

# Accurate and efficient planar near-field measurements: a new perspective from electromagnetic information theory

Junhao ZHENG<sup>1</sup>, Xiaoming CHEN<sup>1\*</sup>, Zhengpeng WANG<sup>2</sup>, Jianxing LI<sup>1</sup>,  
Juan CHEN<sup>1</sup>, Wei E. I. SHA<sup>3</sup> & Lixin GUO<sup>4</sup>

<sup>1</sup>School of Information and Communications Engineering, Xi'an Jiaotong University, Xi'an 710049, China;

<sup>2</sup>School of Electronic Information Engineering, Beihang University, Beijing 100191, China;

<sup>3</sup>College of Information Science and Electronic Engineering, Zhejiang University, Hangzhou 310027, China;

<sup>4</sup>School of Physics, Xidian University, Xi'an 710071, China

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Near-field measurement can obtain the far-field radiation pattern of the antenna under test (AUT) by performing near-field-to-far-field transformation (NFT) [1]. Generally, the minimum bounding for AUTs with different form-factors should be convex surfaces and the optimal bounding should be a spheroidal surface [2]. However, considering the practicality and complexity, planar near-field measurement is often used instead of the spheroidal one. In order to reduce the impact of the truncation errors, it is necessary to increase the reliable region of the near-field scanning plane, but the time cost will increase significantly. Hence, how to obtain the far-field AUT pattern with less sampling time and truncation errors is an important research topic.

It can be known that the sample characteristics distribution near the geometric center of the scanning plane is much denser than that near the edge of the scanning plane [2]. Consequently, the conventional half-wavelength sampling rule [1] is not theoretically optimal for planar near-field measurement resulting in a large amount of time. Besides, in order to reduce the time cost while maintaining far-field accuracy, a clustering and interpolation method was proposed to reconstruct the non-uniformly sampled near-field data and obtain the radiation pattern in [3,4]. What is more, the bandlimited signal extrapolation method was further applied in the Gerchberg-Papoulis (GP) algorithm, decreasing the truncation errors in reliable regions [5]. In this article, we cluster and interpolate the non-uniformly sampled initial dataset with a greater than half-wavelength interval on the near-field scanning plane, and obtain the AUT far-field pattern with reduced truncation error through an iterative algorithm. The measurement results demonstrate the effectiveness of the proposed method, and the measurement time cost is just one-third of that of the conventional planar near-field measurement.

*Sampling theorem.* The 2D Fourier transform of the planar near-field data is expressed as

$$P(k_x, k_y) = \iint E(x, y, d) e^{j(k_x x + k_y y + k_z d)} dx dy, \quad (1)$$

where  $E(x, y, d)$  is the planar  $E$ -field,  $P(k_x, k_y)$  is the plane-wave spectrum (PWS),  $d$  is the near-field scanning distance, and  $k_0 = 2\pi/\lambda_0 = \sqrt{k_x^2 + k_y^2 + k_z^2}$  is the wavenumber. If the scanning plane and the AUT aperture are marked as  $S_1$  and  $S_2$ , then the samples on  $S_1$  are densely distributed in the center and are sparsely distributed in the edge, and some data are not mapped onto  $S_1$  from  $S_2$ , causing data loss and truncation errors [2]. Therefore, the sampling interval near the edges of  $S_1$  can be sparser than that of the center [2], leading to a non-uniformly sampled initial dataset  $A_{\text{init}}$ , and an under-sampling scheme with sampling interval larger than  $\lambda_0/2$  can be used to reduce the time cost.

