

• Supplementary File •

# A Bi-level Optimization Approach for Joint Rack Sequencing and Storage Assignment in Robotic Mobile Fulfillment Systems

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## Appendix A Main notation declaration

The main notations employed in this paper are listed in Table S1.

**Table S1** Main notation declaration

$D_g$	The generalized distance matrix in the upper level optimization
$D_t, D_a, D_{ar}, D_c$	The cost matrix in LBH, LBH-A, RTS and STR modes and LBH-C
$d_{ij}^t, d_{ij}^a, d_{ij}^{ar}, d_{ij}^c$	The one item of $D_t, D_a, D_{ar}, D_c$
$G_t, G_a, G_{ar}, G_c$	The graph in LBH, LBH-A, RTS and STR modes and LBH-C
$k_{max}$	The maximum number of neighborhoods
$l_0, l_0'$	The starting point and ending point
$m$	Number of retrieval racks
$m_1$	Number of retrieval racks
$n$	Number of open storage locations
$Os$	A set of open storage locations
$p_i$	The fixed picking station of the rack $r_i$
$R$	A set of retrieval racks
$RS$	The retrieval sequence obtained by lower level optimization
$S$	The storage assignment set obtained by upper level optimization
$TP$	The travel time matrix between all picking stations and all locations $\Gamma$
$Rl_i$	The index of the retrieval rack for the $i$ th delivery task, $\forall i \in \{1, \dots, m\}$
$Sl_i$	The index of the storage location for the $i$ th delivery task, $\forall i \in \{1, \dots, m\}$
$Tlr$	The travel time between the $l_0$ and $Rl_1$
$Tsr_i$	The travel time between the $Sl_{i-1}$ and $Rl_i$ , $\forall i \in \{2, \dots, m\}$
$Trp_i$	The travel time between the $Rl_i$ and $p_i$ , $\forall i \in \{1, \dots, m\}$
$Tps_i$	The travel time between the $p_i$ and $Sl_i$ , $\forall i \in \{1, \dots, m\}$
$Tsl$	The travel time between the $l_0'$ and $Sl_m$
$Ts_i$	The time to arrive at $x_i$ for the $i$ th delivery task, $\forall i \in \{1, \dots, m\}$
$Te_i$	The time to arrive at $y_i$ for the $i$ th delivery task, $\forall i \in \{1, \dots, m\}$
$v_i^t, v_i^a, v_i^{ar}, v_i^c$	The node set of $G_t, G_a, G_{ar}, G_c$
$x_{ij}$	Binary decision variables: 1, if the rack $j$ is delivered in the $i$ task, otherwise, 0, $\forall i, j \in \{1, \dots, m\}$
$y_{iw}$	Binary decision variables: 1, if the rack $i$ stored in the $w$ location, otherwise, 0, $\forall i \in \{1, \dots, m\}, w \in \{1, \dots, m+n\}$
$\Gamma$	A set of position of all locations consisting of retrieval locations and storage locations
$\varepsilon_i^t, \varepsilon_i^a, \varepsilon_i^{ar}, \varepsilon_i^c$	The edge set of $G_t, G_a, G_{ar}, G_c$
$\delta$	The list of the number of perturbation locations in the upper level optimization

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**Appendix B The details of the preliminary experiment**

In the preliminary experiment, we test the performance of the transformation heuristic in the lower-level optimization of BiJSR, especially the influence of different re-identification strategies. To analyze the solution quality and runtime on the identified ATSP or ATSP-PC,  $m$  storage locations are preselected from the  $O_s$  or  $O_s \cup R$  to construct the lower-level optimization problem. At least one storage location should be selected from the retrieval location for the ATSP-PC. We randomly generate 20 cases in each instance with different storage locations for the identified ATSP or ATSP-PC. For the identified TSP, there exists one case in each instance since all storage locations are selected as retrieval locations.

Table S2 displays the comparison results between our LBH strategy and Gurobi solver on all instances of identified TSP. The  $Ave(\%)$  denotes the average deviation from the optimal solution on all 25\*1 cases with the same  $m$ , and the  $Best(\%)$  and  $Worst(\%)$  denote the best and worst deviation. The  $Count(\%)$  denotes the ratio that the number of instances reaching the optimal solution divided by all cases. As shown in this table, we can see that our proposed LBH can discover the optimal result with an average of 19.01 times less time cost on 122 instances. Although there is a slight deviation of about 0.02% for largest-sized instances, the average running time of LBH is approximately 21.34 times less than that of Gurobi.

**Table S2** Comparison results with respect to the Gurobi for the LBH algorithm

Ins.	Ave(%)	Best(%)	Worst(%)	Count(%)	T-opt(s)	T-LBH(s)
$m = 10$	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>100.00</b>	0.79	<b>0.03</b>
$m = 15$	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>100.00</b>	3.18	<b>0.17</b>
$m = 20$	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>100.00</b>	3.57	<b>0.42</b>
$m = 25$	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>100.00</b>	29.17	<b>1.62</b>
$m = 30$	0.02	<b>0.00</b>	0.28	88.00	52.91	<b>2.48</b>

For the identified ATSP and ATSP-PC problems, the number of cases in a group with the same  $m$  is 25\*20. Table S3 displays the comparison results on identified ATSP. In terms of the solution quality, the performance of the LBH using STR mode is better than that of RTS mode, especially in medium- and large-sized cases. Both re-identification modes have the same average running time, about 41.73 times less than Gurobi.

