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Adaptive leader-following attitude consensus of multiple rigid body systems with resilience to communication link faults

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Appendix A Mathematical preliminaries

A digraph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ contains a finite set of nodes $\mathcal{V} = \{0, 1, \dots, N\}$ with node 0 as the leader system and an edge set $\mathcal{E} \in \mathcal{V}\times\mathcal{V}$. The node i is the neighbor of node j if there exists an edge $\mathcal{E}$ from node i to node j denoted by (i, j). Thus $\mathcal{N}_i = \{j|(j, i) \in \mathcal{E}\}$ denotes the neighbor set of node i. The weighted adjacency matrix $A = [a_{ij}]$, with $a_{ij} > 0$ if and only if $(v_i, v_j) \in \mathcal{E}$ and $a_{ij} = 0$, otherwise. The information flow from node i to node j is captured by a subsequence of edges satisfying $\{(m_i, m_k), (m_k, m_l), \dots, (m_v, m_j)\}$. If information flows from one node to another, then a digraph is said to have a spanning tree. The matrix $G = diag\{g_i\}$ $(i = 1, \dots, N)$ denotes the influence of the leader received by other nodes, and $g_i > 0$ if agent i can receive information from node 0. The definition of degree matric $\Delta(\mathcal{G})$ of $\mathcal{G}$ is as follows $\Delta(\mathcal{G}) = diag(\, a_{ii})$, where $a_{ii} = \sum_{j=1,\, j\neq i}^{N} a_{ij}, i = 1,\ 2,\ \cdots,\ N$. The Laplacian matrix of $\mathcal{G}$ is defined as $L = \Delta(\mathcal{G}) - A$. L_G is obtained by removing the 1st row and the 1st column of Laplacian associated with $\mathcal{G}$. Obviously, we can get such an equation $L_G = L + G$, where L is the Laplacian of subgraph $\bar{\mathcal{G}}$ that only includes followers and the matrix $G = diag\{g_i\}$. The Kronecker product operation is denoted by $\otimes$. The n-dimensional Euclidean space is denoted by $\mathbb{R}^n$. The set of $m \times n$ dimensional real matrices is denoted by $\mathbb{R}^{m\times n}$. The set of all unit quaternions is denoted by $\mathbb{Q}_u$. Leader body and the ith rigid body frames are denoted by $\mathcal{B}_k,\ k = 0, 1, \dots, N$. The matrix function $(\cdot)^\times : \mathbb{R}^3 \mapsto \mathbb{R}^{3\times3}$ is defined as follows: For each $m = col(m_1, m_2, m_3) \in \mathbb{R}^3$,

$$(m)^\times = \begin{bmatrix} 0 & -m_3 & m_2 \\ m_3 & 0 & -m_1 \\ -m_2 & m_1 & 0 \end{bmatrix}.$$

Euclidean norm of a vector or a matrix is denoted by $\|\cdot\|$. $\odot$ denotes the quaternion product, for $q_i, q_j \in \mathbb{Q}_u$,

$$q_i \odot q_j = \begin{bmatrix} \bar{q}_i\hat{q}_j + \bar{q}_j\hat{q}_i + \hat{q}_i^\times\hat{q}_j \\ \bar{q}_i\bar{q}_j - \hat{q}_i^T\hat{q}_j \end{bmatrix}.$$

In addition, q^{-1} denotes quaternion inverse, for $q \in \mathbb{Q}_u$, $q^{-1} = col(-\hat{q}, \bar{q})$. The vector function $q(\cdot)$ is defined as follows: For each $m = col(m_1, m_2, m_3) \in \mathbb{R}^3$, $q(m) = col(m, 0) \in \mathbb{R}^4$. The quaternion function $C(\cdot)$ is defined as: For $\lambda \in \mathbb{Q}_u$, $C(\lambda) = 2\hat{\lambda}\hat{\lambda}^T - 2\bar{\lambda}\hat{\lambda}^\times + (\bar{\lambda}^2 - \hat{\lambda}^T\hat{\lambda})I_3$.

Appendix B Proof of Theorem 1

First, we need to state some properties for later proof. Under the Lemma 1, $L_G(t)$ must be an invertible matrix. Then, we can obtain the following inequality

$$\|\tilde{\eta}\| \leqslant \frac{\|\Gamma\|}{\sigma_{min}(L_G(t) \otimes I_4)}, \tag{B1}$$

we can also have

$$\|\tilde{\xi}\| \leqslant \frac{\|H\|}{\sigma_{min}(L_G(t) \otimes I_3)}, \tag{B2}$$

where $\tilde{\xi} = [\tilde{\xi}_1^{\mathrm{T}}, \tilde{\xi}_2^{\mathrm{T}}, \dots, \tilde{\xi}_N^{\mathrm{T}}] = [\xi_1^{\mathrm{T}} - \omega_0^{\mathrm{T}}, \xi_2^{\mathrm{T}} - \omega_0^{\mathrm{T}}, \dots, \xi_N^{\mathrm{T}} - \omega_0^{\mathrm{T}}]^{\mathrm{T}}$, $H = (L_G(t) \otimes I_3)\tilde{\xi}$.

Proof. The communication topology of MRBSs is time-varying due to the existence of CLFs. This means that at some times the system may have a directed spanning tree with the leader as the root node, and at other times the whole system may be divided into several independent subsystems. In order to include communication topology at each time, we consider two situations. In *Case 1*, we study the error variation of state observers when the digraph has a spanning tree. In *Case 2*, we consider all agents to be divided into subgraphs under CLFs. We design two Lyapunov candidate functions to obtain the observed error properties in *Case 1* and *Case 2*.

Case 1: The digraph $\mathcal{G}$ contains a spanning tree with the leader as its root.

