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The capacity of *k*-connectivity *d*-dimensional wireless networks with node failure

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In large-scale wireless networks, network structure plays a critical role in the transport process of information, especially when wireless networks encounter node failure [1]. The capability of wireless networks to carry information reliably, efficiently and timely is directly influenced by the connectivity of network structures. Meanwhile, the deployment of wireless networks has expanded from plane to space. However, the impact of increasing the dimension on the network capacity of large-scale wireless networks is unclear. Therefore, this work primarily focuses on the relationship between network dimension and network capacity when the network structure encounters node failure. However, it is difficult to determine the exact network capacity of large-scale wireless networks due to the factors including a shared transmission medium, complex scheduling, interference, complex cooperation among nodes, and unpredictable failures. In light of this, an alternative method for analyzing the network capacity, which is related to the number of nodes, has been proposed to comprehend the capabilities of large-scale wireless networks [2]. Unfortunately, the results are only suitable for explaining the network capacity of wireless networks without node failure deployed in plane, instead of in three-dimensional space. To fill this gap, it is necessary to analyze the network capacity of wireless networks in d-dimension, especially for $d = 2$, 3. In general, the k-connectivity structure can be applied to combat node failure by adjusting the transmit power of nodes [3]. However, most studies only focus on the network capacity under simple connectivity ($k = 1$) without considering node failure [2, 4, 5]. Therefore, how node failure impacts the network capacity scaling law remains to be further investigated.

In this letter, we give the network capacity of k-connectivity wireless network, which is deployed in the hypercube $[0, 1]^d, d \geqslant 2$, when the network suffers from node failure. Specifically, the network capacity is revealed from three aspects: network dimensionality, the connectivity of network structure, and the intensity of initial node failures. It is shown that increasing network dimension can reduce the cost of network capacity to improve the robustness of network structure. This is because the consumption of valuable volume by each transmission is reduced. Meanwhile, supposing that the number of nodes is n and the number of the initial failure nodes is $m = n^{\frac{1}{\beta}}$, where $\beta\,(> 1)$ is the initial failure exponent, the effect of node failure on the network capacity is also quantified.

Notations. $f(n) = O(g(n))$ if there exist $c > 0$ and $n_0 > 0$ such that $f(n) < cg(n)$ for $n > n_0$. $f(n) = \Omega(g(n))$ if $g(n) = O(f(n))$. $f(n) = \Theta(g(n))$ if $f(n) = O(g(n))$ and $g(n) = O(f(n))$.

Node failure model. For a wireless network with a k-connectivity structure, n fixed nodes are distributed uniformly in the hypercube $[0, 1]^d, (d \geqslant 2)$ and randomly divided into $N(n) := \frac{n}{2}$ distinct source-destination (S-D) pairs. The locations of the source node and destination node of S-D pair i are denoted as $X_i^S = (x_{i,1}^S, x_{i,2}^S, \ldots, x_{i,d}^S)$ and $X_i^D = (x_{i,1}^D, x_{i,2}^D, \ldots, x_{i,d}^D)$, respectively. Each node can use the same critical transmission radius $r_d(n, k) = (\frac{\log n+(k-1)\log\log n-\log\Gamma(k)+\mu_n}{n})^{1/d}$ to generate the k-connectivity structure, where $\Gamma(k) = (k-1)!$, $k\,(\geqslant 1)$ is the connectivity parameter which measures the robustness of the network structure. When $\lim_{n\to+\infty}\mu_n = +\infty$, the probability that the k-connectivity structure is generated is asymptotic convergence one [3]. With a greater k, a stronger connectivity structure can be constructed, which has better resilience against node failure.

The initial failure nodes are uniformly distributed and the number of failure nodes is denoted as m. The relationship between n and m is $m = n^{\frac{1}{\beta}}$, where β is called as the initial failure exponent. Consequently, a smaller β indicates that more initial failure nodes exist. $\beta > 1$ ensures $m < n$. Otherwise, all the nodes cannot communicate with each other. The radius of the failure propagation is $r_d(n, k)$, i.e., all nodes lose the ability to relay data packets with probability one in the region $\cup_{X_j\in\Xi} O_j$, where $O_j = \{x|\ \rho(x, X_j) \leqslant r_d(n, k)\}$ is the failure region, where Ξ is the set of the initial failure nodes. $\rho(\cdot,\cdot)$ is the distance between two nodes in d-dimensional Euclidean space.

Network protocol. For any source-destination pair i, data packets are exchanged between the node X_i^S and the node X_i^D by using the multi-hop strategy. The detail is described

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as follows:

• Cell partition. Divide the hypercube $[0,1]^d, (d \geqslant 2)$ into cells. The side length of each cell is $a = \frac{r_d(n,k)}{2}$.