Table S4 summarizes the comparison results of identified ATSP-PC. Here the positive percentage deviation indicates that our algorithm has an optimality gap compared with the Gurobi, and the negative deviation indicates that our algorithm can explore a better result than it within the time limit imposed. The  $Count(\%)$  of LBH-C represents the ratio that cases whose deviation is zero or negative value. From the Table V, we can see that both modes have similar runtimes, while the solution quality of STR mode has a great advantage over RTS. The STR mode can find the same or better results in all cases, and the best solution obtained is 26.40% better than Gurobi. For the RTS mode, there is a deviation for small- and medium-sized instances, and the best deviation is 22.61%. In summary, our proposed lower level algorithm has excellent performance and the proposed STR mode is more effective than the RTS mode both in LBH-A and LBH-C.

**Table S3** Comparison results with respect to the Gurobi for the LBH-A algorithm

ATSP	Ave(%)		Best(%)		Worst(%)		Count(%)		T-opt(s)	Time(s)	
	RTS	STR	RTS	STR	RTS	STR	RTS	STR		RTS	STR
$m = 10$	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>100.00</b>	<b>100.00</b>	0.72	<b>0.01</b>	<b>0.01</b>
$m = 15$	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>100.00</b>	<b>100.00</b>	0.56	<b>0.02</b>	<b>0.02</b>
$m = 20$	0.12	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	1.43	<b>0.00</b>	87.20	<b>100.00</b>	1.13	<b>0.02</b>	<b>0.02</b>
$m = 25$	0.47	<b>0.02</b>	0.19	<b>0.00</b>	2.72	<b>0.48</b>	0.00	<b>94.40</b>	1.84	<b>0.05</b>	<b>0.05</b>
$m = 30$	0.81	<b>0.05</b>	0.37	<b>0.00</b>	3.67	<b>1.15</b>	0.00	<b>93.20</b>	2.01	<b>0.07</b>	<b>0.07</b>

**Table S4** Comparison results with respect to the Gurobi for the identified ATSP-PC problems

ATSP-PC	Ave(%)		Best(%)		Worst(%)		Count(%)		T-opt(s)	Time(s)	
	RTS	STR	RTS	STR	RTS	STR	RTS	STR		RTS	STR
$m = 10$	0.05	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	0.67	<b>0.00</b>	86.20	<b>100.00</b>	1763.21	<b>0.01</b>	0.02
$m = 15$	-0.05	<b>-0.24</b>	-3.18	<b>-3.38</b>	0.26	<b>0.00</b>	95.80	<b>100.00</b>	3600.00	<b>0.04</b>	<b>0.04</b>
$m = 20$	-1.92	<b>-6.26</b>	-4.79	<b>-9.27</b>	0.00	<b>-0.53</b>	<b>100.00</b>	<b>100.00</b>	3600.00	<b>0.08</b>	<b>0.08</b>
$m = 25$	-7.29	<b>-8.32</b>	-18.34	<b>-22.69</b>	-3.07	<b>-6.23</b>	<b>100.00</b>	<b>100.00</b>	3600.00	<b>0.13</b>	<b>0.13</b>
$m = 30$	-11.40	<b>-14.98</b>	-22.61	<b>-26.40</b>	-7.06	<b>-8.96</b>	<b>100.00</b>	<b>100.00</b>	3600.00	<b>0.21</b>	0.22

**Appendix C The details of the comparison experiments**

Table S5 to S7 display the comparison results among the Gurobi solver, the BiJSR and three comparison algorithms for small-, medium- and large-sized instances. In these tables,  $UB$  is the upper bound as reported by the Gurobi solver for the formulas (1)-(12), and the  $LB$  is larger value between the lower bound reported by the Gurobi solver and that solved by our proposed relaxation formulation. The  $LB$  values are shown as  $UB$  if the optimal problem is found. If the relaxation model cannot directly obtain the optimal solution within the time limit, its lower bound will be used as  $LB$ . The  $D_U(\%)$  and  $D_L(\%)$  denotes the deviation from the upper and lower bound, where the negative value indicates that the solution obtained by our method is better than the solver within the time limit.

From this table, we find that the BiJSR can discover the optimal solutions for all small-sized instances and better results on all medium- and large-sized instances. Specifically, the objective values obtained by our algorithm are an average of 6.08% on medium-sized instances and 13.88% on large-sized instances, better than that by the Gurobi solver within its time limit. In terms

of the runtime performance, the BiJSR can solve the RS-MTMS problem much more efficiently, especially in larger-sized instances. It can be seen that the computation time of BiJSR averaged about 7.14, 20.16, and 6.29 times less than that of Gurobi on small-, medium- and large-sized instances. Moreover, the average *Ave-L* is about 16.68% for large-sized instances. The result implies that the average gap of BiJSR to the optimal solution is moderate, which is no more than 16.68%.

**Table S5** Comparison results among the Gurobi solver, the BiJSR and three comparison algorithms for small-sized instances.