First, let a_{ξ_i} denotes $\int_0^t H_i^{\mathrm{T}} H_i\, ds$ and $\dot{a}_{\xi_i}$ denotes $\|H_i\|^2$. From the definition of $\tilde{\xi}_i$, we have that

$$\dot{\tilde{\xi}}_i = S_i\tilde{\xi}_i - (a_{\xi i} + \dot{a}_{\xi i})H_i. \tag{B3}$$

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From $H=(L_G(t)\otimes I_3)\tilde{\xi}$ and (B3), one obtains that

$$\dot{H}=(\dot{L}_G(t)\otimes I_n)\tilde{\xi}+\{\mathcal{S}-L_G(t)(A_\xi+\dot{A}_\xi)\otimes I_3\}H, \tag{B4}$$

where $\mathcal{S}=$ block diag$\{S_1,S_2,\ldots,S_N\}$, $\dot{A}_\xi=\mathrm{diag}(\dot{a}_{\xi i})$, $A_\xi=\mathrm{diag}(a_{\xi i})$. According to Assumption 3, $\dot{L}_G(t)=\dot{L}(t)+\dot{G}(t)$ must be bounded. $\mathcal{S}$ is bounded because of Assumption 2.

Consider the following Lyapunov function

$$V_H=\sum_{i=1}^{N}(\dot{a}_{\xi i}+2a_{\xi i})a_i(t)H_i^{\mathrm{T}}H_i+\sum_{i=1}^{N}(a_{\xi i}-\alpha_H)^2, \tag{B5}$$

where $a_i(t)$ is defined in (E2), α_H is a constant that will be designed later. We can derive the derivative of V_H as

$$\begin{aligned}\dot{V}_H=&\,2H^{\mathrm{T}}(\dot{A}_\xi Q(t)\otimes I_3)H+H^{\mathrm{T}}[(\dot{A}_\xi+2A_\xi)\dot{Q}_i(t)\otimes I_3]H\\&+4H^{\mathrm{T}}\{(\dot{A}_\xi+A_\xi)Q(t)\otimes I_3\}\{(\dot{L}_G(t)\otimes I_3)\tilde{\xi}\\&+[\mathcal{S}-L_G(t)(A_\xi+\dot{A}_\xi)\otimes I_3]H\}\\&+2H^{\mathrm{T}}[(A_\xi-\alpha_H I_N)\otimes I_3]H.\end{aligned} \tag{B6}$$

Considering Lemma 1, Lemma 2 and the inequality of matrix norm ($\|A+B\|\leqslant\|A\|+\|B\|$, where $A,B\in\mathbb{R}^{m\times n}$), we have that $\|P(t)\|\leqslant 2\|Q(t)\|\|L_G(t)\|$. So we can further obtain

$$\begin{aligned}\dot{V}_H\leqslant&-2\lambda_0H^{\mathrm{T}}[(\dot{A}_\xi+A_\xi)^2\otimes I_3]H+\lambda_1H^{\mathrm{T}}(\dot{A}_\xi\otimes I_3)H\\&+\lambda_3H^{\mathrm{T}}[(\dot{A}_\xi+A_\xi)\otimes I_3]H+\lambda_2H^{\mathrm{T}}[(\dot{A}_\xi+2A_\xi)\otimes I_3]H\\&+4H^{\mathrm{T}}[(\dot{A}_\xi+A_\xi)Q(t)\dot{L}_G(t)\otimes I_3]\tilde{\xi}+2H^{\mathrm{T}}(A_\xi\otimes I_3)H\\&-2\alpha_HH^{\mathrm{T}}H,\end{aligned} \tag{B7}$$

where $\lambda_0=min_{\forall t\geqslant 0}\sigma_{min}\left(P(t)\right)$, $\lambda_1=max_{\forall t\geqslant0}\ 2\|Q(t)\|_F$, $\lambda_2=max_{\forall t\geqslant0}\ \|\dot{Q}(t)\|_F$, and $\lambda_3=max_{i,\forall t\geqslant0}\ 2\|S_i\|_F\lambda_1$. Note that λ_i $(i=0,1,2,3)$ are positive and bounded constants. Considering Young inequality, and performing identity transformation on part of inequality, we can obtain

$$\begin{aligned}\lambda_1H^{\mathrm{T}}(\dot{A}_\xi\otimes I_3)H=&\,\frac{\sqrt{2}\sqrt{\lambda_0}}{\sqrt{\lambda_0}\sqrt{2}}\lambda_1H^{\mathrm{T}}(\dot{A}_\xi\otimes I_3)H\\\leqslant&\,\frac{\lambda_1^2}{\lambda_0}H^{\mathrm{T}}H+\frac{\lambda_0}{4}H^{\mathrm{T}}(\dot{A}_\xi^2\otimes I_3)H.\end{aligned} \tag{B8}$$

Similarly, one has

$$\begin{aligned}\lambda_2H^{\mathrm{T}}[(\dot{A}_\xi+2A_\xi)\otimes I_3]H\leqslant&\,\frac{4\lambda_2^2}{\lambda_0}H^{\mathrm{T}}H+\frac{\lambda_0}{4}H^{\mathrm{T}}(A_\xi^2\otimes I_3)H\\&+\frac{\lambda_0}{4}H^{\mathrm{T}}(\dot{A}_\xi^2\otimes I_3)H+\frac{\lambda_2^2}{\lambda_0}H^{\mathrm{T}}H,\end{aligned} \tag{B9}$$

$$\begin{aligned}\lambda_3H^{\mathrm{T}}[(\dot{A}_\xi+A_\xi)\otimes I_3]H\leqslant&\,\frac{\lambda_0}{4}H^{\mathrm{T}}((\dot{A}_\xi+A_\xi)^2\otimes I_3)H\\&+\frac{\lambda_3^2}{\lambda_0}H^{\mathrm{T}}H,\end{aligned} \tag{B10}$$

$$2H^{\mathrm{T}}(A_\xi\otimes I_3)H\leqslant\frac{\lambda_0}{4}H^{\mathrm{T}}(A_\xi^2\otimes I_3)H+\frac{4}{\lambda_0}H^{\mathrm{T}}H. \tag{B11}$$

