \frac{\varepsilon_{j,1}}{2}\zeta^{\mathrm{T}}_{j,1}\gamma^{-1}_{j,1}\zeta_{j,1} + \tilde{\eta}^{f\mathrm{T}}_{j,1}P_{j,1}\tilde{\eta}^f_{j,1} \tag{19}$$

for the $(j,1)$th subsystem where $\varepsilon_{j,1} > 0$ is a positive constant, and $P_{j,1} > 0$ is the solution of the equation $(W_{j,1} - \Gamma_{j,1}C_{j,1})^{\mathrm{T}}P_{j,1} + P_{j,1}(W_{j,1} - \Gamma_{j,1}C_{j,1}) = -Q_{j,1}$ with $Q_{j,1} > 0$. The time-derivative of $V_{j,1}$ is calculated using (16)–(18) as

$$\begin{aligned}\dot{V}_{j,1} = {}& z^f_{j,1}(-k_{j,1}z^f_{j,1} + z^f_{j,2} - \varphi^{f\mathrm{T}}_{j,1}\zeta_{j,1} + C_{j,1}\tilde{\eta}^f_{j,1}) + \varepsilon_{j,1}\zeta^{\mathrm{T}}_{j,1}(-\varphi^f_{j,1}\varphi^{f\mathrm{T}}_{j,1}\zeta_{j,1} + \varphi^f_{j,1}C_{j,1}\tilde{\eta}^f_{j,1} \\ & - \sigma_{j,1}(\theta_{j,1} + \zeta_{j,1})) - \tilde{\eta}^{f\mathrm{T}}_{j,1}Q_{j,1}\tilde{\eta}^f_{j,1} + 2\tilde{\eta}^{f\mathrm{T}}_{j,1}P_{j,1}\Gamma_{j,1}\zeta^{\mathrm{T}}_{j,1}\varphi^f_{j,1} + 2\tilde{\eta}^{f\mathrm{T}}_{j,1}P_{j,1}\varpi^f_{j,1}.\end{aligned} \tag{20}$$

Using Young's inequality,

$$\begin{aligned}& z^f_{j,1}\varphi^{f\mathrm{T}}_{j,1}\zeta_{j,1} \leqslant \frac{1}{2}z^{f2}_{j,1} + \frac{1}{2}(\varphi^{f\mathrm{T}}_{j,1}\zeta_{j,1})^2, \\ & z^f_{j,1}C_{j,1}\tilde{\eta}^f_{j,1} \leqslant \frac{1}{2}\|C_{j,1}\|^2\|\tilde{\eta}^f_{j,1}\|^2 + \frac{1}{2}z^{f2}_{j,1}, \\ & -\zeta^{\mathrm{T}}_{j,1}\sigma_{j,1}(\theta_{j,1} + \zeta_{j,1}) \leqslant -\frac{\sigma_{j,1}}{2}\|\zeta_{j,1}\|^2 + \frac{\sigma_{j,1}}{2}\|\theta_{j,1}\|^2, \\ & \varepsilon_{j,1}\zeta^{\mathrm{T}}_{j,1}\varphi^f_{j,1}C_{j,1}\tilde{\eta}^f_{j,1} \leqslant \frac{\varepsilon_{j,1}}{2\rho_{j,1}}(\varphi^{f\mathrm{T}}_{j,1}\zeta_{j,1})^2 + \frac{\varepsilon_{j,1}\rho_{j,1}}{2}\|C_{j,1}\|^2\|\tilde{\eta}^f_{j,1}\|^2, \\ & 2\tilde{\eta}^{f\mathrm{T}}_{j,1}P_{j,1}\Gamma_{j,1}\zeta^{\mathrm{T}}_{j,1}\varphi^f_{j,1} \leqslant \|P_{j,1}\Gamma_{j,1}\|^2(\varphi^{f\mathrm{T}}_{j,1}\zeta_{j,1})^2 + \|\tilde{\eta}^f_{j,1}\|^2, \\ & 2\tilde{\eta}^{f\mathrm{T}}_{j,1}P_{j,1}\varpi^f_{j,1} \leqslant \|P_{j,1}\|^2\|\varpi^f_{j,1}\|^2 + \|\tilde{\eta}^f_{j,1}\|^2,\end{aligned} \tag{21}$$

with $\rho_{j,1} > \frac{1}{2}$, then we have

$$\begin{aligned}\dot{V}_{j,1} \leqslant {}& -(k_{j,1} - 1)z^{f2}_{j,1} + z^f_{j,1}z^f_{j,2} - \left(\frac{\varepsilon_{j,1}(2\rho_{j,1} - 1)}{2\rho_{j,1}} - \|P_{j,1}\Gamma_{j,1}\|^2 - \frac{1}{2}\right)(\varphi^{f\mathrm{T}}_{j,1}\zeta_{j,1})^2 - \frac{\varepsilon_{j,1}\sigma_{j,1}}{2}\|\zeta_{j,1}\|^2 \\ & - \left(\lambda_{\min}(Q_{j,1}) - \frac{\varepsilon_{j,1}\rho_{j,1} + 1}{2}\|C_{j,1}\|^2 - 2\right)\|\tilde{\eta}^f_{j,1}\|^2 + \frac{\varepsilon_{j,1}\sigma_{j,1}}{2}\|\theta_{j,1}\|^2 + \|P_{j,1}\|^2\|\varpi^f_{j,1}\|^2.\end{aligned} \tag{22}$$

Now we shall recover the virtual control law $\alpha_{j,1}$. First the derivative of $\bar{\alpha}^f_{j,1}$ is calculated as

$$\dot{\bar{\alpha}}^f_{j,1} = -k_{j,1}\dot{z}^f_{j,1} - (\dot{\hat{\theta}}_{j,1} + \gamma_{j,1}\dot{\varphi}^f_{j,1}z^f_{j,1} + \gamma_{j,1}\varphi^f_{j,1}\dot{z}^f_{j,1})^{\mathrm{T}}\varphi^f_{j,1} - (\hat{\theta}_{j,1} + \beta_{j,1})^{\mathrm{T}}\dot{\varphi}^f_{j,1} - C_{j,1}\dot{\hat{\eta}}^f_{j,1}, \tag{23}$$

which is available due to the defined low-pass filters and the adaptive laws. From (6), (13), and (23), $\alpha_{j,1}$ can be obtained as

$$\alpha_{j,1} = \tau\bar{\alpha}^f_{j,1} + \dot{\bar{\alpha}}^f_{j,1} - \epsilon_j c_{j,1}\xi_{j,1} - f_{j,1} + \dot{y}_{j,r}. \tag{24}$$

The obtained $\alpha_{j,1}$ is then passed through the command filter (6) to get the filtered signals $\alpha^f_{j,1}$ and $\dot{\alpha}^f_{j,1}$ which will be used in the next step.

Remark 3. With the filter operation, the regressor $\varphi_{j,1}$ is replaced by the filtered variable $\varphi^f_{j,1}$, thus avoiding the requirement of solving PDEs, as displayed in the constructed I&I adaptive law. A similar operation will be applied to the following design steps $(j,2)$–(j,n_j), and this largely releases the pressure due to the tedious calculation, particularly for regressors with miscellaneous arguments. Therefore, the structural conditions on the regressors required in [3] can be removed for systems in the parametric strict feedback form.