No.	Gurobi		BIJOR		PHA			ALNS		DWCA				
	UB	LB	Obj	$D_U(\%)$	$D_L(\%)$	Obj	$D_U(\%)$	$D_L(\%)$	Obj	$D_U(\%)$	$D_L(\%)$	Obj	$D_U(\%)$	$D_L(\%)$
1	328.31	UB	<b>328.31</b> ± 0.00	0.00	0.00	357.08 ± 0.00	8.76	8.76	331.69 ± 0.00	1.03	1.03	337.15 ± 1.56	2.69	2.69
2	357.08	UB	<b>357.08</b> ± 0.00	0.00	0.00	390.92 ± 0.00	9.48	9.48	368.42 ± 1.33	3.18	3.18	370.87 ± 2.00	3.86	3.86
3	299.54	UB	<b>299.54</b> ± 0.00	0.00	0.00	322.30 ± 3.44	7.60	7.60	301.23 ± 0.00	0.56	0.56	324.92 ± 0.00	8.47	8.47
4	353.69	UB	<b>353.69</b> ± 0.00	0.00	0.00	374.00 ± 0.00	5.74	5.74	373.11 ± 1.70	5.49	5.49	360.46 ± 3.95	1.91	1.91
5	328.31	UB	<b>328.31</b> ± 0.00	0.00	0.00	344.30 ± 2.28	4.87	4.87	335.08 ± 0.00	2.06	2.06	349.97 ± 0.80	6.60	6.60
6	272.46	UB	<b>272.46</b> ± 0.00	0.00	0.00	302.92 ± 0.00	11.18	11.18	272.46 ± 0.00	0.00	0.00	293.83 ± 4.75	7.84	7.84
7	314.77	UB	<b>314.77</b> ± 0.00	0.00	0.00	348.62 ± 0.00	10.75	10.75	327.46 ± 1.29	4.03	4.03	348.78 ± 6.18	10.81	10.81
8	353.69	UB	<b>353.69</b> ± 0.00	0.00	0.00	397.69 ± 0.00	12.44	12.44	356.95 ± 1.41	0.92	0.92	393.12 ± 3.54	11.15	11.15
9	338.46	UB	<b>338.46</b> ± 0.00	0.00	0.00	352.00 ± 0.00	4.00	4.00	345.99 ± 1.28	2.22	2.22	357.03 ± 3.49	5.49	5.49
10	326.62	UB	<b>326.62</b> ± 0.00	0.00	0.00	330.00 ± 0.00	1.04	1.04	332.24 ± 3.51	1.72	1.72	368.80 ± 8.44	12.91	12.91
11	314.77	UB	<b>314.77</b> ± 0.00	0.00	0.00	385.85 ± 0.00	22.58	22.58	318.32 ± 1.25	1.13	1.13	349.21 ± 6.40	10.94	10.94
12	387.54	UB	<b>387.54</b> ± 0.00	0.00	0.00	421.38 ± 0.00	8.73	8.73	387.54 ± 0.00	0.00	0.00	413.13 ± 6.77	6.60	6.60
13	311.38	UB	<b>311.38</b> ± 0.00	0.00	0.00	324.46 ± 1.95	4.20	4.20	314.77 ± 0.00	1.09	1.09	369.98 ± 2.31	18.82	18.82
14	330.00	UB	<b>330.00</b> ± 0.00	0.00	0.00	362.15 ± 0.00	9.74	9.74	331.69 ± 0.00	0.51	0.51	372.35 ± 3.32	12.83	12.83
15	316.46	UB	<b>316.46</b> ± 0.00	0.00	0.00	389.23 ± 0.00	22.99	22.99	318.15 ± 0.00	0.53	0.53	353.57 ± 5.12	11.72	11.72
16	313.08	UB	<b>313.08</b> ± 0.00	0.00	0.00	331.69 ± 0.00	5.95	5.95	319.80 ± 1.04	2.15	2.15	353.14 ± 4.45	12.80	12.80
17	294.46	UB	<b>294.46</b> ± 0.00	0.00	0.00	331.69 ± 0.00	12.64	12.64	301.23 ± 0.00	2.30	2.30	334.82 ± 0.40	13.71	13.71
18	365.54	UB	<b>365.54</b> ± 0.00	0.00	0.00	418.21 ± 3.73	14.41	14.41	365.54 ± 0.00	0.00	0.00	398.79 ± 2.28	9.10	9.10
19	374.00	UB	<b>374.00</b> ± 0.00	0.00	0.00	451.85 ± 0.00	20.81	20.81	374.00 ± 0.00	0.00	0.00	424.18 ± 0.83	13.42	13.42
20	346.92	UB	<b>346.92</b> ± 0.00	0.00	0.00	360.46 ± 0.00	3.90	3.90	349.12 ± 0.80	0.63	0.63	377.81 ± 3.62	8.90	8.90
21	328.31	UB	<b>328.31</b> ± 0.00	0.00	0.00	384.15 ± 0.00	17.01	17.01	331.86 ± 0.97	1.08	1.08	362.20 ± 3.10	10.32	10.32
22	304.62	UB	<b>304.62</b> ± 0.00	0.00	0.00	345.23 ± 0.00	13.33	13.33	306.31 ± 0.00	0.56	0.56	331.82 ± 4.98	8.93	8.93
23	402.77	UB	<b>402.77</b> ± 0.00	0.00	0.00	436.62 ± 0.00	8.40	8.40	404.46 ± 0.00	0.42	0.42	426.21 ± 5.94	5.82	5.82
24	370.62	UB	<b>370.62</b> ± 0.00	0.00	0.00	411.23 ± 0.00	10.96	10.96	370.62 ± 0.00	0.00	0.00	391.26 ± 2.25	5.57	5.57
25	314.77	UB	<b>314.77</b> ± 0.00	0.00	0.00	355.38 ± 0.00	12.90	12.90	320.10 ± 0.95	1.69	1.69	350.82 ± 1.00	11.45	11.45

For the three comparison algorithms, the proposed BiJSR performs significantly better than comparison algorithms in most instances and the advantage is further expanded when the size of the instance is larger. Specifically, the objective values obtained by BiJSR are on average 10.58%, 1.33% and 9.31% on small-sized, 10.26%, 6.31%, 12.58% on medium-sized and 12.98%, 7.17%, 11.56% on large-sized instances. To prove the stability of each algorithm's solution results, we use the Wilcoxon rank-sum test to evaluate the statistical performance. In terms of runtime, the ALNS and DWCA will terminate when they run out of the allowable 1800 seconds, and the computation time of the PHA is much more than that of BiJSR for the same instance. Specifically, the computation time of BiJSR averaged on all instances is about 12.15 times less than that of PHA and 20.77 times less than that of ALNS and DWCA.

