SCIENCE CHINA
Information Sciences

• LETTER •

November 2023, Vol. 66 219302:1–219302:2
https://doi.org/10.1007/s11432-022-3814-8

A novel sparse SAR unambiguous imaging method based on mixed-norm optimization

Hui BI[1,2*], Yufan SONG[1,2], Yanjie YIN[1,2], Jingjing ZHANG[1,2], Wen HONG[3] & Yirong WU[3]

[1] *College of Electronic and Information Engineering, Nanjing University of Aeronautics and Astronautics, Nanjing* 211106*, China;*
[2] *Key Laboratory of Radar Imaging and Microwave Photonics, Ministry of Education, Nanjing University of Aeronautics and Astronautics, Nanjing* 211106*, China;*
[3] *Aerospace Information Research Institute, Chinese Academy of Sciences, Beijing* 100094*, China*

Received 28 December 2022/Revised 7 May 2023/Accepted 26 June 2023/Published online 7 October 2023

Citation Bi H, Song Y F, Yin Y J, et al. A novel sparse SAR unambiguous imaging method based on mixed-norm optimization. Sci China Inf Sci, 2023, 66(11): 219302, https://doi.org/10.1007/s11432-022-3814-8

Sparse synthetic aperture radar (SAR) systems always obtain a wide swath by reducing the pulse repetition frequency (PRF). However, the reduction of PRF will cause azimuth ambiguity, and even lead to failed reconstruction of the observation scenes in SAR imaging because traditional sparse SAR imaging models do not consider the azimuth ambiguity terms and cannot effectively suppress ambiguity. To solve this problem, an unambiguous sparse SAR imaging model was proposed, which achieves unambiguous sparse reconstruction by solving an $L_{2,1}$-norm regularization problem [1]. However, it is not highly effective in suppressing azimuth ambiguity [2, 3]. As the reduction of PRF is theoretically equivalent to the down-sampling of echo data, a novel $L_{2,1/2}$-norm regularization-based unambiguous sparse SAR imaging method is proposed to further improve SAR imaging performance with down-sampled data and applied for the high-quality recovery of large-scale sparse surveillance area.

Method. The center frequency f_a of ambiguity areas can be written as $f_a = f_{dc} + i \cdot \mathrm{PRF}$, $i \in \mathbb{Z}$ and $i \neq 0$, where f_{dc} is the Doppler center frequency, and $i \in \mathbb{Z}^+$ and $i \in \mathbb{Z}^-$ denote the azimuth ambiguity terms in the right and left sides of the main imaging area in the azimuth direction, respectively. Incorporating the azimuth ambiguity terms into the two-dimensional (2-D) unambiguous sparse SAR imaging model, since the main image term and azimuth ambiguity terms have the same support set, the group sparsity between them could be used for the scene recovery by solving [3]

$$\hat{\boldsymbol{X}} = \min_{\boldsymbol{X}} \left\{ \left\| \boldsymbol{Y} - \boldsymbol{\Xi}_a \circ \left(\mathcal{M}(\boldsymbol{X}) + \sum_i \mathcal{M}_i(\boldsymbol{X}_i) \right) \right\|_F^2 + \beta_1 \|\boldsymbol{X}_{\mathrm{all}}\|_{2,q}^q + \beta_2 \|\boldsymbol{X}\|_q^q \right\} \quad (1)$$

where $\boldsymbol{Y} \in \mathbb{C}^{N_a \times N_r}$ is the 2-D down-sampled data with N_r and N_a being the number of samples in the range and azimuth directions, respectively, $\boldsymbol{\Xi}_a$ is the azimuth sampling matrix, $\boldsymbol{X}$ and $\boldsymbol{X}_i$ are the backscattering coefficients of the surveillance region and the ith azimuth ambiguity term, respectively, and $\circ$ is the Hadamard product. $\mathcal{M}(\cdot)$ and $\mathcal{M}_i(\cdot)$ are the echo simulation operators, which are the inverse procedures of typically matched filtering (MF) imaging algorithm $\mathcal{R}(\cdot)$ and $\mathcal{R}_i(\cdot)$, respectively [3]. $\hat{\boldsymbol{X}}$ is the recovered sparse SAR image and $\|\boldsymbol{X}_{\mathrm{all}}\|_{2,q}^q$ is defined as

$$\|\boldsymbol{X}_{\mathrm{all}}\|_{2,q}^q \triangleq \left(\sum_{n_a=1}^{N_a} \left(|\boldsymbol{X}_{n_a}| + \sum_i |\boldsymbol{X}_{i,n_a}| \right)^q \right)^{1/q}, \quad (2)$$

where $\boldsymbol{X}_{n_a}$ and $\boldsymbol{X}_{i,n_a}$ are the n_ath row of $\boldsymbol{X}$ and $\boldsymbol{X}_i$, respectively. $n_a = 1, 2, \ldots, N_a$. β_1 and β_2 are the regularization parameters which control the sparsity of the entire scenes $\boldsymbol{X}_{\mathrm{all}}$ and $\boldsymbol{X}$. For the model in (1), iterative thresholding [4] is used for scene recovery. Considering $q = 1/2$, which is more advantageous than $q = 1$ [5], and the thresholding operator $F_{\mu,\beta,q}(\boldsymbol{Z})$ for matrix $\boldsymbol{Z} \in \mathbb{C}^{N_a \times N_r}$ can be written as

$$F_{\mu,\beta,q}(\boldsymbol{Z}) = [f_{\mu,\beta,q}(z)]_{N_a \times N_r}, \quad (3)$$

where $z = \boldsymbol{Z}(n_a, n_r)$, $n_r = 1, 2, \ldots, N_r$, the thresholding function $f_{\mu,\beta,q}(\cdot)$ is [5]

$$f_{\mu,\beta,1/2}(z) = \begin{cases} \frac{2}{3} z (1 + g_{\mu,\beta}(z)), & \text{if } |z| \geqslant \frac{3\sqrt[3]{2}}{4} (\mu\beta)^{\frac{2}{3}}, \\ 0, & \text{otherwise}, \end{cases} \quad (4)$$