*Interpolation method.* Since the sampling interval is larger than  $\lambda_0/2$ , the under-sampled regions need to be supplemented by an appropriate method. K-means method can be used to cluster  $A_{\text{init}}$  into  $k$  clusters, and the cluster center  $c_j$  can be calculated as

$$c_j = \frac{1}{n_j} \sum_{i \in C_j} a_i, \quad (2)$$

where  $n_j$  is the number of the samples  $a_i$  in the  $j$ -th cluster  $C_j$ . Then the sum of squares of errors (SSE) of all the  $k$  clusters is obtained as

$$\text{SSE}_k = \sum_{j=1}^k \sum_{i \in C_j} \|a_i - c_j\|^2. \quad (3)$$

Changing the clustering number  $k$  and recalculating (2) and (3), and the optimal clustering number  $k$  can be

\* Corresponding author (email: xiaoming.chen@mail.xjtu.edu.cn)

acquired with a significant inflection point along the SSE-curve. Then, use the Voronoi cell classification to calculate the normalized cell area  $U(a_n)$  and gradient  $V(a_n)$  as

$$U(a_n) = \frac{u(a_n)}{u(a_1) + u(a_2) + \cdots + u(a_{N_{\text{samp}}})}, \quad (4)$$

$$V(a_n) = \frac{v(a_n)}{v(a_1) + v(a_2) + \cdots + v(a_{N_{\text{samp}}})}, \quad (5)$$

where  $u(a_n)$  is the cell area of each sample,  $v(a_n) = \sum_{m=1}^{M_{\text{samp}}} |\nabla E_{(a_n, a_m)}|$ ,  $N_{\text{samp}}$  is the number of the near-field samples,  $M_{\text{samp}}$  is the number of adjacent samples having common cell walls and vertices with  $a_n$ . Combining the two parameters with coefficients  $z_1$  and  $z_2$  ( $z_1 + z_2 = 1$ ),

$$Z(a_n) = z_1(1 + U(a_n)) + z_2(1 + V(a_n)). \quad (6)$$

Then a large  $Z(a_n)$  means 24 samples are added around  $a_n$  (deep interpolation), and small  $Z(a_n)$  only adds 8 samples (shallow interpolation). The interpolated dataset is noted as  $A_{\text{inter}}$ . Notably, another uniformly under-sampled dataset  $A_{\text{add}}$  is needed to supplement the additional samples to  $A_{\text{init}}$ , because  $A_{\text{init}}$  is non-uniformly sampled whose incomplete dataset is insufficient to achieve self-interpolation.

**Iterative algorithm.** GP algorithm is introduced to reduce the truncation error of  $A_{\text{inter}}$ . The reliable region in the wavenumber domain is [5]

$$\eta_0 = \left\{ \frac{k_x^2}{(k \sin \theta_x)^2} + \frac{k_y^2}{k^2} < \xi_x \right\} \cap \left\{ \frac{k_x^2}{k^2} + \frac{k_y^2}{(k \sin \theta_y)^2} < \xi_y \right\}, \quad (7)$$

where  $\xi_x$  and  $\xi_y$  are larger than one to take more wavenumber modes into account,  $\theta_x$  and  $\theta_y$  are the angles of the reliable region between  $S_1$  and  $S_2$  [3]. The data in  $\eta_0$  should be kept, and the filtered PWS can be derived as

$$F_1 = \begin{cases} 1, & (k_x, k_y \in \eta_0), \\ 0, & (k_x, k_y \notin \eta_0), \end{cases} \quad (8)$$

$$P_{(F_1)}^{n+1}(k_x, k_y) = F_1 P^0(k_x, k_y) + P^n(k_x, k_y)[1 - F_1], \quad (9)$$

where  $P^0(k_x, k_y)$  is the initial spectrum of  $A_{\text{inter}}$ ,  $n$  is the iterative times of the filtering. The  $E$ -field of the filtered PWS can be obtained as

$$E^{n+1}(x, y) = \frac{1}{4\pi^2} \iint P_{(F_1)}^{n+1}(k_x, k_y) e^{-j(k_x x + k_y y)} dk_x dk_y. \quad (10)$$

Then, the spatial domain filtering on the AUT aperture  $S_2$  can be defined as

$$F_2 = \begin{cases} 1, & (x, y \in S_2), \\ 0, & (x, y \notin S_2), \end{cases} \quad (11)$$

$$E_{(F_2)}^{n+1}(x, y) = F_2 E^{n+1}(x, y). \quad (12)$$

Consequently, the  $(n+1)$ -th filtered PWS is obtained by the Fourier transform of (12)

$$P^{n+1}(k_x, k_y) = \iint E_{(F_2)}^{n+1}(x, y) e^{j(k_x x + k_y y)} dx dy. \quad (13)$$