**Table S6** Comparison results among the Gurobi solver, the BiJSR and three comparison algorithms for medium-sized instances.

No.	Gurobi		BIJOR			PHA			ALNS			DWCA		
	UB	LB	Obj	$D_U$ (%)	$D_L$ (%)	Obj	$D_U$ (%)	$D_L$ (%)	Obj	$D_U$ (%)	$D_L$ (%)	Obj	$D_U$ (%)	$D_L$ (%)
26	534.77	525.46	<b>534.35</b> ± 0.75	<b>-0.08</b>	<b>1.69</b>	556.77 ± 0.00	4.11	5.96	547.29 ± 1.91	2.34	4.15	564.09 ± 8.17	5.48	7.35
27	524.62	509.38	<b>523.35</b> ± 0.93	<b>-0.24</b>	<b>2.74</b>	548.31 ± 0.00	4.52	7.64	529.99 ± 1.11	1.02	4.04	564.38 ± 9.75	7.58	10.80
28	480.62	477.23	<b>474.35</b> ± 0.80	<b>-1.30</b>	<b>-0.60</b>	492.46 ± 0.00	2.46	3.19	483.41 ± 2.62	0.58	1.29	513.40 ± 7.77	6.82	7.58
29	443.38	439.15	<b>443.38</b> ± 0.00	<b>0.00</b>	<b>0.96</b>	490.98 ± 1.55	10.73	11.80	461.96 ± 1.56	4.19	5.19	482.14 ± 0.85	8.74	9.79
30	490.77	485.69	<b>486.79</b> ± 0.83	<b>-0.81</b>	<b>0.23</b>	516.15 ± 0.00	5.17	6.27	500.92 ± 0.00	2.07	3.14	526.05 ± 2.27	7.19	8.31
31	477.23	469.62	<b>477.23</b> ± 0.00	<b>0.00</b>	<b>1.62</b>	506.00 ± 0.00	6.03	7.75	497.12 ± 0.93	4.17	5.86	533.33 ± 9.89	11.76	13.57
32	494.15	479.77	<b>494.15</b> ± 0.00	<b>0.00</b>	<b>3.00</b>	548.31 ± 0.00	10.96	14.29	507.99 ± 1.68	2.80	5.88	566.29 ± 11.71	14.60	18.03
33	477.23	463.69	<b>476.64</b> ± 0.83	<b>-0.12</b>	<b>2.79</b>	529.69 ± 0.00	10.99	14.23	489.58 ± 0.80	2.59	5.58	506.85 ± 4.15	6.21	9.31
34	480.62	477.23	<b>477.23</b> ± 0.00	<b>-0.70</b>	<b>0.00</b>	509.38 ± 0.00	5.99	6.74	509.09 ± 6.30	5.92	6.68	552.67 ± 13.13	14.99	15.81
35	521.23	512.77	<b>517.76</b> ± 0.38	<b>-0.67</b>	<b>0.97</b>	600.77 ± 0.00	15.26	17.16	552.07 ± 8.32	5.92	7.67	558.04 ± 10.97	7.06	8.83
36	544.92	533.92	<b>542.89</b> ± 0.69	<b>-0.37</b>	<b>1.68</b>	565.23 ± 0.00	3.73	5.86	547.72 ± 2.18	0.51	2.58	564.64 ± 4.46	3.62	5.75
37	526.31	517.00	<b>524.62</b> ± 0.00	<b>-0.32</b>	<b>1.47</b>	541.54 ± 0.00	2.89	4.75	543.99 ± 2.02	3.36	5.22	599.08 ± 7.96	13.83	15.88
38	402.77	392.62	<b>402.77</b> ± 0.00	<b>0.00</b>	<b>2.59</b>	460.31 ± 0.00	14.29	17.24	410.85 ± 0.64	2.01	4.64	441.52 ± 3.98	9.62	12.46
39	434.92	424.77	<b>434.67</b> ± 0.62	<b>-0.06</b>	<b>2.33</b>	463.69 ± 0.00	6.61	9.16	438.35 ± 1.44	0.79	3.20	462.68 ± 5.04	6.38	8.92
40	495.00	488.23	<b>495.76</b> ± 0.38	<b>0.15</b>	<b>1.54</b>	577.08 ± 0.00	16.58	18.20	520.22 ± 4.66	5.09	6.55	531.77 ± 3.12	7.43	8.92
41	478.92	470.46	<b>474.18</b> ± 1.04	<b>-0.99</b>	<b>0.79</b>	493.52 ± 0.95	3.05	4.90	488.95 ± 1.49	2.09	3.93	524.87 ± 3.54	9.59	11.56
42	517.85	511.31	<b>514.46</b> ± 0.00	<b>-0.65</b>	<b>0.62</b>	531.38 ± 0.00	2.61	3.93	522.58 ± 1.56	0.92	2.21	568.57 ± 4.30	9.80	11.20
43	528.00	509.38	<b>527.75</b> ± 0.62	<b>-0.05</b>	<b>3.60</b>	561.85 ± 0.00	6.41	10.30	540.18 ± 1.83	2.31	6.05	546.40 ± 7.82	3.49	7.27
44	528.00	522.92	<b>528.00</b> ± 0.00	<b>0.00</b>	<b>0.97</b>	555.08 ± 0.00	5.13	6.15	554.82 ± 1.69	5.08	6.10	593.70 ± 10.42	12.44	13.54
45	460.31	455.23	<b>455.74</b> ± 1.11	<b>-0.99</b>	<b>0.11</b>	485.69 ± 0.00	5.51	6.69	484.89 ± 6.88	5.34	6.51	525.72 ± 14.69	14.21	15.48
46	502.62	494.15	<b>502.62</b> ± 0.00	<b>0.00</b>	<b>1.71</b>	522.92 ± 0.00	4.04	5.82	525.76 ± 5.65	4.60	6.40	523.26 ± 5.09	4.11	5.89
47	544.92	539.00	<b>542.98</b> ± 0.62	<b>-0.36</b>	<b>0.74</b>	561.85 ± 0.00	3.11	4.24	558.84 ± 1.66	2.55	3.68	602.59 ± 9.17	10.58	11.80
48	488.23	478.92	<b>488.40</b> ± 0.85	<b>0.03</b>	<b>1.98</b>	517.85 ± 0.00	6.07	8.13	505.32 ± 2.19	3.50	5.51	527.20 ± 4.38	7.98	10.08
49	538.15	529.69	<b>537.82</b> ± 0.69	<b>-0.06</b>	<b>1.53</b>	566.92 ± 0.00	5.35	7.03	543.27 ± 1.04	0.95	2.56	603.14 ± 3.74	12.08	13.87