As for $4H^{\mathrm{T}}[(\dot{A}_\xi+A_\xi)Q(t)\dot{L}_G(t)\otimes I_3]\tilde{\xi}$, notice that $(\dot{A}_\xi+A)^2\leqslant 2(\dot{A}_\xi^2+A^2)$, and $\|\tilde{\xi}\|\leqslant\frac{\|H\|}{\sigma_{min}(L_G(t)\otimes I_3)}$, we can obtain that

$$\begin{aligned}4H^{\mathrm{T}}[(\dot{A}_\xi+A_\xi)Q(t)\dot{L}_G(t)\otimes I_3]\tilde{\xi}\leqslant&\,\frac{\lambda_0}{4}H^{\mathrm{T}}[(\dot{A}_\xi^2+A_\xi^2)\otimes I_3]H\\&+\frac{2\lambda_4^2H^{\mathrm{T}}H}{\lambda_0\sigma_{min}^2(L_G(t)\otimes I_3)},\end{aligned} \tag{B12}$$

with $\lambda_4=max_{\forall t\geqslant0}\ 4\|Q(t)\dot{L}_G(t)\|_F$. Then, we recombine the inequality by extracting common factors

$$\begin{aligned}\dot{V}_H\leqslant&-2\lambda_0H^{\mathrm{T}}[(\dot{A}_\xi+A_\xi)^2\otimes I_3]H+\frac{\lambda_0}{4}H^{\mathrm{T}}[(\dot{A}_\xi+A_\xi)^2\otimes I_3]H\\&+\frac{3}{4}\lambda_0H^{\mathrm{T}}[(\dot{A}_\xi^2+A_\xi^2)\otimes I_3]H+\lambda_{sum}H^{\mathrm{T}}H-2\alpha_HH^{\mathrm{T}}H,\end{aligned} \tag{B13}$$

where $\lambda_{sum}=(4+\lambda_1^2+5\lambda_2^2+\lambda_3^2+\frac{\lambda_4^2}{\sigma_{min}^2(L_G(t)\otimes I_3)})/\lambda_0$. Because of the boundedness of λ_i, $Q(t)$ and $\dot{L}_G(t)$, obviously λ_{sum} is bounded. Then we can chose the constant α_H satisfying $\alpha_H\geqslant\frac{1}{2}\lambda_{sum}$ and define a certain constant λ_{L0} which satisfies $0<\lambda_{L0}<\lambda_0$. Finally, (B13) changes into

$$\dot{V}_H\leqslant-\lambda_{L0}H^{\mathrm{T}}[(\dot{A}_\xi+A_\xi)^2\otimes I_3]H, \tag{B14}$$

which illustrates that all the variables in ξ_i including H_i, $a_{\xi i}$ and $\dot{a}_{\xi i}$ are bounded.

To prove the convergence of the leader angular state, we solve (B14) as

$$\begin{aligned} V_H(t) - V_H(0) &\leqslant -\int_0^t \lambda_{L0} H^{\mathrm{T}}[(\dot{A}_\xi + A_\xi)^2 \otimes I_3] H d\tau \\ &\leqslant -\int_0^t \lambda_{L0} H^{\mathrm{T}}(A_\xi \otimes I_3) H d\tau. \end{aligned} \tag{B15}$$

By combining (B5), (B15) is changed to

$$a_{min} a_{\xi min} H^{\mathrm{T}} H \leqslant -\lambda_{L0} a_{\xi min} \int_0^t H^{\mathrm{T}} H d\tau + V_H(0), \tag{B16}$$

with $a_{min} = min_{\forall t \geqslant 0}\{a_{i(t)} \| i = 1, \ldots, N\}$ and $a_{\xi min} = min_{\forall t \geqslant 0}\{a_{\xi i(t)} \| i = 1, \ldots, N\}$. This means that $\int_0^t H^{\mathrm{T}} H d\tau$ is bounded. According to Barbalat's Lemma, we can get that $H^{\mathrm{T}} H$ converges to zero. Therefore, by (B1), the leader angular observer globally converges to the defined angular state ω_0.

Case 2:

The CLFs cause some edges of the communication network topology $\mathcal{G}(t)$ to be disconnected at some time instants, so that the communication network topology $\mathcal{G}(t)$ does not contain a spanning tree. It is easy to get that $H(t) = -\big(L_G(t) \otimes I_3\big)\tilde{\xi}(t) = \mathbf{0}$ and $\dot{V}_H = 0$ if $L_G(t) = \mathbf{0}$, which meets the goal of the first step that $H(t) \to \mathbf{0}$ when $t \to \infty$. Therefore, the case that the graph $\mathcal{G}$ loses all its edges at some time instants will not be considered in the following analysis.

First, we divide the communication network topology $\mathcal{G}(t)$ into some connected subgraphs, where the no-leader subgraphs contain zero in-degree nodes. Without loss of generality, suppose that the directed graph $\mathcal{G}(t)$ is divided into c connected subgraphs $\bar{\mathcal{G}}_k$, where any two subgraphs are disconnected, and there exist m $(1 \leqslant m \leqslant N)$ zero in-degree nodes. Let $\bar{\mathcal{N}}_0$ denote the set of all zero in-degree nodes, and $\bar{\mathcal{N}}_k$ denote the set of non-zero in-degree nodes in connected subgraph $\bar{\mathcal{G}}_k$ $(k = 1, \ldots, c)$. Then, we reorder the labels of all followers as follows

$$\begin{aligned} &\bar{\mathcal{N}}_0 = \left\{1, \ldots, \bar{N}_0\right\}, \\ &\bar{\mathcal{N}}_1 = \left\{\bar{N}_0 + 1, \ldots, \bar{N}_0 + \bar{N}_1\right\}, \ldots, \\ &\bar{\mathcal{N}}_c = \left\{N - \bar{N}_c + 1, \ldots, N\right\}, \end{aligned}$$

