

# Adaptive leader-following attitude consensus of multiple rigid body systems with resilience to communication link faults

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Distributed coordination in multi-agent systems has been widely utilized in attitude alignment of spacecraft and cooperative control of unmanned vehicles, which has attracted extensive attention from biology, physics, and engineering researchers [1]. With the growing complexity of tasks, an increase in space missions and engineering projects needs to be completed by the cooperative operation of multi-spacecraft or multi-robot systems. Thus, the attitude consensus of multiple rigid body systems (MRBSs) has received progressive attention from the control field.

In light of this, researchers studied the attitude consensus of MRBSs from two perspectives: leaderless attitude consensus and leader-following attitude consensus (LFAC). The leaderless attitude consensus problems aim to achieve the attitude consensus of all individuals through the information exchange of networked MRBSs [2]. In contrast, LFAC aims to design attitude control protocols for each RBS to reach the predetermined trajectory generated by an external system called the target system [3–6]. Cai et al. [3,4] demonstrated observer stability under the condition that the communication topology is static and connected, based on the attitude consensus between unknown parameter MRBSs. Liu et al. [5,6] further investigated observer stability in jointly connected switching networks under the assumption that communication link weights are piecewise constants.

In comparison with control laws that directly utilize neighboring agent state information [7], observer-based approaches transform MRBSs coordinate control problems into single RBS tracking control problems. Because this approach uses observer estimations directly to create control protocols, unquestionably, the performance of control protocols is highly related to the performance of observers.

Moreover, current studies revealed that achieving consensus or synchronization of multi-agent systems (MASs) using distributed information under specific graph conditions is feasible. Because agents are only allowed to exchange information with neighboring agents, unknown faults (actuator

failures, communication failures, etc.) may not be considered in the control law design, making the resilience control of MASs a critical problem.

Nonetheless, several methods have been proposed to dispose of the resilience problems in multi-agent systems. In particular, some researchers have modeled communication link faults (CLFs) as uncertain communication link weights and studied resilience problems [8,9]. Li et al. [8] assumed that each agent obtains relative information from its neighbors via ambiguous channels, which are ideal channels with unit transfer functions disturbed by additive uncertainties. Further, Chen et al. [9] studied these failed information transmission behaviors and proposed a novel CLF model that could cover the fault model used in [8]. However, current research on resilience problems is based on linear models, which cannot be trivially extended to MRBSs.

This study addresses the LFAC problem of MRBSs subject to CLFs. Specifically, we model CLFs as time-varying functions that affect the fixed communication link weights. By utilizing an adaptive approach, fully distributed control protocols that achieve the LFAC of MRBSs under CLFs are designed. In comparison with the highly related studies, the main contributions of this study are threefold. First, from the content point of view, this study is the first attempt to study the resilience problem of LFAC of MRBSs. Second, different from the existing studies whose results were obtained assuming that the weights of communication links are piecewise constants, communication link weights under CLFs pose a specific challenge that cannot be addressed by approaches in [5,6]. Here, fully distributed adaptive controllers are created to solve resilience problems, and two kinds of Lyapunov functions are constructed to prove the convergence of proposed control protocols. Finally, the pseudo-linear representation of the attitude kinematics of MRBSs is used to overcome the nonlinearity of MRBSs, whose representation allows the design of fully distributed control laws.

The attitude of RBSs to the inertial frame is represented

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using unit quaternions. MRBSs consisting of  $N$  agents are considered, and the attitude kinematics and dynamics of RBSs can be written as follows:

$$\dot{q}_i = \frac{1}{2} q_i \odot q(\omega_i), \quad (1a)$$

$$j_i \dot{\omega}_i = -\omega_i^\times j_i \omega_i + \tau_i, \quad i = 1, \dots, N, \quad (1b)$$

where  $q_i \in \mathbb{Q}_u$  is a unit quaternion representation of the attitude from the own reference system to the inherent reference system;  $\omega_i \in \mathbb{R}^3$  represents angular velocity; and  $j_i \in \mathbb{R}^{3 \times 3}$  and  $\tau_i \in \mathbb{R}^3$  represent the inertia tensor and control moment of the  $i$ th rigid body, respectively.

Presumably, the desired angular velocity  $\omega_0 \in \mathbb{R}^3$  and the attitude  $q_0 \in \mathbb{Q}_u$  of the leader rigid body frame  $\mathcal{B}_0$  to the inertial frame  $\mathcal{I}$  are generated by the following equations:  $\dot{\omega}_0 = S_0 \omega_0$  and  $\dot{q}_0 = \frac{1}{2} q_0 \odot q(\omega_0)$ , where  $S_0 \in \mathbb{R}^{3 \times 3}$  is a constant matrix.

**Assumption 1.**  $\mathcal{G}$  is an acyclic graph and contains a spanning tree with the leader as its root.

**Assumption 2.** The system  $\dot{\omega}_0 = S_0 \omega_0$  is marginally stable.

This study models CLFs as  $\bar{a}_{ij}(t)$  and  $\bar{g}_i(t)$  with unknown corrupted weights,  $\bar{a}_{ij}(t) = a_{ij} + \delta_{ij}^a(t)$ ,  $\bar{g}_i(t) = g_i + \delta_i^g(t)$ , where  $a_{ij}$  and  $g_i$  are the primary elements of adjacency matrix and matrices  $G(t)$ ;  $\delta_{ij}^a(t)$  and  $\delta_i^g(t)$  denote the unknown corrupted weights.

**Assumption 3.** The CLFs ( $\delta_{ij}^a(t)$ ,  $\delta_i^g(t)$ ) and their derivatives are bounded.

**Assumption 4.**  $\delta_{ij}^a(t)$  and  $\delta_i^g(t)$  do not change the adjacency weight  $\bar{a}_{ij}(t)$  into negative.