Step j, i_j ($j = 1, \ldots, m,\ i_j = 2, \ldots, n_j$). The process for the second step to the n_jth step of the jth subsystem will be illustrated here. Differentiating z_{j,i_j} using (1) and (8) gives

$$\begin{aligned}\dot{z}_{j,i_j} &= \dot{x}_{j,i_j} - \dot{\alpha}^f_{j,i_j-1} - \dot{\xi}_{j,i_j}\\ &= z_{j,i_j+1} + \alpha^c_{j,i_j} + \xi_{j,i_j+1} + \theta^{\mathrm{T}}_{j,i_j}\varphi_{j,i_j} + f_{j,i_j} + d_{j,i_j} + \delta_{j,i_j} - \dot{\alpha}^c_{j,i_j-1} - \dot{\xi}_{j,i_j}\\ &= z_{j,i_j+1} + \theta^{\mathrm{T}}_{j,i_j}\varphi_{j,i_j} + d_{j,i_j} + \delta_{j,i_j} + \bar{\alpha}_{j,i_j}, \quad i_j = 2, \ldots, n_j - 1,\end{aligned} \tag{25}$$

where $\delta_{j,1} = \cdots = \delta_{j,n_j-1} = 0$, $\delta_{j,n_j} = d_{j,s}(v_j)$, and the equivalent virtual control $\bar{\alpha}_{j,i_j}$ is given by

$$\bar{\alpha}_{j,i_j} = \alpha_{j,i_j} + \epsilon^{i_j} c_{j,i_j}\xi_{j,1} + f_{j,i_j} - \dot{\alpha}^c_{j,i_j-1}. \tag{26}$$

Filtering both sides of (25) by using (6) yields

$$\dot{z}^f_{j,i_j} = z^f_{j,i_j+1} + \theta^{\mathrm{T}}_{j,i_j}\varphi^f_{j,i_j} + d^f_{j,i_j} + \bar{\alpha}^f_{j,i_j} + \delta^f_{j,i_j}, \quad i_j = 2, \ldots, n_j, \tag{27}$$

where d^f_{j,i_j} is generated by the filtered exosystem,

$$\begin{aligned}\dot{\eta}^f_{j,i_j} &= W_{j,i_j}\eta^f_{j,i_j} + \varpi^f_{j,i_j},\\ d^f_{j,i_j} &= C_{j,i_j}\eta^f_{j,i_j}.\end{aligned} \tag{28}$$

Assign $\bar{\alpha}^f_{j,i_j}$ as

$$\bar{\alpha}^f_{j,i_j} = -z^f_{j,i_j-1} - k_{j,i_j} z^f_{j,i_j} - (\hat{\theta}_{j,i_j} + \beta_{j,i_j})^{\mathrm{T}}\varphi^f_{j,i_j} - \hat{d}^f_{j,i_j} \quad i_j = 2, \ldots, n_j - 1, \tag{29}$$

where $k_{j,i_j} > 0$ is a positive control parameter, $(\hat{\theta}_{j,i_j} + \beta_{j,i_j})$ is the I&I estimate for θ_{j,i_j} updated by

$$\begin{aligned}\dot{\hat{\theta}}_{j,i_j} &= -\gamma_{j,i_j}\varphi^f_{j,i_j}(-z^f_{j,i_j-1} - k_{i,i_j} z^f_{j,i_j} + z^f_{j,i_j+1}) - \gamma_{j,i_j}\dot{\varphi}^f_{j,i_j} z^f_{j,i_j} - \gamma_{j,i_j}\sigma_{j,i_j}(\hat{\theta}_{j,i_j} + \beta_{j,i_j}),\\ \beta_{j,i_j} &= \gamma_{j,i_j}\varphi^f_{j,i_j} z^f_{j,i_j},\end{aligned} \tag{30}$$

where $\gamma_{j,i_j} > 0$ and $\sigma_{j,i_j} > 0$ are positive design parameters, and $\hat{d}^f_{j,i_j}$ is the disturbance observer updated by

$$\begin{aligned}\dot{\hat{\eta}}^f_{j,i_j} &= W_{j,i_j}(\hat{\eta}^f_{j,i_j} + \Gamma_{j,1} z^f_{j,i_j}) - \Gamma_{j,i_j}(-z^f_{j,i_j-1} - k_{j,i_j} z^f_{j,i_j} + z^f_{j,i_j+1}),\\ \hat{d}^f_{j,i_j} &= C_{j,i_j}(\hat{\eta}^f_{j,i_j} + \Gamma_{j,1} z^f_{j,i_j}),\end{aligned} \tag{31}$$

with Γ_{j,i_j} a suitable design parameter vector. Substituting (29) into (27) yields

$$\dot{z}^f_{j,i_j} = -z^f_{j,i_j-1} - k_{j,i_j} z^f_{j,i_j} + z^f_{j,i_j+1} - \varphi^{f\mathrm{T}}_{j,i_j}\zeta_{j,i_j} + C_{j,i_j}\tilde{\eta}^f_{j,i_j} + \delta^f_{j,i_j}, \tag{32}$$

with $\zeta_{j,i_j} = \hat{\theta}_{j,i_j} - \theta_{j,i_j} + \beta_{j,i_j}$ and $\tilde{\eta}^f_{j,i_j} = \eta^f_{j,i_j} - \hat{\eta}^f_{j,i_j} - \Gamma_{j,1} z^f_{j,i_j}$ is the observer error. The time-derivative of ζ_{j,i_j} and $\tilde{\eta}^f_{j,i_j}$ can be calculated respectively as

$$\begin{aligned}\dot{\zeta}_{j,i_j} &= -\gamma_{j,i_j}\varphi^f_{j,i_j}\varphi^{f\mathrm{T}}_{j,i_j}\zeta_{j,i_j} + \gamma_{j,i_j}\varphi^f_{j,i_j} C_{j,i_j}\tilde{\eta}^f_{j,i_j} + \gamma_{j,i_j}\varphi^f_{j,i_j}\delta^f_{j,i_j} - \gamma_{j,i_j}\sigma_{j,i_j}(\theta_{j,i_j} + \zeta_{j,i_j}),\\ \dot{\tilde{\eta}}^f_{j,i_j} &= (W_{j,i_j} - \Gamma_{j,i_j} C_{j,i_j})\tilde{\eta}^f_{j,i_j} + \varpi^f_{j,i_j} + \Gamma_{j,i_j}\zeta^{\mathrm{T}}_{j,i_j}\varphi^f_{j,i_j} - \Gamma_{j,i_j}\delta^f_{j,i_j}.\end{aligned} \tag{33}$$

Now consider the following function:

$$V_{j,i_j} = \frac{1}{2} z^{f2}_{j,i_j} + \frac{\varepsilon_{j,i_j}}{2}\zeta^{\mathrm{T}}_{j,i_j}\gamma^{-1}_{j,i_j}\zeta_{j,i_j} + \tilde{\eta}^{f\mathrm{T}}_{j,i_j} P_{j,i_j}\tilde{\eta}^f_{j,i_j} \tag{34}$$

for the (j, i_j)th subsystem where $\varepsilon_{j,i_j} > 0$ is a positive constant and $P_{j,i_j} > 0$ is the solution of the equation $(W_{j,i_j} - \Gamma_{j,i_j} C_{j,i_j})^{\mathrm{T}} P_{j,i_j} + P_{j,i_j}(W_{j,i_j} - \Gamma_{j,i_j} C_{j,i_j}) = -Q_{j,i_j}$ with $Q_{j,i_j} > 0$. The time-derivative of V_{j,i_j} is calculated using (32) and (33) as