50	499.23	490.77	<b>499.23</b> ± 0.00	<b>0.00</b>	<b>1.72</b>	526.31 ± 0.00	5.42	7.24	516.20 ± 3.22	3.40	5.18	532.36 ± 3.73	6.64	8.47
51	638.00	473.00	<b>494.66</b> ± 0.89	<b>-22.47</b>	<b>4.58</b>	608.38 ± 0.00	-4.64	28.62	589.43 ± 0.43	-7.61	24.62	691.39 ± 7.16	8.37	46.17
52	619.38	454.38	<b>481.97</b> ± 0.89	<b>-22.19</b>	<b>6.07</b>	634.62 ± 0.00	2.46	39.66	579.28 ± 2.37	-6.48	27.49	596.24 ± 10.72	-3.74	31.22
53	710.77	628.69	<b>673.45</b> ± 0.38	<b>-5.25</b>	<b>7.12</b>	707.38 ± 0.00	-0.48	12.52	705.57 ± 3.00	-0.73	12.23	761.24 ± 2.99	7.10	21.08
54	604.15	450.15	<b>486.37</b> ± 0.52	<b>-19.50</b>	<b>8.05</b>	654.08 ± 0.00	8.26	45.30	570.43 ± 0.74	-5.58	26.72	581.27 ± 4.51	-3.79	29.13
55	668.46	616.85	<b>642.32</b> ± 0.86	<b>-3.91</b>	<b>4.13</b>	673.54 ± 0.00	0.76	9.19	650.52 ± 1.54	-2.68	5.46	691.69 ± 10.38	3.47	12.13
56	627.85	499.23	<b>531.81</b> ± 1.08	<b>-15.30</b>	<b>6.53</b>	584.69 ± 0.00	-6.87	17.12	631.10 ± 0.74	0.52	26.42	573.35 ± 6.93	-8.68	14.85
57	797.08	610.08	<b>635.29</b> ± 1.01	<b>-20.30</b>	<b>4.13</b>	680.31 ± 0.00	-14.65	11.51	670.62 ± 7.42	-15.87	9.92	705.27 ± 5.98	-11.52	15.60
58	668.46	485.69	<b>522.50</b> ± 1.08	<b>-21.84</b>	<b>7.58</b>	632.08 ± 0.00	-5.44	30.14	612.53 ± 1.92	-8.37	26.11	672.48 ± 5.01	0.60	38.46
59	624.46	570.31	<b>602.38</b> ± 0.38	<b>-3.54</b>	<b>5.62</b>	665.08 ± 0.00	6.50	16.62	631.53 ± 3.31	1.13	10.73	672.02 ± 6.59	7.62	17.83
60	614.31	533.08	<b>551.95</b> ± 1.14	<b>-10.15</b>	<b>3.54</b>	630.38 ± 0.00	2.62	18.25	620.15 ± 0.72	0.95	16.33	574.37 ± 6.73	-6.50	7.75
61	646.46	537.31	<b>557.62</b> ± 1.40	<b>-13.74</b>	<b>3.78</b>	666.77 ± 0.00	3.14	24.09	637.83 ± 1.77	-1.34	18.71	660.89 ± 2.27	2.23	23.00
62	666.77	534.77	<b>561.59</b> ± 0.83	<b>-15.77</b>	<b>5.02</b>	617.69 ± 0.00	-7.36	15.51	642.15 ± 1.37	-3.69	20.08	668.46 ± 2.25	0.25	25.00
63	602.46	563.54	<b>583.85</b> ± 0.00	<b>-3.09</b>	<b>3.60</b>	627.85 ± 0.00	4.21	11.41	618.37 ± 6.82	2.64	9.73	693.59 ± 8.10	15.13	23.08
64	876.62	643.08	<b>669.90</b> ± 0.62	<b>-23.58</b>	<b>4.17</b>	734.46 ± 0.00	-16.22	14.21	707.34 ± 6.58	-19.31	9.99	796.32 ± 16.66	-9.16	23.83
65	624.46	501.77	<b>539.34</b> ± 1.11	<b>-13.63</b>	<b>7.49</b>	678.62 ± 0.00	8.67	35.24	581.90 ± 1.51	-6.82	15.97	651.58 ± 8.68	4.34	29.86
66	668.46	616.85	<b>642.23</b> ± 1.50	<b>-3.92</b>	<b>4.12</b>	660.00 ± 0.00	-1.27	7.00	661.90 ± 2.93	-0.98	7.30	692.49 ± 9.06	3.59	12.26
67	698.92	609.23	<b>659.92</b> ± 0.38	<b>-5.58</b>	<b>8.32</b>	698.92 ± 1.23	0.00	14.72	681.75 ± 2.41	-2.46	11.90	757.27 ± 9.42	8.35	24.30
68	600.77	474.69	<b>510.57</b> ± 1.24	<b>-15.01</b>	<b>7.56</b>	580.46 ± 0.00	-3.38	22.28	549.15 ± 0.00	-8.59	15.69	563.54 ± 3.04	-6.20	18.72
69	670.15	525.46	<b>563.12</b> ± 1.08	<b>-15.97</b>	<b>7.17</b>	665.08 ± 0.00	-0.76	26.57	615.11 ± 1.56	-8.21	17.06	619.17 ± 3.49	-7.61	17.83
70	702.31	635.46	<b>658.90</b> ± 0.83	<b>-6.18</b>	<b>3.69</b>	709.08 ± 0.00	0.96	11.58	680.22 ± 1.64	-3.14	7.04	726.21 ± 6.18	3.40	14.28
71	665.08	595.69	<b>617.52</b> ± 0.52	<b>-7.15</b>	<b>3.66</b>	676.92 ± 0.00	1.78	13.64	668.46 ± 0.87	0.51	12.22	752.32 ± 4.41	13.12	26.29
72	673.54	628.69	<b>648.15</b> ± 0.00	<b>-3.77</b>	<b>3.10</b>	734.46 ± 0.00	9.05	16.82	680.31 ± 0.00	1.01	8.21	726.13 ± 3.23	7.81	15.50
73	705.69	547.46	<b>584.78</b> ± 1.40	<b>-17.13</b>	<b>6.82</b>	692.15 ± 0.00	-1.92	26.43	661.73 ± 1.15	-6.23	20.87	748.34 ± 3.66	6.04	36.69
74	648.15	597.38	<b>619.55</b> ± 1.73	<b>-4.41</b>	<b>3.71</b>	658.31 ± 0.00	1.57	10.20	645.70 ± 6.51	-0.38	8.09	680.86 ± 4.23	5.05	13.97
75	622.77	569.46	<b>605.25</b> ± 0.83	<b>-2.81</b>	<b>6.29</b>	653.23 ± 0.00	4.89	14.71	646.84 ± 0.80	3.87	13.59	670.79 ± 7.61	7.71	17.79