After several times of iteration of (9)–(13), the reliable region is enlarged and the planar near-field dataset after the GP algorithm is acquired as  $A_{\text{GP}}$ . Accordingly, the far-field pattern can be calculated as

$$E(\theta, \varphi) = j \frac{e^{-jk_r}}{2\pi r} k \cos \theta P^{n+1}(k_x, k_y). \quad (14)$$

In order to obtain the optimal iteration times for the far-field pattern  $E_{n_1}^1(\theta, \varphi)$ ,  $n_1 = 1, \dots, N_1$  of  $A_{\text{GP}}$ , a subset is

extracted from the interpolated dataset  $A_{\text{inter}}$ , and the far-field pattern of the subset after GP algorithm is  $E_{n_2}^2(\theta, \varphi)$ ,  $n_2 = 1, \dots, N_1$ . Then the energy difference is

$$E_{(n_1, n_2)} = \iint |E_{n_1}^1(\theta, \varphi) - E_{n_2}^2(\theta, \varphi)|^2 \sin \theta d\theta d\varphi. \quad (15)$$

Since the accurate components dominate  $E_{(n_1, n_2)}$  before the optimal termination time  $N_{\text{op}}$ , which is gradually surpassed by the erroneous components, we can find the minimum value of  $E_{(n_1, n_2)}$ , where the number  $n_1$  is the optimal iteration time  $N_{\text{op}}$ .

**Experiment results.** A mm-wave array antenna working at 29 GHz with gain of 21.0 dBi is measured with  $d = 3\lambda_0$  and  $S_1 = 0.16 \text{ m} \times 0.16 \text{ m}$ . The sampling intervals for  $A_{\text{init}}$  and  $A_{\text{add}}$  are  $0.6\lambda_0$  and  $0.7\lambda_0$ . A completely sampled dataset  $A_{\text{comp}}$  with intervals of  $0.3\lambda_0$  is obtained for comparison with the NFT pattern of  $A_{\text{GP}}$ . The experimental results are provided in Appendix B, and we give the comparisons of the final reconstructed pattern accuracy in  $E$ - and  $H$ -plane by the relative errors

$$D = \frac{\sum_{\theta=\theta_1}^{\theta_2} |E_{\text{theor}}(\theta, \varphi) - E_{\text{recon}}(\theta, \varphi)|^2}{\sum_{\theta=\theta_1}^{\theta_2} |E_{\text{theor}}(\theta, \varphi)|^2}, \quad (16)$$

where  $E_{\text{theor}}(\theta, \varphi)$  is the theoretical pattern,  $E_{\text{recon}}(\theta, \varphi)$  is the NFT pattern of  $A_{\text{GP}}$  or  $A_{\text{comp}}$ ,  $\theta_1 = -65^\circ$ ,  $\theta_2 = 65^\circ$ . The errors  $D$  between theoretical pattern and  $A_{\text{GP}}$  NFT pattern in  $E$ - and  $H$ -plane are calculated as 7.21% and 5.15%, while the values  $D$  between theoretical pattern and  $A_{\text{comp}}$  NFT pattern in  $E$ - and  $H$ -plane are 7.93% and 5.37%. Besides, the further theory of the proposed method is illustrated in Appendix A, and the sampling interval for the bandlimited signal is detailed discussed in Appendix C.

**Conclusion.** This letter introduces an interpolation and bandlimited signal extrapolation method to reconstruct the non-uniformly distributed samples with sampling intervals larger than  $\lambda_0/2$ . The proposed method saves the time cost compared with the conventional measurement (sampling interval smaller than  $\lambda_0/2$ ). Besides, the truncation errors are decreased, and the testing accuracy is improved. Therefore, the proposed method is accurate and efficient for planar near-field measurements of mm-wave antennas.

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**Supporting information** Appendixes A–C. The supporting information is available online at [info.scichina.com](http://info.scichina.com) and [link.springer.com](http://link.springer.com). The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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