$$\begin{aligned}\dot{V}_{j,i_j} &= z^f_{j,i_j}(-z^f_{j,i_j-1} - k_{j,i_j}z^f_{j,i_j} + z^f_{j,i_j+1} - \varphi^{f\mathrm{T}}_{j,i_j}\zeta_{j,i_j} + C_{j,i_j}\tilde{\eta}^f_{j,i_j}) + \varepsilon_{j,i_j}\zeta^{\mathrm{T}}_{j,i_j}(-\varphi^f_{j,i_j}\varphi^{f\mathrm{T}}_{j,i_j}\zeta_{j,i_j} \\ &\quad + \varphi^f_{j,i_j}C_{j,i_j}\tilde{\eta}^f_{j,i_j} + \varphi^f_{j,i_j}\delta^f_{j,i_j} - \sigma_{j,i_j}(\theta_{j,i_j} + \zeta_{j,i_j})) - \tilde{\eta}^{f\mathrm{T}}_{j,i_j}Q_{j,i_j}\tilde{\eta}^f_{j,i_j} + 2\tilde{\eta}^{f\mathrm{T}}_{j,i_j}P_{j,i_j}\Gamma_{j,i_j}\zeta^{\mathrm{T}}_{j,i_j}\varphi^f_{j,i_j} \\ &\quad + 2\tilde{\eta}^{f\mathrm{T}}_{j,i_j}P_{j,i_j}\varpi^f_{j,i_j} - 2\tilde{\eta}^{f\mathrm{T}}_{j,i_j}P_{j,i_j}\Gamma_{j,i_j}\delta^f_{j,i_j} \\ &\leqslant -z^f_{j,i_j-1}z^f_{j,i_j} - (k_{j,i_j}-1)z^{f2}_{j,i_j} + z^f_{j,i_j}z^f_{j,i_j+1} - \left(\frac{\varepsilon_{j,i_j}(2\rho_{j,i_j}-1)}{2\rho_{j,i_j}} - \|P_{j,i_j}\Gamma_{j,i_j}\|^2 - \frac{1}{2}\right) \\ &\quad \times(\varphi^{f\mathrm{T}}_{j,i_j}\zeta_{j,i_j})^2 - \frac{\varepsilon_{j,i_j}\sigma_{j,i_j}}{2}\|\zeta_{j,i_j}\|^2 - \left(\lambda_{\min}(Q_{j,i_j}) - \frac{\varepsilon_{j,i_j}\rho_{j,i_j}+1}{2}\|C_{j,i_j}\|^2 - 2\right)\|\tilde{\eta}^f_{j,i_j}\|^2 \\ &\quad + \frac{\varepsilon_{j,i_j}\sigma_{j,i_j}}{2}\|\theta_{j,i_j}\|^2 + \|P_{j,i_j}\|^2\|\varpi^f_{j,i_j}\|^2 + \varepsilon_{j,i_j}\zeta^{\mathrm{T}}_{j,i_j}\varphi^f_{j,i_j}\delta^f_{j,i_j} - 2\tilde{\eta}^{f\mathrm{T}}_{j,i_j}P_{j,i_j}\Gamma_{j,i_j}\delta^f_{j,i_j},\end{aligned} \tag{35}$$

with $\rho_{j,1},\ldots,\rho_{j,n_j-1} > \frac{1}{2}$ and $\rho_{j,n_j} > \frac{3}{2}$. Note that the virtual control law α_{j,i_j} should be recovered according to (29),

$$\alpha_{j,i_j} = \tau\bar{\alpha}^f_{j,i_j} + \dot{\bar{\alpha}}^f_{j,i_j} - \epsilon^{i_j}_j c_{j,i_j}\xi_{j,1} - f_{j,i_j} + \dot{\alpha}^f_{j,i_j-1}, \tag{36}$$

where the derivative of $\bar{\alpha}^f_{j,i_j}$ is

$$\begin{aligned}\dot{\bar{\alpha}}^f_{j,i_j} &= -\dot{z}^f_{j,i_j-1} - k_{j,i_j}\dot{z}^f_{j,i_j} - (\dot{\hat{\theta}}_{j,i_j} + \gamma_{j,i_j}\dot{\varphi}^f_{j,i_j}z^f_{j,i_j} + \gamma_{j,i_j}\varphi^f_{j,i_j}\dot{z}^f_{j,i_j})^{\mathrm{T}}\varphi^f_{j,i_j} \\ &\quad - (\hat{\theta}_{j,i_j} + \beta_{j,i_j})^{\mathrm{T}}\dot{\varphi}^f_{j,i_j} - C_{j,i_j}\dot{\hat{\eta}}^f_{j,i_j},\end{aligned} \tag{37}$$

which is available due to the filters and adaptive law. By passing α_{j,i_j} through the command filter, the filtered signals α^f_{j,i_j} and $\dot{\alpha}^f_{j,i_j}$ can be obtained.

Step j, n_j+1. This is the final step of the design. Differentiating z_n using (5) and (8) gives

$$\begin{aligned}\dot{z}_{j,n_j+1} &= \frac{\partial g_j(v_j)}{\partial v_j}(-h_jv_j + w_j) - \dot{\alpha}^c_{j,n_j} - \dot{\xi}_{j,n_j+1} \\ &= \chi_j(-h_jv_j + w_j) - \dot{\alpha}^c_{j,n_j} + \epsilon^{n_j+1}_j c_{j,n_j+1}\xi_{j,1},\end{aligned} \tag{38}$$

with $\chi_j = \frac{\partial g_j(v_j)}{\partial v_j} > 0$. In [26], the Nussbaum-based approach is used to deal with the time-varying term χ_j. In this paper, inspired by [38, 39], we use an update law with a smooth function for the lower bound of χ_j, defined as $\bar{\chi}_j = \inf_{t\geqslant 0}\{\chi_j(t)\}$.

The control law is designed as

$$\begin{aligned} w_j &= -\hat{\chi}^2_j\bar{w}^2_j z_{j,n_j+1}/(\hat{\chi}^2_j\bar{w}^2_j z^2_{j,n_j+1} + \vartheta^2)^{\frac{1}{2}}, \\ \bar{w}_j &= k_{j,n_j+1}z_{j,n_j+1} - \chi_jh_jv_j - \dot{\alpha}^f_{j,n_j} + \epsilon^{n_j+1}_j c_{j,n_j+1}\xi_{j,1}, \\ \dot{\hat{\chi}}_j &= \psi_j\bar{w}_j z_{j,n_j+1} - \psi_j\Psi_j\hat{\chi}_j,\end{aligned} \tag{39}$$

with $k_{j,n_j+1}, \psi_j, \Psi_j, \vartheta > 0$. Note that the following holds [38, 39]:

$$\begin{aligned}\chi_jw_jz_{j,n_j+1} &= -\chi_j\hat{\chi}^2_j\bar{w}^2_jz^2_{j,n_j+1}/(\hat{\chi}^2_j\bar{w}^2_jz^2_{j,n_j+1} + \vartheta^2)^{\frac{1}{2}} \\ &\leqslant -\bar{\chi}_j\hat{\chi}^2_j\bar{w}^2_jz^2_{j,n_j+1}/(\hat{\chi}^2_j\bar{w}^2_jz^2_{j,n_j+1} + \vartheta^2)^{\frac{1}{2}} \\ &\leqslant \bar{\chi}_j\vartheta - \bar{\chi}_j\hat{\chi}_jz_{j,n_j+1}\bar{w}_j \\ &\leqslant \bar{\chi}_j\vartheta - z_{j,n_j+1}\bar{w}_j + \bar{\chi}_j\tilde{\chi}_jz_{j,n_j+1}\bar{w}_j,\end{aligned} \tag{40}$$

with the estimation error $\tilde{\chi}_j = \frac{1}{\bar{\chi}_j} - \hat{\chi}_j$.