**Table S7** Comparison results among the Gurobi solver, the BiJSR and three comparison algorithms for large-sized instances.

No.	Gurobi		BIJOR			PHA			ALNS			DWCA		
	UB	LB	Obj	$D_U$ (%)	$D_L$ (%)	Obj	$D_U$ (%)	$D_L$ (%)	Obj	$D_U$ (%)	$D_L$ (%)	Obj	$D_U$ (%)	$D_L$ (%)
76	859.69	765.77	<b>807.82 ± 1.38</b>	<b>-6.03</b>	<b>5.49</b>	877.46 ± 0.00	2.07	14.59	857.20 ± 1.72	-0.29	11.94	877.46 ± 0.00	3.40	16.08
77	910.46	751.38	<b>843.78 ± 1.59</b>	<b>-7.32</b>	<b>12.30</b>	941.77 ± 0.00	3.44	25.34	892.18 ± 1.18	-2.01	18.74	941.77 ± 0.00	5.98	28.41
78	795.38	632.92	<b>637.75 ± 0.83</b>	<b>-19.82</b>	<b>0.76</b>	774.23 ± 0.00	-2.66	22.33	761.41 ± 2.47	-4.27	20.30	774.23 ± 0.00	-11.23	11.55
79	930.77	716.69	<b>751.38 ± 3.06</b>	<b>-19.27</b>	<b>4.84</b>	846.15 ± 0.00	-9.09	18.06	785.23 ± 0.00	-15.64	9.56	846.15 ± 0.00	-8.36	19.01
80	827.54	671.85	<b>769.92 ± 0.67</b>	<b>-6.96</b>	<b>14.60</b>	865.62 ± 0.00	4.60	28.84	829.06 ± 3.27	0.18	23.40	865.62 ± 0.00	0.67	24.00
81	1001.85	778.46	<b>825.42 ± 4.28</b>	<b>-17.61</b>	<b>6.03</b>	883.38 ± 0.00	-11.82	13.48	846.96 ± 1.66	-15.46	8.80	883.38 ± 0.00	-3.88	23.71
82	785.23	605.85	<b>717.62 ± 1.40</b>	<b>-8.61</b>	<b>18.45</b>	816.54 ± 0.00	3.99	34.78	790.35 ± 1.66	0.65	30.45	816.54 ± 0.00	-1.79	27.29
83	912.15	802.15	<b>831.68 ± 1.16</b>	<b>-8.82</b>	<b>3.68</b>	922.31 ± 0.00	1.11	14.98	878.56 ± 2.68	-3.68	9.53	922.31 ± 0.00	8.16	23.00
84	832.62	705.69	<b>772.12 ± 1.08</b>	<b>-7.27</b>	<b>9.41</b>	885.08 ± 0.00	6.30	25.42	778.97 ± 3.19	-6.44	10.38	885.08 ± 0.00	13.19	33.55
85	793.69	572.85	<b>648.83 ± 0.52</b>	<b>-18.25</b>	<b>13.26</b>	764.92 ± 0.00	-3.62	33.53	794.20 ± 2.72	0.06	38.64	764.92 ± 0.00	-5.18	31.38
86	812.31	573.69	<b>750.12 ± 0.75</b>	<b>-7.66</b>	<b>30.75</b>	806.38 ± 0.00	-0.73	40.56	780.62 ± 2.46	-3.90	36.07	806.38 ± 0.00	11.23	57.50
87	795.38	666.77	<b>754.85 ± 1.02</b>	<b>-5.10</b>	<b>13.21</b>	820.13 ± 0.95	3.11	23.00	798.81 ± 2.68	0.43	19.80	820.13 ± 0.95	9.09	30.13
88	871.54	700.62	<b>798.09 ± 1.01</b>	<b>-8.43</b>	<b>13.91</b>	867.31 ± 0.00	-0.49	23.79	840.70 ± 1.59	-3.54	19.99	867.31 ± 0.00	4.72	30.27
89	807.23	659.15	<b>752.06 ± 0.76</b>	<b>-6.83</b>	<b>14.09</b>	853.77 ± 0.00	5.77	29.53	787.09 ± 1.36	-2.49	19.41	853.77 ± 0.00	0.59	23.19
90	817.38	678.62	<b>756.29 ± 3.34</b>	<b>-7.47</b>	<b>11.45</b>	846.15 ± 0.00	3.52	24.69	799.62 ± 0.95	-2.17	17.83	846.15 ± 0.00	6.46	28.23
91	832.62	629.54	<b>683.27 ± 0.93</b>	<b>-17.94</b>	<b>8.53</b>	812.31 ± 0.00	-2.44	29.03	819.71 ± 1.31	-1.55	30.21	812.31 ± 0.00	-7.18	22.76