μ controls the algorithm convergence speed and is usually set to a constant, β is the regularization parameter, and

* Corresponding author (email: bihui@nuaa.edu.cn)

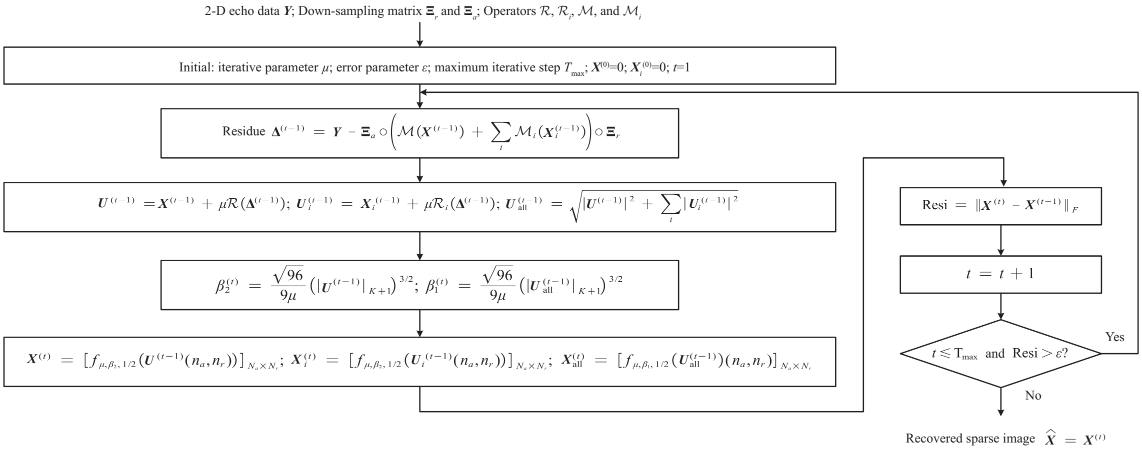

Figure 1 Flow diagram of the proposed method.

$g_{\mu,\beta}(z)$ can be written as

$$g_{\mu,\beta}(z) = \cos\left(\frac{2\pi}{3} - \frac{2}{3}\arccos\left(\frac{\mu\beta}{8}\left(\frac{|z|}{3}\right)^{-3/2}\right)\right). \quad (5)$$

The detailed steps are shown in Figure 1. $|\boldsymbol{U}^{(t-1)}|_{K+1}$ is the $(K+1)$th largest component of amplitude $|\boldsymbol{U}^{(t-1)}|$ in descending order.

Let $M = N_a \times N_r$ and $N = N_p \times N_q$, the number of iterative steps required by the proposed method is T, and the number of azimuth ambiguity terms considered in the model is I. The computational complexity of the proposed method primarily comprises three parts. Consider a single iteration process, wherein the first part is the MF and inverse MF processes of the main image term and I azimuth ambiguity terms, and the computational complexity is $\mathcal{O}((I+1)M\log M)$ [1]. The second part is the computational complexity of the threshold iterative operation, given by $\mathcal{O}(2N)$. The third part is the computational complexity of the iteration operation of the whole scene, given by $\mathcal{O}(N)$. Therefore, the computational complexity of the proposed method can be expressed as $\mathcal{O}(T((I+1)M\log M + 3N))$.

Experiments and results. For the azimuth ambiguity suppression in SAR imaging, the proposed method is compared with MF-based, L_q-norm regularization-based and existing $L_{2,1}$-norm regularization-based methods. Detailed results are presented in Appendix A due to space limitations.

Conclusion. A novel $L_{2,1/2}$-norm regularization-based sparse imaging method is proposed and applied to the unambiguous sparse reconstruction of large-scale areas. It has good performance in enhancing the quality of the recovered image, especially in terms of azimuth ambiguity suppression.

Acknowledgements This work was supported by National Natural Science Foundation of China (Grant Nos. 62271248, 61901213) and Guangdong Basic and Applied Basic Research Foundation (Grant No. 2020B1515120060).

Supporting information Appendix A. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

References

1 Zhang B C, Hong W, Wu Y R. Sparse microwave imaging: Principles and applications. Sci China Inf Sci, 2012, 55: 1722–1754
2 Liu M, Zhang B, Xu Z, et al. Ambiguities suppression for azimuth multichannel SAR based on $L_{2,q}$ regularization with application to Gaofen-3 ultra-fine stripmap mode. IEEE J Sel Top Appl Earth Observations Remote Sens, 2021, 14: 1532–1544
3 Bi H, Lu X, Yin Y, et al. Sparse SAR imaging based on periodic block sampling data. IEEE Trans Geosci Remote Sens, 2022, 60: 1–12
4 Daubechies I, Defrise M, de Mol C. An iterative thresholding algorithm for linear inverse problems with a sparsity constraint. Comm Pure Appl Math, 2004, 57: 1413–1457
5 Zeng J S, Fang J, Xu Z B. Sparse SAR imaging based on $L_{1/2}$ regularization. Sci China Inf Sci, 2012, 55: 1755–1775