Consider the following function:

$$V_{j,n_j+1} = \frac{1}{2}z^2_{j,n_j+1} + \frac{\bar{\chi}_j}{2\psi_j}\tilde{\chi}^2_j, \tag{41}$$

whose time-derivative is derived using (38)–(40) as

$$\begin{aligned}\dot{V}_{j,n_j+1} &= z_{j,n_j+1}(\chi_j(-h_jv_j - \hat{\chi}_j^2\bar{w}_j^2 z_{j,n_j+1}/(\hat{\chi}_j^2\bar{w}_j^2 z_{j,n_j+1}^2+\eta^2)^{\frac{1}{2}}) - \dot{\alpha}_{j,n_j}^f + \epsilon^{n_j+1}c_{j,n_j+1}\xi_{j,1}) - \frac{\bar{\chi}_j}{\psi_j}\tilde{\chi}_j\dot{\hat{\chi}}_j \\ &\leqslant -k_{j,n_j+1}z_{j,n_j+1}^2 + \bar{\chi}_j\vartheta + \bar{\chi}_j\tilde{\chi}_j z_{j,n_j+1}\bar{w}_j - \frac{\bar{\chi}_j}{\psi_j}\tilde{\chi}_j\dot{\hat{\chi}}_j \\ &\leqslant -k_{j,n_j+1}z_{j,n_j+1}^2 + \bar{\chi}_j\vartheta + \tilde{\chi}_j\bar{\chi}_j\Psi_j\hat{\chi}_j.\end{aligned} \tag{42}$$

Using Young's inequality,

$$\tilde{\chi}_j\bar{\chi}_j\Psi_j\hat{\chi}_j \leqslant -\frac{\bar{\chi}_j\Psi_j}{2}\tilde{\chi}_j^2 + \frac{\Psi_j}{2\bar{\chi}_j}, \tag{43}$$

then Eq. (42) becomes

$$\dot{V}_{j,n_j+1} \leqslant -k_{j,n_j+1}z_{j,n_j+1}^2 - \frac{\bar{\chi}_j\Psi_j}{2}\tilde{\chi}_j^2 + \bar{\chi}_j\vartheta + \frac{\Psi_j}{2\bar{\chi}_j}. \tag{44}$$

4 Stability analysis

The stability of the closed-loop system under the designed control scheme will be illustrated in the following theorem.

Theorem 1. Consider the uncertain MIMO nonlinear system (1) subject to the external disturbance (2) and the input constraint (3). It is closed with the control laws (24), (36), and (39), the I&I adaptive laws (14) and (30), and the disturbance observers (15) and (31). Under Assumptions 1 and 2, it can be guaranteed that (1) all the signals in the closed-loop system are ultimately bounded and, (2) the output tracking error $e_j(t) = x_{j,1}(t) - y_{j,r}(t)$ finally converges to a small residual bound around zero that can be adjusted by choosing appropriate design parameters.

Proof. Consider the function $V_n = \sum_{j=1}^m V_{j,n_j+1}$ whose derivative can be calculated using (44) as

$$\begin{aligned}\dot{V}_n &\leqslant \sum_{j=1}^m\left(-k_{j,n_j+1}z_{j,n_j+1}^2 - \frac{\bar{\chi}_j\Psi_j}{2}\tilde{\chi}_j^2 + \bar{\chi}_j\vartheta + \frac{\Psi_j}{2\bar{\chi}_j}\right) \\ &\leqslant -\kappa_n V_n + \Delta_n,\end{aligned} \tag{45}$$

where $\kappa_n = \min\{2k_{j,n_j+1}, \psi_j\Psi_j\}$ and $\Delta_n = \sum_{j=1}^m(\bar{\chi}_j\vartheta + \frac{\Psi_j}{2\bar{\chi}_j})$. This implies that $V_n(t)$ is bounded and $z_{j,n_j+1}(t)$ and $\tilde{\chi}_j(t)$ are also bounded. Now consider the function $\bar{V} = \sum_{j=1}^m\sum_{i_j=1}^{n_j} V_{j,i_j}$, whose time derivative is calculated by using (22), (35) and choosing $\frac{\varepsilon_{j,i_j}(2\rho_{j,i_j}-1)}{2\rho_{j,i_j}} - \|P_{j,i_j}\Gamma_{j,i_j}\|^2 - \frac{1}{2} \geqslant 0$, $\lambda_{\min}(Q_{j,i_j}) - \frac{\varepsilon_{j,i_j}\rho_{j,i_j}+1}{2}\|C_{j,i_j}\|^2 - 2 = 1$, $i_j = 1,\ldots,n_j-1$, $\frac{3\varepsilon_{j,n_j}\rho_{j,n_j}-2\varepsilon_{j,n_j}}{4\rho_{j,n_j}} - \|P_{j,n_j}\Gamma_{j,n_j}\|^2 - \frac{1}{2} \geqslant 0$, and

$\lambda_{\min}(Q_{j,n_j}) - \frac{\varepsilon_{j,n_j}\rho_{j,n_j}+1}{2}\|C_{j,n_j}\|^2 - 3 = 1$,

$$\begin{aligned}
\dot{V} \leqslant{}& \sum_{j=1}^{m}\Bigg(-(k_{j,1}-1)z_{j,1}^{f2} + z_{j,1}^{f}z_{j,2}^{f} - \left(\frac{\varepsilon_{j,1}(2\rho_{j,1}-1)}{2\rho_{j,1}} - \|P_{j,1}\Gamma_{j,1}\|^2 - \frac{1}{2}\right)(\varphi_{j,1}^{f\mathrm{T}}\zeta_{j,1})^2 - \frac{\varepsilon_{j,1}\sigma_{j,1}}{2}\|\zeta_{j,1}\|^2 \\
&- \left(\lambda_{\min}(Q_{j,1}) - \frac{\varepsilon_{j,1}\rho_{j,1}+1}{2}\|C_{j,1}\|^2 - 2\right)\|\tilde{\eta}_{j,1}^{f}\|^2 + \frac{\varepsilon_{j,1}\sigma_{j,1}}{2}\|\theta_{j,1}\|^2 + \|P_{j,1}\|^2\|\varpi_{j,1}^{f}\|^2\Bigg) \\
&+ \sum_{j=1}^{m}\sum_{i_j=2}^{n_j}\Bigg(-z_{j,i_j-1}^{f}z_{j,i_j}^{f} - (k_{j,i_j}-1)z_{j,i_j}^{f2} + z_{j,i_j}^{f}z_{j,i_j+1}^{f} - \left(\frac{\varepsilon_{j,i_j}(2\rho_{j,i_j}-1)}{2\rho_{j,i_j}} - \|P_{j,i_j}\Gamma_{j,i_j}\|^2 - \frac{1}{2}\right) \\
&\times (\varphi_{j,i_j}^{f\mathrm{T}}\zeta_{j,i_j})^2 - \frac{\varepsilon_{j,i_j}\sigma_{j,i_j}}{2}\|\zeta_{j,i_j}\|^2 - \left(\lambda_{\min}(Q_{j,i_j}) - \frac{\varepsilon_{j,i_j}\rho_{j,i_j}+1}{2}\|C_{j,i_j}\|^2 - 2\right)\|\tilde{\eta}_{j,i_j}^{f}\|^2 \\
&+ \frac{\varepsilon_{j,i_j}\sigma_{j,i_j}}{2}\|\theta_{j,i_j}\|^2 + \|P_{j,i_j}\|^2\|\varpi_{j,i_j}^{f}\|^2 + \varepsilon_{j,i_j}\zeta_{j,i_j}^{\mathrm{T}}\varphi_{j,i_j}^{f}\delta_{j,i_j}^{f} - 2\tilde{\eta}_{j,i_j}^{f\mathrm{T}}P_{j,i_j}\Gamma_{j,i_j}\delta_{j,i_j}^{f}\Bigg) \\
\leqslant{}& \sum_{j=1}^{m}\sum_{i_j=1}^{n_j-1}\left(-(k_{j,i_j}-1)z_{j,i_j}^{f2} - \frac{\varepsilon_{j,i_j}\sigma_{j,i_j}}{2}\|\zeta_{j,i_j}\|^2 - \|\tilde{\eta}_{j,i_j}^{f}\|^2\right) + \sum_{j=1}^{m}\Bigg(-\left(k_{j,n_j} - \frac{3}{2}\right)z_{j,n_j}^{f2} \\
&- \frac{\varepsilon_{j,n_j}\sigma_{j,n_j}}{2}\|\zeta_{j,n_j}\|^2 - \|\tilde{\eta}_{j,n_j}^{f}\|^2\Bigg) + \bar{\Delta} \\
\leqslant{}& -\bar{\kappa}\bar{V} + \bar{\Delta},
\end{aligned} \tag{46}$$