92	783.54	696.38	<b>735.31 ± 0.87</b>	<b>-6.16</b>	<b>5.59</b>	820.77 ± 0.00	4.75	17.86	797.42 ± 4.37	1.77	14.51	820.77 ± 0.00	-2.68	9.50
93	905.38	708.23	<b>816.88 ± 0.97</b>	<b>-9.78</b>	<b>15.34</b>	897.77 ± 0.00	-0.84	26.76	871.71 ± 2.12	-3.72	23.08	897.77 ± 0.00	-6.17	19.95
94	940.92	685.38	<b>740.38 ± 1.16</b>	<b>-21.31</b>	<b>8.02</b>	801.31 ± 0.00	-14.84	16.91	782.48 ± 2.85	-16.84	14.17	801.31 ± 0.00	-19.34	10.73
95	825.85	673.54	<b>768.14 ± 2.26</b>	<b>-6.99</b>	<b>14.05</b>	844.46 ± 0.00	2.25	25.38	810.40 ± 0.91	-1.87	20.32	844.46 ± 0.00	-5.40	15.99
96	852.92	734.46	<b>796.23 ± 1.03</b>	<b>-6.65</b>	<b>8.41</b>	869.00 ± 0.00	1.88	18.32	849.50 ± 2.98	-0.40	15.66	869.00 ± 0.00	-0.88	15.11
97	903.69	706.54	<b>792.08 ± 1.78</b>	<b>-12.35</b>	<b>12.11</b>	885.92 ± 0.61	-1.97	25.39	838.50 ± 0.89	-7.21	18.68	885.92 ± 0.61	-2.96	24.12
98	820.77	648.15	<b>759.93 ± 2.09</b>	<b>-7.41</b>	<b>17.25</b>	840.23 ± 0.00	2.37	29.63	817.26 ± 2.59	-0.43	26.09	840.23 ± 0.00	5.49	33.59
99	847.85	688.77	<b>792.85 ± 1.94</b>	<b>-6.49</b>	<b>15.11</b>	870.69 ± 0.00	2.69	26.41	854.62 ± 2.39	0.80	24.08	870.69 ± 0.00	5.14	29.43
100	820.77	654.92	<b>778.46 ± 2.20</b>	<b>-5.15</b>	<b>18.86</b>	874.08 ± 0.00	6.49	33.46	819.08 ± 0.00	-0.21	25.06	874.08 ± 0.00	6.44	33.40
101	1100.00	775.92	<b>908.60 ± 3.00</b>	<b>-17.40</b>	<b>17.10</b>	1042.46 ± 0.00	-5.23	34.35	965.93 ± 1.56	-12.19	24.49	1060.65 ± 3.45	-3.58	36.70
102	1137.23	815.69	<b>937.28 ± 0.62</b>	<b>-17.58</b>	<b>14.91</b>	1062.77 ± 0.00	-6.55	30.29	1004.17 ± 5.08	-11.70	23.11	1083.63 ± 12.70	-4.71	32.85
103	1216.77	825.00	<b>974.01 ± 1.02</b>	<b>-19.95</b>	<b>18.06</b>	1113.54 ± 0.00	-8.48	34.97	1036.03 ± 4.69	-14.85	25.58	1074.32 ± 3.08	-11.71	30.22
104	1177.85	842.77	<b>973.75 ± 1.39</b>	<b>-17.33</b>	<b>15.54</b>	1105.92 ± 0.00	-6.11	31.22	1054.94 ± 4.13	-10.43	25.18	1098.52 ± 8.92	-6.73	30.35
105	1347.08	874.92	<b>991.52 ± 1.73</b>	<b>-26.39</b>	<b>13.33</b>	1118.62 ± 0.00	-16.96	27.85	1028.20 ± 4.12	-23.67	17.52	1114.13 ± 11.18	-17.29	27.34
106	1150.77	769.15	<b>902.00 ± 0.00</b>	<b>-21.62</b>	<b>17.27</b>	1044.15 ± 0.00	-9.26	35.75	962.97 ± 1.97	-16.32	25.20	1008.11 ± 2.34	-12.40	31.07
107	1098.31	804.69	<b>949.38 ± 1.55</b>	<b>-13.56</b>	<b>17.98</b>	1075.46 ± 1.23	-2.08	33.65	1017.46 ± 5.76	-7.36	26.44	1066.20 ± 13.14	-2.92	32.50
108	1235.38	890.15	<b>981.54 ± 0.00</b>	<b>-20.55</b>	<b>10.27</b>	1094.92 ± 0.00	-11.37	23.00	1056.13 ± 1.24	-14.51	18.65	1113.96 ± 8.09	-9.83	25.14