with $\bar{N}_k$ being the cardinality of $\bar{\mathcal{N}}_k$ $(k = 0, 1, \ldots, c)$. Then, $L_G(t)$ can be rewritten as

$$L_G(t) = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ L_{\mathrm{r}1}(t) & L_{\bar{\mathcal{G}}_1}(t) & \mathbf{0} & \mathbf{0} \\ \vdots & \mathbf{0} & \ddots & \mathbf{0} \\ L_{\mathrm{r}c}(t) & \mathbf{0} & \mathbf{0} & L_{\bar{\mathcal{G}}_c}(t) \end{bmatrix}, \tag{B17}$$

where $L_{\bar{\mathcal{G}}_k}(t)$ denotes the Laplacian matrix of the kth connected subgraph except the rows of zero in-degree nodes, and $L_{\mathrm{r}k}(t)$ $(k = 1, \ldots, c)$ are non-zero matrices with appropriate dimensions. Then, $\forall i \in \bar{\mathcal{N}}_k (k = 0, 1, \ldots, c)$, denote the concatenated errors of $\tilde{\xi}_i$ and H_i as X_k and Y_k, respectively. In light of the definition of H_i, we have H_i $(i \in \bar{\mathcal{N}}_0)$ of the zero in-degree nodes are zero vectors, that is $Y_0 = \mathbf{0}$, and those of the non-zero in-degree nodes satisfy that $Y_k(t) = -\big(L_{\mathrm{r}k}(t) \otimes I_3\big)X_0(t) - \big(L_{\bar{\mathcal{G}}_k}(t) \otimes I_3\big)X_k(t)$, $k = 1, \ldots, c$. Since the initial graph $\mathcal{G}$ contains a spanning tree, without loss of generality, suppose that $\mathcal{G}(t)$ changes at $t = t_1$, that is for $t \in [t_0, t_1)$, $L_G(t)$ is a nonsingular M-matrix, and when $t = t_1$, the graph $\mathcal{G}(t)$ changes to *Case 2* from *Case 1*, $L_G(t)$ is shown as (B17). According to $\dot{V}_H \leqslant 0$ in the analysis of *Case 1* and $\dot{X}_0 = \mathbf{0}$ at *Case 2*, it is easy to get that X_0 is bounded at *Case 2*. Moreover, under Assumption 5, though some edges will be disconnected after experiencing CLFs, based on the above Laplacian matrix (B17), $L_{\bar{\mathcal{G}}_k}(t)$ is still a nonsingular M-matrix $(k = 1, \ldots, c)$. Then, we have

$$\|X_k\| \leqslant \frac{\|Y_k\| + \|(L_{\mathrm{r}k}(t) \otimes I_3)X_0\|}{\sigma_{\min}\big(L_{\bar{\mathcal{G}}_k}(t) \otimes I_3\big)}, \ k = 1, \ldots, c. \tag{B18}$$

In light of the definitions of X_k and Y_k, $k = 0, 1, \ldots, c$, we have $\dot{Y}_0 = 0$, and

$$\begin{aligned} \dot{Y}_k = &- \Big(L_{\bar{\mathcal{G}}_k}(t)(A_{\xi\bar{\mathcal{G}}_k} + \dot{A}_{\xi\bar{\mathcal{G}}_k}) \otimes I_3\Big) Y_k \\ &+ \Big(\dot{L}_{\mathrm{r}k}(t) \otimes I_3\Big) X_0 + \Big(\dot{L}_{\bar{\mathcal{G}}_k}(t) \otimes I_3\Big) X_k, \end{aligned} \tag{B19}$$

where $k = 1, \ldots, c$, $A_{\xi\bar{\mathcal{G}}_k}$ represent the block diagonal matrices of $a_{\xi i}$ for $i \in \bar{\mathcal{N}}_k$.

To simplify the analysis process, here we consider a new Lyapunov function as follow

$$V_{H_2} = \sum_{i=1}^{N} (2a_{\xi i} + \dot{a}_{\xi i}) H_i^T H_i + \sum_{i=1}^{N} (a_{\xi i} - \alpha_H)^2, \tag{B20}$$

where $\alpha_H > 0$ will be determined later.

The time derivative of V_{H_2} is calculated as follow

$$\dot{V}_{H_2} = 4\sum_{i=1}^{N} (a_{\xi i} + \dot{a}_{\xi i}) H_i^T \dot{H}_i + 2\sum_{i=1}^{N} \dot{a}_{\xi i} H_i^T H_i + 2\sum_{i=1}^{N} (a_{\xi i} - \alpha_H) \dot{a}_{\xi i}. \tag{B21}$$

Then, in light of the definitions of Y_k with $Y_0 = \dot{Y}_0 = \mathbf{0}$, we have

$$\begin{aligned}\dot{V}_{H_2} = \sum_{k=1}^{c} \Big(& 4Y_k^T\big((A_{\xi\bar{\mathcal{G}}_k} + \dot{A}_{\xi\bar{\mathcal{G}}_k}) \otimes I_3\big)\dot{Y}_k + 2Y_k^T(\dot{A}_{\xi\bar{\mathcal{G}}_k} \otimes I_3)Y_k \\ & + 2Y_k^T(A_{\xi\bar{\mathcal{G}}_k} \otimes I_3)Y_k - 2\alpha_H Y_k^T Y_k\Big).\end{aligned} \tag{B22}$$

Substituting (B19) into (B22) results that

$$\begin{aligned}\dot{V}_{H_2} = \sum_{k=1}^{c} \Big(& -4Y_k^T\big((A_{\xi\bar{\mathcal{G}}_k} + \dot{A}_{\xi\bar{\mathcal{G}}_k})L_{\bar{\mathcal{G}}_k}(t)(A_{\xi\bar{\mathcal{G}}_k} + \dot{A}_{\xi\bar{\mathcal{G}}_k}) \otimes I_3\big)Y_k \\ & + 4Y_k^T\big((A_{\xi\bar{\mathcal{G}}_k} + \dot{A}_{\xi\bar{\mathcal{G}}_k})\dot{L}_{\mathrm{r}k}(t) \otimes I_3\big)X_0 \\ & + 4Y_k^T\big((A_{\xi\bar{\mathcal{G}}_k} + \dot{A}_{\xi\bar{\mathcal{G}}_k})\dot{L}_{\bar{\mathcal{G}}_k}(t) \otimes I_3\big)X_k \\ & + 2Y_k^T\big((A_{\xi\bar{\mathcal{G}}_k} + \dot{A}_{\xi\bar{\mathcal{G}}_k}) \otimes I_3\big)Y_k - 2\alpha_H Y_k^T Y_k\Big).\end{aligned} \tag{B23}$$