where the inequalities $\varepsilon_{j,n_j}\zeta_{j,n_j}^{\mathrm{T}}\varphi_{j,n_j}^{f}\delta_{j,n_j}^{f} \leqslant \frac{\varepsilon_{j,n_j}}{4}(\zeta_{j,n_j}^{\mathrm{T}}\varphi_{j,n_j}^{f})^2 + \varepsilon_{j,n_j}\delta_{j,n_j}^{f2}$ and $-2\tilde{\eta}_{j,n_j}^{f\mathrm{T}}P_{j,n_j}\Gamma_{j,n_j}\delta_{j,n_j}^{f} \leqslant \|\tilde{\eta}_{j,n_j}^{f}\|^2 + \|P_{j,n_j}\Gamma_{j,n_j}\delta_{j,n_j}^{f}\|^2$ have been used and $\bar{\kappa} = \min_{j=1,\ldots,m}\{2k_{j,1}-2,\ldots,2k_{j,n_j-1}-2, 2k_{j,n_j}-3, \frac{\sigma_{j,1}}{\lambda_{\max}(\gamma_{j,1}^{-1})},\ldots,\frac{\sigma_{j,n_j}}{\lambda_{\max}(\gamma_{j,n_j}^{-1})}, \frac{1}{\lambda_{\max}(P_{j,1})},\ldots,\frac{1}{\lambda_{\max}(P_{j,n_j})}\}$ and $\bar{\Delta} = \sum_{j=1}^{m}\sum_{i_j=1}^{n_j}(\frac{1}{2}\sup\{z_{j,n_j+1}^{f2}(t)\} + \frac{\varepsilon_{j,i_j}\sigma_{j,i_j}}{2} \times\|\theta_{j,i_j}\|^2 + \|P_{j,i_j}\|^2\|\varpi_{j,i_j}^{f}\|^2 + \varepsilon_{j,n_j}\delta_{j,n_j}^{f2} + \|P_{j,n_j}\Gamma_{j,n_j}\delta_{j,n_j}^{f}\|^2)$. Note that since $z_{j,n_j+1}(t)$ is bounded from the previous analysis, from Assumption 2, $\varpi_{j,i_j}(t)$ is bounded and $\delta_{j,n_j} = g_j(v_j)$ is bounded; then the filtered variables $z_{j,n_j+1}^{f}(t)$, $\varpi_{j,i_j}^{f}(t)$, and δ_{j,n_j}^{f} are also bounded which implies that from (44) $\bar{V}(t)$ is bounded and thus $z_{j,i_j}^{f}(t)$, $\zeta_{j,i_j}(t)$, and $\tilde{\eta}_{j,i_j}(t)$ are all bounded. Immediately the estimates $\hat{\theta}_{j,i_j}(t) + \beta_{j,i_j}(t)$ and $\hat{\eta}_{j,i_j}(t) + \Gamma_{j,i_j}z_{j,i_j}^{f}(t)$ are also bounded. Because $z_{j,i_j}^{f}(t)$ are all generated by the stable filters with the input $z_{j,i_j}(t)$, then $z_{j,i_j}(t)$ is also bounded.

Next, to analyze the boundedness of ξ_{j,i_j} we define the scaled variable $\xi_j = \bar{\epsilon}_j^{-1}[\xi_{j,1},\ldots,\xi_{j,n_j+1}]^{\mathrm{T}}$ with $\bar{\epsilon}_j = \mathrm{diag}\{\epsilon_j, \epsilon_j^2,\ldots,\epsilon_j^{n_j+1}\}$; then the dynamics of ξ_j in (8) can be rewritten in a compact form as

$$\dot{\xi}_j = \epsilon_j A_j\xi_j + \bar{\epsilon}_j^{-1}B_j, \tag{47}$$

where $B_j = [\alpha_{j,1}^{f} - \alpha_{j,1},\ldots,\alpha_{j,n_j}^{f} - \alpha_{j,n_j}, 0]^{\mathrm{T}}$ with

$$A_j = \begin{bmatrix} -c_{j,1} & 1\ 0\ \ldots\ 0 \\ -c_{j,2} & 0\ 1\ \ldots\ 0 \\ \vdots & \vdots\ \vdots\ \ddots\ \vdots \\ -c_{j,n_j} & 0\ 0\ \ldots\ 1 \\ -c_{j,n_j+1} & 0\ 0\ \ldots\ 0 \end{bmatrix}. \tag{48}$$

Consider the function $V_\xi = \sum_{j=1}^m \xi_j^{\mathrm{T}} P_j \xi_j$ with $A_j^{\mathrm{T}} P_j + P_j A_j = -I,\ P_j > 0$ whose time derivative is

$$\begin{aligned}\dot{V}_\xi &= \sum_{j=1}^m (-\epsilon_j \|\xi_j\|^2 + 2\xi_j^{\mathrm{T}} P_j \bar{\epsilon}_j^{-1} B_j) \\ &\leqslant \sum_{j=1}^m (-(\epsilon_j - 1)\|\xi_j\|^2 + \|P_j \bar{\epsilon}_j^{-1} B_j\|^2) \\ &\leqslant -\kappa_\xi V_\xi + \Delta_\xi,\end{aligned} \tag{49}$$

with $\kappa_\xi = \min_{j=1,\ldots,m}\{\frac{\epsilon_j - 1}{\lambda_{\max}(P_j)}\}$, $\Delta_\xi = \sup\{\sum_{j=1}^m \|P_j \bar{\epsilon}_j^{-1} B_j(t)\|^2\}$. Since from [14] the command filter errors $\alpha_{j,i_j}^f - \alpha_{j,i_j}, \ldots, i_j = 1, \ldots, n_j$ are bounded, then Δ_ξ is bounded. As a result we can deduce the boundedness of $V_\xi(t)$ and thus $\xi_j(t)$ is bounded. Given the boundedness of $z_{j,1}(t)$, $\xi_{j,1}(t)$, and Assumption 2 we know $x_{j,1}$ is also bounded according to (7). Then immediately the virtual control law $\alpha_{j,1}$ is bounded in (24) and the command filtered variables $\alpha_{j,1}^f,\ \dot{\alpha}_{j,1}^f$ are also bounded. According to (7) again then $x_{j,2}$ is bounded. By recursive procedures we have $\alpha_{j,2}, \ldots, \alpha_{j,n_j}, \alpha_{j,2}^f, \ldots, \alpha_{j,n_j}^f$, and $x_{j,3}, \ldots, x_{j,n_j}$ all bounded. Furthermore, the boundedness of the control law w_j in (39) can be established. Consequently, all the closed-loop signals are ultimately bounded.