• Circumvented routing strategy. The S-D line of the S-D pair i is constructed in the order $X_i^S \rightarrow X_{i,1}^R \rightarrow X_{i,2}^R \rightarrow \cdots X_{i,j}^R \cdots \rightarrow X_{i,d-1}^R \rightarrow X_{i,d}^R = X_i^D$, where $X_{i,j}^R = (x_{i,1}^S, x_{i,2}^S, \ldots, x_{i,d-j}^S, x_{i,d-j+1}^D, \ldots, x_{i,d}^D)$, $j = 1, \ldots, d$, $i = 1, \ldots, N(n)$. Then, a source X_i^S delivers data packets to its destination X_i^D by hops along the adjacent cells lying on its S-D line. Especially, when the S-D line is blocked due to node failure, the circumvented routing strategy is used to rebuild the S-D line. To understand the transport process of data packets, an example is given in Appendix A.1 at $d = 2$.

In the above process, one single hop transmission is successful if and only if Condition 1 is satisfied. This condition ensures that there exists an interference-free schedule such that each cell becomes active regularly once in τ time slots. This ensures that other simultaneous transmission cells in the same time slot are interference-free.

Condition 1 (Protocol model for successful transport [4])**.** A transport between transmission node X_i and receive node X_j is successful if and only if $\rho(X_k, X_j) \geqslant (1+\triangle)\rho(X_i, X_j), k \neq j$ is satisfied for any other simultaneously transmission node X_k, where $\triangle\ (>0)$ is the interference parameter.

Definition 1 (Network capacity)**.** In d-dimensional Euclidean space, deploy a k-connectivity network in the unit hypercube $[0,1]^d$. Network capacity is defined by

$$S_d(n,m,k) = (1 - \varepsilon_d(n,m,k))N(n)T_d(n,m,k),$$

where $\varepsilon_d(n,m,k)$ is the fraction of unserved S-D pairs, $N(n)$ is the total number of S-D pairs and $T_d(n,m,k)$ (bps/Hz) is the feasible throughput of each S-D pair.

Condition 1 and Definition 1 are further explained in Appendixes A.2 and A.3, respectively.

Analysis of network capacity. We analyze the order of network capacity in d-dimensional Euclidean space. To this end, two preliminary propositions are provided. According to the node failure model, S-D pairs may not be serviced. The fraction of unserved S-D pairs, which is dependent on the number of nodes, number of initial failure nodes, and connectivity parameter, is given in Proposition 1.

Proposition 1. The order of the fraction of unserved S-D pairs is

$$\varepsilon_{d,0}(n,m,k) = O\left(\frac{(k\log n)^{\frac{d}{d-1}}}{n^{1-\beta^{-1}}}\right),\ d \geqslant 2,$$

where $\beta > 1$ and $n > k \geqslant 1$. The proof is given in Appendix B.1.

As the number of data paths carried by each cell is an essential factor limiting network capacity, Proposition 2 will provide the number of data paths carried by each cell. We define the non-failures cells around the failure regions as loaded cells and the others as regular cells. Compared with the regular cell, each loaded cell not only carries the traffic of its data paths but also carries the traffic of the re-build data paths.

Proposition 2. Each regular cell can carry at most $\varDelta = 2nda^{d-1}$ data paths and each loaded cell can carry at most $(8\zeta+1)\varDelta$ data paths w.h.p., where $\zeta = \log m + (k-1)\log\log m - \log\Gamma(k)$. The proof is given in Appendix B.2.

From Proposition 2, there are significant differences in the number of data paths carried by non-failures cells because the circumvented routing strategy is used to combat node failure. Based on Propositions 1 and 2, we obtain network capacity in Theorem 1.

Theorem 1. For a k-connectivity wireless network in d-dimensional Euclidean space, network capacity is given by

$$S_d(n,m,k) = \begin{cases} (1-\varepsilon_d(n,m,k))\left(\frac{n}{k\log n}\right)^{\frac{d-1}{d}}, & \beta > 2, \\ (1-\varepsilon_d(n,m,k))\frac{1}{k\log n}\left(\frac{n}{k\log n}\right)^{\frac{d-1}{d}}, & 1 < \beta \leqslant 2, \end{cases}$$

where

$$\varepsilon_d(n,m,k) = \begin{cases} O\left(\frac{(k\log n)^{\frac{d-1}{d}}}{n^{\frac{d-1}{d}-\beta^{-1}}}\right), & \beta > 2, \\ O\left(\frac{(k\log n)^{\frac{d}{d-1}}}{n^{1-\beta^{-1}}}\right), & 1 < \beta \leqslant 2. \end{cases}$$

The proof is given in Appendix B.3.

Remark 1. In the sense of order, the network capacity at $\beta > 2$ is $O(k\log n)$ times that at $1 < \beta \leqslant 2$. When $\beta > 2$, network capacity is $\Theta((\frac{n}{k\log n})^{\frac{d-1}{d}})$. The circumvented routing strategy is an effective strategy to rebuild data paths. Otherwise, network capacity is significantly degraded under $1 < \beta \leqslant 2$. This means that other strategies should be designed to combat node failure.

Remark 2. To improve the robustness of network structure, the consumed network capacity can be reduced by increasing the network dimension when the number of nodes is constant. This is because increasing the network dimension reduces the effective volume consumed of per transmission.

Conclusions. In this letter, the impact of network dimension, connectivity parameter, and the number of failure nodes on network capacity is quantified from the viewpoint of order. These results can provide some suggestions for designing the large-scale wireless networks with the ability to combat node failure.

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Supporting information Appendixes A and B. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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