Denote that $\bar{\sigma}_{2k} = \min_{\forall t \geqslant t_1}\left\{\sigma_{\min}\big(L_{\bar{\mathcal{G}}_k}(t) \otimes I_3\big)\right\}$, $\bar{\sigma}_{3k} = \max_{\forall t \geqslant t_1}\left\{\left\|\dot{L}_{\bar{\mathcal{G}}_k}(t)\right\|\right\}$, $\bar{\sigma}_{4k} = \max_{\forall t \geqslant t_1}\left\{\left\|\dot{L}_{\mathrm{r}k}(t)\right\|\right\}$, based on the Young inequality, one has

$$\begin{aligned}\dot{V}_{H_2} \leqslant \sum_{k=1}^{c} \Big(& -4\bar{\sigma}_{2k}Y_k^T\big((A_{\xi\bar{\mathcal{G}}_k} + \dot{A}_{\xi\bar{\mathcal{G}}_k})^2 \otimes I_3\big)Y_k \\ & + 4\bar{\sigma}_{4k}\left\|Y_k^T\big((A_{\xi\bar{\mathcal{G}}_k} + \dot{A}_{\xi\bar{\mathcal{G}}_k}) \otimes I_3\big)\right\| \|X_0\| \\ & + 4\bar{\sigma}_{3k}\left\|Y_k^T\big((A_{\xi\bar{\mathcal{G}}_k} + \dot{A}_{\xi\bar{\mathcal{G}}_k}) \otimes I_3\big)\right\| \|X_k\| \\ & + 2\left\|Y_k^T\big((A_{\xi\bar{\mathcal{G}}_k} + \dot{A}_{\xi\bar{\mathcal{G}}_k}) \otimes I_3\big)\right\| \|Y_k\| - 2\alpha_H Y_k^T Y_k\Big).\end{aligned} \tag{B24}$$

Furthermore, according to the inequality (B18) and Young inequality, and denoting that $\bar{\sigma}_{5k} = \max_{\forall t \geqslant t_1}\{\|L_{\mathrm{r}k}(t)\|\}$, we have

$$\begin{aligned}\dot{V}_{H_2} \leqslant & \sum_{k=1}^{c} \Big(-4\bar{\sigma}_{2k}Y_k^T\big((A_{\xi\bar{\mathcal{G}}_k} + \dot{A}_{\xi\bar{\mathcal{G}}_k})^2 \otimes I_3\big)Y_k \\ & + 4\bar{\sigma}_{4k}\left\|Y_k^T\big((A_{\xi\bar{\mathcal{G}}_k} + \dot{A}_{\xi\bar{\mathcal{G}}_k}) \otimes I_3\big)\right\| \|X_0\| \\ & + \frac{4\bar{\sigma}_{3k}}{\bar{\sigma}_{2k}}\left\|Y_k^T\big((A_{\xi\bar{\mathcal{G}}_k} + \dot{A}_{\xi\bar{\mathcal{G}}_k}) \otimes I_3\big)\right\| \|Y_k\| \\ & + \frac{4\bar{\sigma}_{3k}\bar{\sigma}_{5k}}{\bar{\sigma}_{2k}}\left\|Y_k^T\big((A_{\xi\bar{\mathcal{G}}_k} + \dot{A}_{\xi\bar{\mathcal{G}}_k}) \otimes I_3\big)\right\| \|X_0\| \\ & + 2\left\|Y_k^T\big((A_{\xi\bar{\mathcal{G}}_k} + \dot{A}_{\xi\bar{\mathcal{G}}_k}) \otimes I_3\big)\right\| \|Y_k\| - 2\alpha_H Y_k^T Y_k\Big) \\ \leqslant & \sum_{k=1}^{c} \Big(-2\bar{\sigma}_{2k}Y_k^T\big((A_{\xi\bar{\mathcal{G}}_k} + \dot{A}_{\xi\bar{\mathcal{G}}_k})^2 \otimes I_3\big)Y_k \\ & + 4\bar{\sigma}_{4k}\left\|Y_k^T\big((A_{\xi\bar{\mathcal{G}}_k} + \dot{A}_{\xi\bar{\mathcal{G}}_k}) \otimes I_3\big)\right\| \|X_0\| \\ & + \frac{4\bar{\sigma}_{3k}\bar{\sigma}_{5k}}{\bar{\sigma}_{2k}}\left\|Y_k^T\big((A_{\xi\bar{\mathcal{G}}_k} + \dot{A}_{\xi\bar{\mathcal{G}}_k}) \otimes I_3\big)\right\| \|X_0\| \\ & + \frac{4\bar{\sigma}_{3k}^2}{\bar{\sigma}_{2k}^3}Y_k^T Y_k + \frac{1}{\bar{\sigma}_{2k}}Y_k^T Y_k - 2\alpha_H Y_k^T Y_k\Big).\end{aligned} \tag{B25}$$