On the other hand, solving (46) and (49) respectively yields

$$\begin{aligned}\bar{V}(t) &\leqslant \bar{V}(0)\mathrm{e}^{-\bar{\kappa}t} + \frac{\bar{\Delta}}{\bar{\kappa}}(1 - \mathrm{e}^{-\bar{\kappa}t}), \\ V_\xi(t) &\leqslant V_\xi(0)\mathrm{e}^{-\kappa_\xi t} + \frac{\Delta_\xi}{\kappa_\xi}(1 - \mathrm{e}^{-\kappa_\xi t});\end{aligned} \tag{50}$$

then it can be concluded that $\lim_{t\to\infty} |z_{j,1}^f(t)| \leqslant \sqrt{\frac{2\bar{\Delta}}{\bar{\kappa}}}$ and $\lim_{t\to\infty} |\xi_{j,i_j}(t)| \leqslant \sqrt{\frac{\Delta_\xi}{\lambda_{\min}(P_j)\kappa_\xi}}$. From (6) the ultimate bound of $z_{j,1}(t)$ can be calculated as $\lim_{t\to\infty} |z_{j,1}^f(t)| = \lim_{t\to\infty} |\tau \dot{z}_{j,1}^f + z_{j,1}^f| \leqslant \sqrt{\frac{2\bar{\Delta}}{\bar{\kappa}}} + \tau\varpi_j$ with $\varpi_j > 0$ the ultimate bound of $\dot{z}_{j,1}^f$. Consequently, from (7) the output tracking error $e_j(t) = x_{j,1}(t) - y_{j,r}(t)$ satisfies $\lim_{t\to\infty} |e_j(t)| = \lim_{t\to\infty} |z_{j,1}(t) + \xi_{j,1}(t)| \leqslant \sqrt{\frac{2\bar{\Delta}}{\bar{\kappa}}} + \tau\varpi_j + \sqrt{\frac{\Delta_\xi}{\lambda_{\min}(P_j)\kappa_\xi}}$. The proof is thus completed.

Remark 4. Note that in the development process, the command filters for the tracking error variable z_{j,i_j}, the regressor φ_{j,i_j}, and the virtual control α_{j,i_j} are actually needed to be implemented to generate the filtered variables z_{j,i_j}^f, φ_{j,i_j}^f, and α_{j,i_j}^f for $j = 1, \ldots, m,\ i_j = 1, \ldots, n_j$. The filtered exosystems (12) and (28) are only presented for analysis because their states need to be estimated by the disturbance observer.

Remark 5. Although the result in [40] can completely dominate the command filter error by introducing a continuous robust function in the auxiliary system, the upper bound of the command filter error may be hard to know in practice. Alternatively, in this paper the auxiliary system in the command-filtered backstepping method is designed in a high-gain form, which restrains the command filter error by simply increasing the high-gain parameter.

Remark 6. In the suggested control scheme, the main control parameters $k_{j,i_j},\ c_{j,i_j},\ \tau,\ \epsilon_j,\ \gamma_{j,i_j}$, $\Gamma_{j,i_j},\ \sigma_{j,i_j},\ \psi_j$, and Ψ_j, $j = 1, \ldots, m,\ i_j = 1, \ldots, n_j$ are needed to be chosen appropriately. Besides, theoretically speaking, the larger values of k_{j,i_j}, ϵ_j, ψ_j, and γ_{j,i_j} contribute to faster convergence rates and smaller residual tracking errors. However, too high values for them can also result in an increase in control effort and may activate unmodeled dynamics. In particular, the large value of the high-gain parameter ϵ_j implies higher sensitivity to noises such as input and sensor noises. As a result, when choosing these parameters, users should design and adjust based on the actual situation to determine the more appropriate values. The Γ_{j,i_j}s and c_{j,i_j}s need to be chosen such that the system matrices in (18), (33), and (48) are Hurwitz. The parameters σ_{j,i_j}, Ψ_j of the leakage terms in the adaptive laws should be fixed to the appropriate values since their sizes provide the trade-off between the boundedness of estimates and the size of ultimate bounds. Furthermore, the constant τ in the command filter for filtering the virtual control laws should be selected small for a good approximation, while the τ for filtering the tracking error variable or regressors need not be chosen small essentially. However, it has been indicated from the derived bound of the output error $e_j(t)$ that a small value of τ is beneficial for decreasing the ultimate bound of $e_j(t)$ since it decreases the bound of the term $\tau\varpi_j$.

5 Simulations

5.1 Example 1

Consider the following nonlinear system:

$$\begin{aligned}
\dot{x}_{1,1} &= x_{1,2} + \theta_1 x_{1,1}^2,\\
\dot{x}_{1,2} &= u_1 + d_1(t),\\
\dot{x}_{2,1} &= x_{2,2} + d_2(t),\\
\dot{x}_{2,2} &= u_2 + \theta_2 x_{1,1} x_{2,1},
\end{aligned} \tag{51}$$

where unknown parameters $\theta_1 = \theta_2 = 1$ and $d_1(t)$ and $d_2(t)$ are generated by the following exosystems:

$$\begin{aligned}
\dot{\eta}_j &= \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \eta_j + \varpi_j(t),\\
d_j(t) &= \begin{bmatrix} 1 & 0 \end{bmatrix} \eta_j,
\end{aligned} \tag{52}$$

for $j = 1, 2$, $\varpi_1(t) = [\sin(5t), 0]^{\mathrm{T}}$ and $\varpi_2(t) = [0, \cos(5t)]^{\mathrm{T}}$. For simplicity the controller parameters for the two subsystems are set as the same, given by $k_1 = k_2 = k_3 = c_1 = c_2 = c_3 = h = 5$, $\epsilon = 1.5$, $\gamma_1 = \gamma_2 = 2$, $\vartheta = 0.01$, $\sigma_1 = \sigma_2 = \Phi = 0.01$, $\Gamma_1 = \Gamma_2 = [2, 0]^{\mathrm{T}}$, and $\psi = 0.0001$. The filter constants for generating the filtered tracking error variable and the filtered regressors are set as $\tau = 1$ and the filter constant for filtered virtual control is $\tau = 0.01$. In simulation, the initial states are $x_1(0) = [1, -0.5]^{\mathrm{T}}$ and $x_2(0) = [-1, 0]^{\mathrm{T}}$. The initial values for the update laws are all set as zero. The initial exosystem states are $\eta_1(0) = [1, 2]^{\mathrm{T}}$ and $\eta_2(0) = [-1, 0]^{\mathrm{T}}$. The control saturation magnitude is set as $u_{1,M} = 80$ and $u_{2,M} = 170$. Simulation results for this case are shown in Figures 2 and 3. In Figure 2, the curves of tracking errors and control inputs of the two subsystems are depicted, where good tracking performances can be observed and control saturation occurs shortly in the transient stage. Figure 3 presents the time evolutions of estimated disturbance and parameter, where it can be observed that they are all bounded and very close to their true values. In order to further illustrate the performance of the used smooth function with bounding estimate in (39), we replace the bounding estimation with the Nussbaum-based approach in [26] under the same condition and parameters. The tracking errors and control inputs of this case are shown in Figure 4, where it can be seen that the transient tracking performance is relatively poor. More specifically, the tracking errors increase reversely in the transient process and the control inputs have longer saturation time compared with the previous case, which illustrate the advantage of the proposed method.