109	1162.62	886.77	<b>993.30 ± 0.38</b>	<b>-14.56</b>	<b>12.01</b>	1193.08 ± 0.00	2.62	34.54	1071.48 ± 3.07	-7.84	20.83	1107.95 ± 10.44	-4.70	24.94
110	1228.62	934.15	<b>1042.80 ± 1.70</b>	<b>-15.12</b>	<b>11.63</b>	1176.15 ± 0.00	-4.27	25.91	1093.99 ± 1.94	-10.96	17.11	1182.67 ± 4.48	-3.74	26.60
111	1196.46	773.38	<b>874.42 ± 2.70</b>	<b>-26.92</b>	<b>13.06</b>	1014.54 ± 0.00	-15.21	31.18	914.18 ± 2.38	-23.59	18.21	1020.21 ± 7.06	-14.73	31.91
112	1101.69	844.46	<b>972.91 ± 0.52</b>	<b>-11.69</b>	<b>15.21</b>	1124.54 ± 0.00	2.07	33.17	1035.31 ± 1.18	-6.03	22.60	1074.74 ± 4.40	-2.45	27.27
113	1057.69	789.46	<b>915.45 ± 1.28</b>	<b>-13.45</b>	<b>15.96</b>	1072.08 ± 0.00	1.36	35.80	982.05 ± 1.56	-7.15	24.39	971.64 ± 4.25	-8.14	23.08
114	1309.85	839.38	<b>953.45 ± 1.93</b>	<b>-27.21</b>	<b>13.59</b>	1067.00 ± 0.00	-18.54	27.12	1013.35 ± 1.11	-22.64	20.73	1040.09 ± 5.21	-20.59	23.91
115	1130.46	805.54	<b>910.21 ± 0.62</b>	<b>-19.48</b>	<b>12.99</b>	1059.38 ± 0.00	-6.29	31.51	969.95 ± 1.65	-14.20	20.41	1002.44 ± 9.92	-11.32	24.44
116	1018.77	762.38	<b>912.15 ± 0.00</b>	<b>-10.47</b>	<b>19.64</b>	1022.15 ± 0.00	0.33	34.07	981.96 ± 2.51	-3.61	28.80	1004.30 ± 5.63	-1.42	31.73
117	1096.62	782.69	<b>913.68 ± 0.52</b>	<b>-16.68</b>	<b>16.74</b>	1039.08 ± 0.00	-5.25	32.76	1001.72 ± 3.42	-8.65	27.98	1032.73 ± 10.50	-5.83	31.95
118	1157.54	877.46	<b>979.25 ± 0.83</b>	<b>-15.40</b>	<b>11.60</b>	1111.00 ± 0.00	-4.02	26.62	1062.09 ± 2.85	-8.25	21.04	1060.70 ± 13.51	-8.37	20.88
119	1100.00	797.08	<b>937.12 ± 2.39</b>	<b>-14.81</b>	<b>17.57</b>	1045.85 ± 0.00	-4.92	31.21	1000.70 ± 2.30	-9.03	25.55	1049.32 ± 6.63	-4.61	31.65
120	1071.23	799.62	<b>934.75 ± 3.81</b>	<b>-12.74</b>	<b>16.90</b>	1087.31 ± 0.00	1.50	35.98	1013.90 ± 2.21	-5.35	26.80	1032.31 ± 13.77	-3.63	29.10
121	1186.31	884.23	<b>975.53 ± 2.42</b>	<b>-17.77</b>	<b>10.33</b>	1132.15 ± 0.00	-4.56	28.04	1028.97 ± 3.33	-13.26	16.37	1063.49 ± 4.73	-10.35	20.27
122	1140.62	781.00	<b>913.51 ± 0.89</b>	<b>-19.91</b>	<b>16.97</b>	1044.15 ± 0.00	-8.46	33.69	953.19 ± 0.43	-16.43	22.05	998.17 ± 6.17	-12.49	27.81
123	996.77	782.69	<b>871.03 ± 0.80</b>	<b>-12.61</b>	<b>11.29</b>	977.31 ± 0.00	-1.95	24.86	950.02 ± 1.84	-4.69	21.38	999.18 ± 5.79	0.24	27.66
124	1191.38	835.15	<b>960.22 ± 2.22</b>	<b>-19.40</b>	<b>14.97</b>	1073.77 ± 0.00	-9.87	28.57	995.58 ± 0.43	-16.43	19.21	1046.99 ± 9.36	-12.12	25.36
125	1140.62	815.69	<b>962.84 ± 0.38</b>	<b>-15.59</b>	<b>18.04</b>	1104.23 ± 0.00	-3.19	35.37	1034.55 ± 1.72	-9.30	26.83	1041.83 ± 9.82	-8.66	27.72