Then, choosing an enough large α_H as $\alpha_H > \frac{2\bar{\sigma}_{3k}^2}{\bar{\sigma}_{2k}^3} + \frac{1}{2\bar{\sigma}_{2k}}$, we have

$$\begin{aligned}\dot{V}_{H_2} \leqslant & \sum_{k=1}^{c} \left\|Y_k^T\big((A_{\xi\bar{\mathcal{G}}_k} + \dot{A}_{\xi\bar{\mathcal{G}}_k}) \otimes I_3\big)\right\| \\ & \times \Big(-2\bar{\sigma}_{2k}\left\|\big((A_{\xi\bar{\mathcal{G}}_k} + \dot{A}_{\xi\bar{\mathcal{G}}_k}) \otimes I_3\big)Y_k\right\| + \beta_1\Big) \\ \leqslant & \sum_{k=1}^{c} \left\|Y_k^T\big((A_{\xi\bar{\mathcal{G}}_k} + \dot{A}_{\xi\bar{\mathcal{G}}_k}) \otimes I_3\big)\right\| \\ & \times \Big(\beta_1 - \beta_2\left\|\big((A_{\xi\bar{\mathcal{G}}_k} + \dot{A}_{\xi\bar{\mathcal{G}}_k}) \otimes I_3\big)Y_k\right\|_1\Big),\end{aligned} \tag{B26}$$

where the inequality $\|Y_k\| \leqslant \|Y_k\|_1 \leqslant \sqrt{3\bar{N}_k}\,\|Y_k\|$ has been used, and $\beta_1 = \max_{k=1,\ldots,c}\left\{\left(\frac{4\bar{\sigma}_{3k}\bar{\sigma}_{5k}}{\bar{\sigma}_{2k}} + 4\bar{\sigma}_{4k}\right)\|X_0\|\right\}$, and $\beta_2 = \min_{k=1,\ldots,c}\left\{\frac{2\bar{\sigma}_{2k}}{\sqrt{3\bar{N}_k}}\right\}$. Then, we have

$$\begin{aligned}\dot{V}_{H_2} \leqslant & \sum_{k=1}^{c} \left\|Y_k^T\big((A_{\xi\bar{\mathcal{G}}_k} + \dot{A}_{\xi\bar{\mathcal{G}}_k}) \otimes I_3\big)\right\| \\ & \times \Big(\beta_1 - \beta_2\sum_{i\in\bar{\mathcal{N}}_k}(a_{\xi i} + \dot{a}_{\xi i})\|H_i\|_1\Big).\end{aligned} \tag{B27}$$

Furthermore, we will prove that $\forall i \in \bar{\mathcal{N}}_k$ $(k = 1, \dots, c)$, $a_{\xi i} + \dot{a}_{\xi i}$ are bounded, and $\|H_i\|_1 \to 0$ as $t \to \infty$. By contradiction, assume that there is an agent i such that $a_{\xi i} + \dot{a}_{\xi i} \to \infty$ and $\|H_i\|_1 \neq 0$ as $t \to \infty$, that is there exists a sequence $\{T_j\}$ $(j = 1, 2, 3, \dots)$ of time-values, $T_j \to \infty$ as $j \to \infty$, such that $\|H_i(T_j)\|_1 > H_{\min}$, with $H_{\min}$ being an arbitrary positive constant. Then, according to the definition of $a_{\xi i}$, we know that $a_{\xi i}$ is strictly monotonically increasing over time if and only if $\|H_i\|_1 \neq 0$. Then, since β_1 and β_2 are positive and bounded, there must be a time instant $T^* > T_1$ such that for $T_j > T^*$, $\beta_2(a_{\xi i} + \dot{a}_{\xi i})H_{\min} > c\beta_1$. However, according to (B27), one has $\dot{V}_{H_2}(T_j) \leqslant 0$, $\forall T_j > T^*$. Based on the Lyapunov function (B20), this implies that H_i, $a_{\xi i}$ and $\dot{a}_{\xi i}$ are bounded. Since $a_{\xi i} = \int_0^t H_i^T H_i d\tau$ is bounded, by Barbalat's Lemma, one can obtain that H_i converges to zero, which contradicts with $a_{\xi i} + \dot{a}_{\xi i} \to \infty$ and $\|H_i\|_1 \neq 0$ as $t \to \infty$. Thus, we can get that $\forall i \in \bar{\mathcal{N}}_k$ $(k = 1, \dots, c)$, $a_{\xi i} + \dot{a}_{\xi i}$ are bounded, and $\|H_i\|_1 \to 0$ as $t \to \infty$. Moreover, together with $H_i = \mathbf{0}$, $i \in \bar{\mathcal{N}}_0$, one has

$$\lim_{t\to\infty} H(t) = \lim_{t\to\infty} \big(L_G(t) \otimes I_3\big)\tilde{\xi}(t) = \mathbf{0}.$$

At the second step, we will prove that $\tilde{\xi}(t) = \mathbf{0}$ when $t \to \infty$. According to the definition of ξ_i, we have

$$\lim_{t\to\infty} \big(L_G(t + T_0) \otimes I_3\big)\tilde{\xi}(t) = \mathbf{0}. \tag{B28}$$

Denote that $t = t_{i_k}$ and $t + T_0 = t_{i_k+j}$, $j = 0, 1, \dots, (i_{k+1} - 1 - i_k)$, in light of (B28), one has

$$\lim_{k\to\infty} \big(L_G(t_{i_k+j}) \otimes I_3\big)\tilde{\xi}(t_{i_k}) = \mathbf{0}. \tag{B29}$$

Let $J_k = \sum_{q=i_k}^{i_{k+1}-1} \big(L_G(t_q) \otimes I_3\big)$. From (B29), one has

$$\lim_{k\to\infty} J_k\tilde{\xi}(t_{i_k}) = \mathbf{0}. \tag{B30}$$

Based on the Assumption 5, it is obvious that $\forall k = 1, 2, 3, \dots$, J_k is nonsingular, then we have

$$\lim_{k\to\infty} \tilde{\xi}(t_{i_k}) = \mathbf{0}, \tag{B31}$$

that is $\lim_{t\to\infty} \xi_i(t) \to \omega_0$, $i \in \mathcal{N}$. This complete the proof of the convergence of leader angular velocity observers.