5.2 Example 2

To further test the efficiency of the suggested controller, we apply the scheme to an unmanned surface vehicle (USV). We consider the fully actuated USV modeled by [41, 42],

$$\begin{aligned}
\dot{\eta} &= J(\eta)\nu,\\
M\dot{\nu} &= -C(\nu)\nu - D(\nu)\nu + \tau + \tau_d(t),
\end{aligned} \tag{53}$$

where $\eta = [x, y, \psi]^{\mathrm{T}}$ and $\nu = [u, v, r]^{\mathrm{T}}$ with (x, y) being the position of the vehicle in the earth-fixed frame, ψ being the yaw angle, and u, v and r being the velocities in surge, sway, and yaw in the body-fixed frame. τ and τ_d are the control inputs and external disturbances, respectively. $J(\eta)$ denotes the rotation matrix, $M > 0$ is the inertia matrix, $C(\nu)$ is the total Coriolis and centripetal acceleration matrix, and $D(\nu)$ is

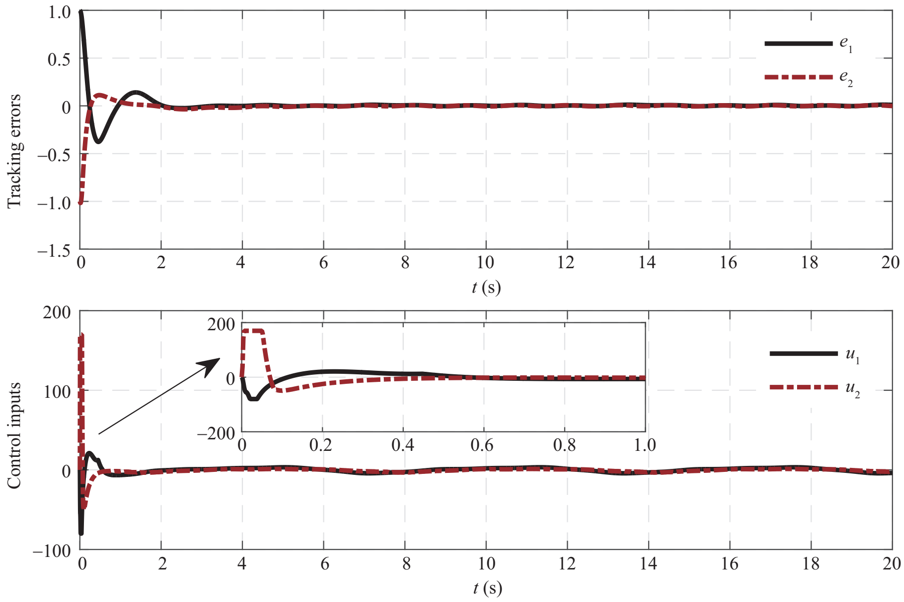

Figure 2 (Color online) Tracking errors and control inputs of the proposed method.

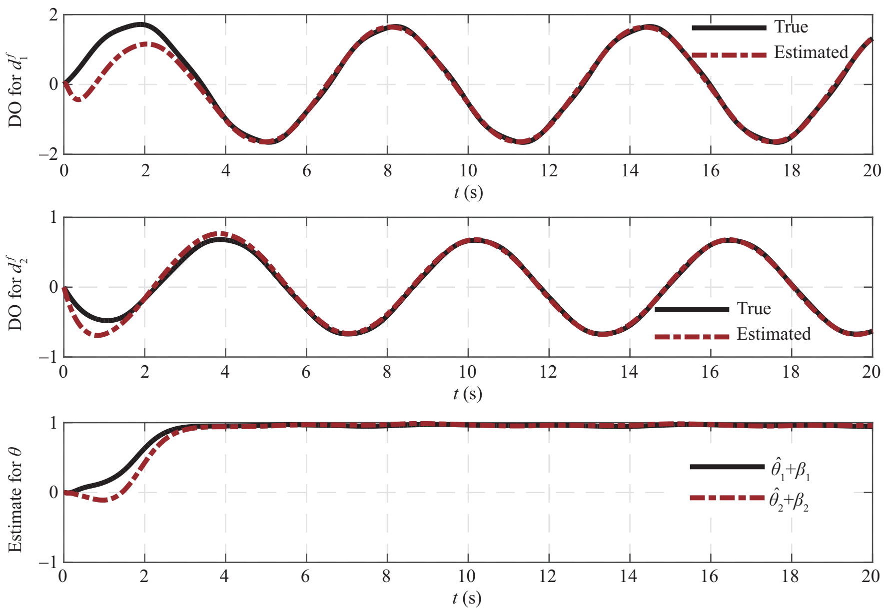

Figure 3 (Color online) Disturbance observer and adaptive law.

the hydrodynamic damping matrix, which are given by

$$
\begin{aligned}
J(\eta) &= \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad M = \begin{bmatrix} M_{11} & 0 & 0 \\ 0 & M_{22} & M_{23} \\ 0 & M_{23} & M_{33} \end{bmatrix}, \\
C(\nu) &= \begin{bmatrix} 0 & 0 & C_{13} \\ 0 & 0 & M_{11}u \\ -C_{13} & -M_{11}u & 0 \end{bmatrix}, \quad D(\nu) = \begin{bmatrix} D_{11}(\nu) & 0 & 0 \\ 0 & D_{22}(\nu) & D_{23}(\nu) \\ 0 & D_{32}(\nu) & D_{33}(\nu) \end{bmatrix},
\end{aligned} \tag{54}
$$

where $C_{13} = -M_{22}v - M_{23}r$, $M_{11} = m - X_{\dot{u}}$, $M_{22} = m - Y_{\dot{v}}$, $M_{23} = mx_g - Y_{\dot{r}}$, $M_{33} = I_z - N_{\dot{r}}$, $D_{11} = -X_u - X_{|u|u}|u| - X_{uuu}u^2$, $D_{22} = -Y_v - Y_{|v|v}|v| - Y_{|r|v}|r|$, $D_{23} = -Y_r - Y_{|v|r}|v| - Y_{|r|r}|r|$, $D_{32} = -N_v - N_{|v|v}|v| - N_{|r|v}|r|$, and $D_{33} = -N_r - N_{|v|r}|v| - N_{|r|r}|r|$ with m being the vehicle mass, I_z

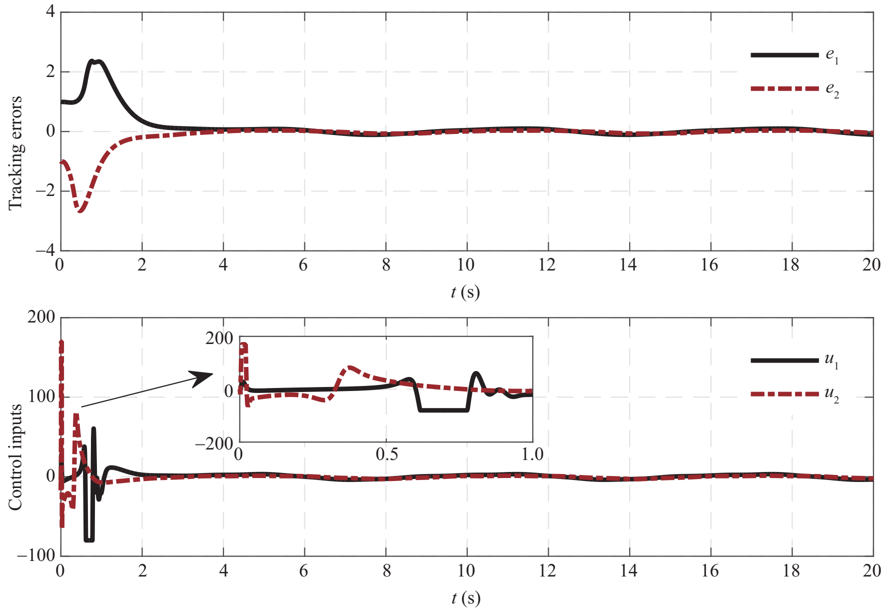

Figure 4 (Color online) Tracking errors and control inputs using Nussbaum function.