**Assumption 5.** There exist a positive constant  $v$  and a subsequence  $i_k$  of  $\mathbb{N}$  such that  $t_{i_{k+1}} - t_{i_k} < v$ . A spanning tree with the leader as its root is included in the union graph  $\bigcup_{j=i_k}^{i_{k+1}-1} \mathcal{G}_j(t_j)$ .

**Lemma 1.** If the digraph  $\mathcal{G}$  contains a spanning tree with the leader as its root at time  $t$ , then there exists a positive definition diagonal matrix  $Q(t)$  such that  $Q(t)L_G(t) + L_G^T(t)Q(t) = P(t)$ , where  $P(t)$  is positive-definite, and  $L_G(t) = L(t) + G(t)$ .

**Lemma 2.** If Assumptions 1–5 hold, Lemma 1 is satisfied by a positive definition diagonal matrix  $Q(t)$ . In the meanwhile,  $Q(t)$  and its derivative are bounded.

The leader attitude and angular velocity observers as  $\eta_i \in \mathbb{Q}_u$ ,  $\xi_i \in \mathbb{R}^3$  are defined. Then, the distributed leader state errors for each agent are defined as  $\Gamma_i = \sum_{j=1}^N \bar{a}_{ij}(\eta_i - \eta_j) + \bar{g}_i(\eta_i - q_0)$  and  $H_i = \sum_{j=1}^N \bar{a}_{ij}(\xi_i - \xi_j) + \bar{g}_i(\xi_i - \omega_0)$ .

The global form of angular errors can be rewritten as  $H = [H_1^T, \dots, H_N^T]^T$ . Furthermore, the global leader attitude observer error is defined as  $\tilde{\eta} = [\tilde{\eta}_1^T, \tilde{\eta}_2^T, \dots, \tilde{\eta}_N^T]^T = [\eta_1^T - q_0^T, \eta_2^T - q_0^T, \dots, \eta_N^T - q_0^T]^T$ . Therefore, the global leader attitude error can be expressed as  $\Gamma = (L_G(t) \otimes I_4)\tilde{\eta}$ . We can also obtain  $H = (L_G(t) \otimes I_3)\tilde{\xi}$ .

**Theorem 1.** Suppose that Assumptions 1–5 hold; all estimated state variables will converge to  $S_0$ ,  $\omega_0$ , and  $q_0$  by the proposed distributed leader state observers as the following forms:

$$\begin{aligned} \text{vec}(\dot{S}_i(t)) &= -\beta \text{sgn} \left( \sum_{j=0}^N \bar{a}_{ij}(t) [\text{vec}(S_i(t)) - \text{vec}(S_j(t))] \right) \\ &\quad - \alpha \sum_{j=0}^N \bar{a}_{ij}(t) [\text{vec}(S_i(t)) - \text{vec}(S_j(t))], \end{aligned} \quad (2a)$$

$$\dot{\xi}_i = S_i \xi_i - \left( \|H_i\|_2 + \int_0^t H_i^T H_i ds \right) H_i, \quad (2b)$$

$$\dot{\eta}_i = \frac{1}{2} \eta_i \odot q(\xi_i) - \left( \|\Gamma_i\|_2 + \int_0^t \Gamma_i^T \Gamma_i ds \right) \Gamma_i, \quad (2c)$$

where  $\alpha, \beta > 0$ , that is,  $\lim_{t \rightarrow \infty} \|S_i - S_0\| = 0$ ,  $\lim_{t \rightarrow \infty} \|\xi_i - \omega_0\| = 0$ ,  $\lim_{t \rightarrow \infty} \|\eta_i - q_0\| = 0$ .

The distributed forms of attitude and angular velocity errors between each follower and leader are defined as follows:  $\phi_i = \eta_i^* \odot q_i$ ,  $\tilde{\omega}_i = \omega_i - C(\phi_i)\xi_i$ , where  $\phi_i = \text{col}(\hat{\phi}_i, \bar{\phi}_i) \in \mathbb{Q}_u$  and  $\tilde{\omega}_i \in \mathbb{R}^3$ . Then, the variable  $x_i$  is defined as  $x_i = \tilde{\omega}_i + k_{i1}\hat{\phi}_i$ , where  $k_{i1}$  is a nonnegative constant. The control torque can be given as follows:

$$\begin{aligned} \tau_i &= \omega_i^\times j_i \omega_i - j_i((x_i - k_{i1}\hat{\phi}_i)^\times C(\phi_i)\xi_i - C(\phi_i)S_i \xi_i) \\ &\quad - \frac{1}{2} k_{i1} j_i (\hat{\phi}_i^\times + \bar{\phi}_i I_3)(x_i - k_{i1}\hat{\phi}_i) - k_{i2} x_i, \end{aligned} \quad (3)$$

where  $k_{i2}$  is a positive constant.

**Theorem 2.** Given a leader-following system and a time-varying digraph with CLFs, under Assumptions 1–5, the distributed consensus of MRBSs is solvable by a fully distributed control law consisting of (2) and (3).

The mathematical preliminaries and necessary symbol interpretations are included in Appendix A. The proofs of Theorems 1 and 2 are included in Appendix B, respectively. The proposed results are illustrated by a numerical simulation in Appendix D. The proofs of Lemmas 1 and 2 are included in Appendix E. Some necessary remarks are embodied in the supplementary material.

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**Supporting information** Appendixes A–E. The supporting information is available online at [info.scichina.com](http://info.scichina.com) and [link.springer.com](http://link.springer.com). The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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