Then, we are ready to analysis the leader attitude observers. First, let $a_{\eta i}$ denotes $\int_0^t \Gamma_i^{\mathrm{T}}\Gamma_i\, ds$ and $\dot{a}_{\eta i}$ denotes $\|\Gamma_i\|^2$, from the definition of $\tilde{\eta}_i$, we have that

$$\dot{\tilde{\eta}}_i = \frac{1}{2}\tilde{\eta}_i \odot q(\omega_0) - (a_{\eta i} + \dot{a}_{\eta i})\Gamma_i + \frac{1}{2}\eta_i \odot q(\tilde{\xi}_i). \tag{B32}$$

To obtain a compact form of (B32), define a matrix operator $M(\cdot) : \mathbb{R}^3 \to \mathbb{R}^{4\times4}$, such that for each $\xi_i = \mathrm{col}(\xi_{i1}, \xi_{i2}, \xi_{i3}) \in \mathbb{R}^3$

$$M(\xi_i) = \begin{bmatrix} 0 & \xi_{i3} & -\xi_{i2} & \xi_{i1} \\ -\xi_{i3} & 0 & \xi_{i1} & \xi_{i2} \\ \xi_{i2} & -\xi_{i1} & 0 & \xi_{i3} \\ -\xi_{i1} & -\xi_{i2} & -\xi_{i3} & 0 \end{bmatrix}, \tag{B33}$$

then the system (B32) is equivalent to

$$\dot{\tilde{\eta}}_i = \frac{1}{2}M(\omega_0)\tilde{\eta}_i - (a_{\eta i} + \dot{a}_{\eta i})\Gamma_i + \frac{1}{2}M(\tilde{\xi}_i)\eta_i. \tag{B34}$$

According to $\lim_{t\to\infty} \xi_i(t) \to \omega_0$, we can have after a long enough time, (B34) can be regarded as

$$\lim_{t\to\infty} \dot{\tilde{\eta}}_i = \frac{1}{2}M(\omega_0)\tilde{\eta}_i - (a_{\eta i} + \dot{a}_{\eta i})\Gamma_i. \tag{B35}$$

It is obvious that the dynamic of (B35) is the same as (B3). Thus, we can obtain the convergence of η_i by the same analysis step of ξ_i.

This completes the proof.

Remark 1. Notice that, as for S_0 observers, we do not use the adaptive method, because S_0 is a constant matrix and its dynamic accords with the first order integral model. We chose a simple observer to reduce the computational burden for each agent without losing performance.

Remark 2. The S_0 observer is another form of [5], whose proof can be extracted from the main result of Theorem 3.1 in [5]. Due to the limited space, the detailed proof is omitted. Because $S_i(t)$ converges to zero in a finite time, by using the certainty equivalence principle, $S_i(t)$ replaces the $S_0(t)$ required for observers in [4], so the proposed observers can be called fully distributed observers.

Appendix C Proof of Theorem 2

Proof. We define the distributed forms of attitude and angular velocity errors between each follower and leader as follows:

$$\epsilon_i = q_0^{-1} \odot q_i, \tag{C1a}$$

$$\breve{\omega}_i = \omega_i - C(\epsilon_i)\omega_0, \quad i = i, \dots, N, \tag{C1b}$$

with $\epsilon_i = col(\hat{\epsilon_i}, \bar{\epsilon_i}) \in \mathbb{Q}_u$ and $\breve{\omega}_i \in \mathbb{R}^3$, whose kinematics and dynamics are described by

$$\dot{\epsilon_i} = \frac{1}{2}\epsilon_i \odot q(\breve{\omega}_i), \tag{C2a}$$

$$j_i\dot{\breve{\omega}}_i = -\omega_i^{\times} j_i\omega_i + j_i(\breve{\omega}_i^{\times} C(\epsilon_i)\omega_0 - C(\epsilon_i)\dot{\omega}_0) + \tau_i, \tag{C2b}$$

where τ_i is the control protocol. Theorem 1 enables us to use certainty equivalence principle to analysis the control torque τ_i. First, let ξ_i and η_i substitute for the error signals ϵ_i and $\breve{\omega}_i$ between each agent and leader as defined in (C2). Referring to the error signals form of [1, 2]:

$$\phi_i = \eta_i^* \odot q_i, \tag{C3a}$$

$$\tilde{\omega}_i = \omega_i - C(\phi_i)\xi_i, \quad i = 1, \dots, N, \tag{C3b}$$

where $\phi_i = col(\hat{\phi}_i, \bar{\phi}_i) \in \mathbb{Q}_u$ and $\tilde{\omega}_i \in \mathbb{R}^3$. We will construct a control torque τ_i depending on ϕ_i and $\tilde{\omega}_i$ for each agent. The set of τ_i $(i = 1, \dots, N)$, together with the observers, forms the overall distributed control laws. First, we derive (C3) as following equations:

$$\dot{\phi}_i = \frac{1}{2}\phi_i \odot q(\tilde{\omega}_i) + \phi_{oi}, \tag{C4a}$$

$$j_i\dot{\tilde{\omega}}_i = -\omega_i^{\times} j_i\omega_i + j_i(\tilde{\omega}_i^{\times} C(\phi_i)\xi_i - C(\phi_i)S_i\xi_i) + j_{oi} + \tau_i, \tag{C4b}$$

where $\phi_{oi} \in \mathbb{Q}_u$ and $j_{oi} \in \mathbb{R}^3$ are given by

$$\phi_{oi} = \frac{1}{2}(\phi_i^{\mathrm{T}}\phi_i - 1)q(\xi_i) \odot \phi_i - (a_{\eta i} + \dot{a}_{\eta i})\Gamma_i^* \odot q_i, \tag{C5a}$$

$$j_{oi} = j_i(-C(\phi_i)(\dot{a}_{\xi i} + a_{\xi i})H_i + C_{oi}\xi_i), \tag{C5b}$$

with $C_{oi} = 2\bar{\phi}_i\bar{\phi}_{oi}I_3 - 2\hat{\phi}_i^{\mathrm{T}}\hat{\phi}_{oi}I_3 + 2\hat{\phi}_i\hat{\phi}_{oi}^{\mathrm{T}} + 2\hat{\phi}_{oi}\hat{\phi}_i^{\mathrm{T}} - 2\bar{\phi}_{oi}\hat{\phi}_i^{\times} - 2\bar{\phi}_i\hat{\phi}_{oi}^{\times}$.