the moment of inertia in yaw direction, $X_{\dot{u}}, Y_{\dot{v}}, Y_{\dot{r}}$, and $N_{\dot{r}}$ the added masses, x_g the x_b-coordination of the center of gravity, and $X_{(\cdot)}, Y_{(\cdot)}, N_{(\cdot)}$ the hydrodynamic parameters. The explicit value of these parameters can be found in [41, 42]. In simulation we assume that the matrices M and $C(\nu)$ are known but $D(\nu)$ is unknown. Note that the auxiliary system (8) and virtual control law (36) for $i_j = n_j$ should be modified to take the known input matrix M into account. The disturbances in this simulation are the outputs of the van der Pol oscillators given by $\dot{\eta}_{j,1} = \eta_{j,2},\ \dot{\eta}_{j,2} = -\mu_j\eta_{j,1} + \eta_{j,2}(1-\eta_{j,1}^2),\ d_j(t) = \eta_{j,1},\ j = 1,2,3$ with the parameter $\mu_j = j$. We assume that the parameter μ_j is known but the nonlinear part $\eta_{j,2}(1-\eta_{j,1}^2)$ is unknown to the control design, acting as the bounded unmodeled error. In simulation the initial exosystem states are $\eta_1(0) = [0.2, 0.5]^{\rm T}$, $\eta_2(0) = [0.6, -0.3]^{\rm T}$, and $\eta_3(0) = [0, 0.7]^{\rm T}$.

The controller parameters are given by $c_{j,k} = 5,\ j,k = 1,2,3,\ k_{j,k} = 5,\ j,k = 1,2,\ k_{j,3} = h_j = 10$, $\epsilon = 1.5,\ \gamma_j = 3,\ \vartheta = 0.01,\ \sigma_j = \Phi_j = 0.01,\ \Gamma_{j,1} = \Gamma_{j,2} = [5,5]^{\rm T},\ \Gamma_{j,3} = [20,10]^{\rm T},\ \psi_1 = 0.0000001$, $\psi_2 = 0.001$, and $\psi_3 = 0.0001$. The filter constants are set as the same with the previous simulation. The initial states are $\eta(0) = [0.8, 1, 0.1]^{\rm T}$ and $\nu(0) = [-0.3, 0.6, 0.2]^{\rm T}$. The initial values for the update laws are all set as zero. The control saturation magnitude are set as $\tau_{1,M} = \tau_{2,M} = 60$ and $\tau_{3,M} = 5$. For the sake of comparison we also implement the controller with the Lyapunov-based adaptive scheme, which is actually the proposed control scheme whose adaptation part is replaced with the Lyapunov-based adaptive approach. The Lyapunov-based adaptive controller is labeled as Controller 1 and the proposed one is labeled as Controller 2. Both controllers are run under the same conditions and parameters. Simulation results for this case are shown in Figures 5 and 6, where it can be observed that the proposed method outperforms the Lyapunov-based adaptive controller during the transient stage and the control inputs saturate less times during 5–10 s, which illustrates the advantage of the filter-based I&I adaptive scheme.

6 Conclusion

In this paper, a command filter-based I&I adaptive controller is developed to cope with parametric uncertainty, external disturbance, and input saturation of a class of MIMO systems. To design I&I adaptive laws more feasibly, low-pass filters are used to avoid solving PDEs. A high-gain auxiliary system is constructed to remove the effect of the command filter error of the virtual controls. A disturbance observer is created and co-designed with the command filter-based adaptive backstepping method. A smooth function is used to approximate the input saturation with an extended state. It is shown that the suggested strategy guarantees the boundedness of closed-loop signals and the convergence to a small residual bound of the tracking error. Numerical simulated examples are finally performed to verify the proposed design. In future research, we consider experimentally confirming the proposed control scheme

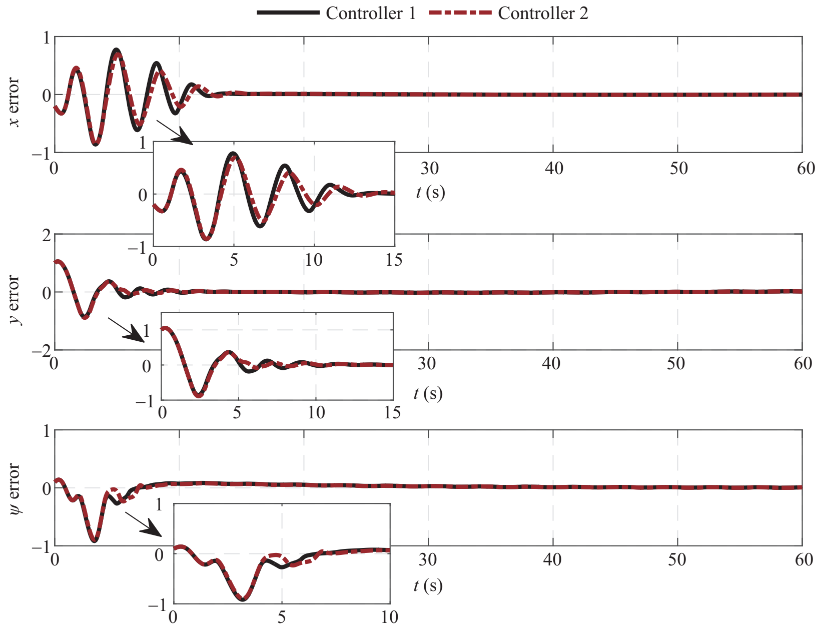

Figure 5 (Color online) Comparison of tracking errors.

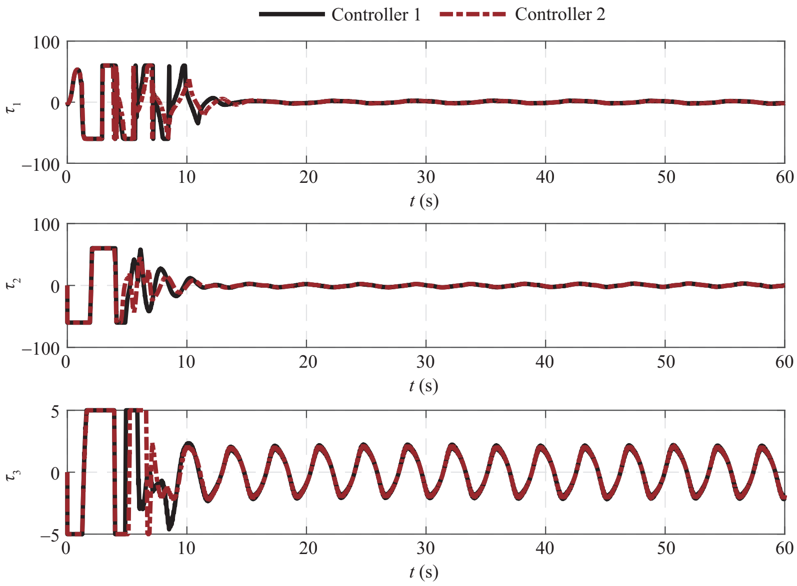

Figure 6 (Color online) Comparison of control inputs.

in actual systems and extending the approach to the multi-agent coordination field.

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