As in [1,2], we introduce a temporary variable to analyze the system (C4). The variable x_i is defined as follows, $x_i = \tilde{\omega}_i + k_{i1}\hat{\phi}_i$, where k_{i1} is a nonnegative constant. Then we have

$$\dot{\hat{\phi}}_i = \frac{1}{2}(\hat{\phi}_i^{\times} + \bar{\phi}_i I_3)(x_i - k_{i1}\hat{\phi}_i) + \hat{\phi}_{oi}, \tag{C6a}$$

$$\dot{\bar{\phi}}_i = -\frac{1}{2}\hat{\phi}_i^{\mathrm{T}}(x_i - k_{i1}\hat{\phi}_i) + \bar{\phi}_{oi}, \tag{C6b}$$

$$\begin{aligned} j_i\dot{x}_i = {} & -\omega_i^{\times} j_i\omega_i + j_i((x_i - k_{i1}\hat{\phi}_i)^{\times} C(\phi_i)\xi_i - C(\phi_i)S_i\xi_i) \\ & + \frac{1}{2}k_{i1}j_i(\hat{\phi}_i^{\times} + \bar{\phi}_i I_3)(x_i - k_{i1}\hat{\phi}_i) + j_{oi}^{'} + \tau_i, \end{aligned} \tag{C6c}$$

where $j_{oi}^{'} = j_{oi} + k_{i1}j_i\hat{\phi}_{oi}$.

Now we are ready to give the control torque for each agent $i = 1, \dots, N$ as following form

$$\begin{aligned} \tau_i = {} & \omega_i^{\times} j_i\omega_i - j_i((x_i - k_{i1}\hat{\phi}_i)^{\times} C(\phi_i)\xi_i - C(\phi_i)S_i\xi_i) \\ & - \frac{1}{2}k_{i1}j_i(\hat{\phi}_i^{\times} + \bar{\phi}_i I_3)(x_i - k_{i1}\hat{\phi}_i) - k_{i2}x_i, \end{aligned} \tag{C7}$$

where k_{i2} is a positive constant.

By the definition of ϕ_i and ϵ_i, for each follower $i = 1, \dots, N$, we have

$$\phi_i - \epsilon_i = (\eta_i - q_0)^* \odot q_i. \tag{C8}$$

By Theorem 1, $\lim_{t\to\infty}(\eta_i(t) - q_0(t)) = \mathbf{0}$. Thus

$$\lim_{t\to\infty}(\phi_i(t) - \epsilon_i(t)) = \mathbf{0}. \tag{C9}$$

With the control torque (C7), system (C6c) becomes

$$j_i\dot{x}_i = -k_{i2}x_i + j_{oi}^{'} \quad i = 1, \dots, N. \tag{C10}$$

Since $\lim_{t\to\infty} j_{oi}^{'} = \mathbf{0}$ and j_i is positive define, systems (C10) are strictly stable linear systems when $t \to \infty$. Therefore, we obtain that $\lim_{t\to\infty} x_i(t) = \mathbf{0}$, $i = 1, \dots, N$. Then, using the Lemma 4.1 of [1], we can obtain that

$$\lim_{t\to\infty} \hat{\epsilon}_i = \lim_{t\to\infty} \hat{\phi}_i = \mathbf{0}. \tag{C11}$$

Consequently, the adaptive leader-following attitude consensus of MRBS with resilience to CLFs under time-varying network is achieved. Furthermore, by the definition of x_i, we can obtain

$$\lim_{t\to\infty} \tilde{\omega}_i(t) = \mathbf{0}. \tag{C12}$$

Combining (C1) and (C3), we have

$$\breve{\omega}_i = \tilde{\omega}_i + C(\phi_i)(\xi_i - \omega_0) + (C(\phi_i) - C(\epsilon_i))\omega_0. \tag{C13}$$

Thus, by Theorem 1 and (C13),

$$\lim_{t\to\infty} \breve{\omega}_i(t) = \mathbf{0}, \tag{C14}$$

which completes the proof.

Remark 3. Theorem 2 uses the certainty equivalence principle and pseudo-linear representation of MRBSs to transform the attitude control problem of MRBSs into the design of stable conventional linear systems (C10). The observer convergence obtained by Theorem 1 ensures that the nonlinear term of the system (C10) converges to zero as time tends to infinity. Note that Theorem 1 of [6] may not hold under CLFs, so we cannot obtain observer convergence under general switched systems.

Appendix D Simulation Examples

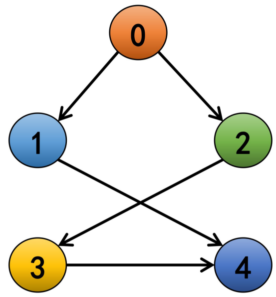

Figure D1 Communication topology

The simulated MRBSs have four followers and a leader subject to a time-varying digraph $\mathcal{G}(t)$. Consider a leader-following network that has the communication topology shown in Fig. D1. Under the Assumption 5, we consider the digraph $\mathcal{G}(t)$ in accord with a jointly connected condition that is denoted by a piecewise function. Note that changes in function at the segmentation point can be considered failures caused by CLFs. Next, we design the corrupted weight δ as the following form

$$\delta = sin(t + 0.1 + 2 * cos(t + 0.2) + sin(cos(t + 0.3))).$$

Moreover, we design the communication weights $a_{ij}(t)$ as piecewise functions which have the following forms

$$a_{01}(t) = \begin{cases} 1+\delta, & if \ \ sT_0 \leqslant t \leqslant (s+\frac{1}{4})T_0, \\ 0, & if \ \ (s+\frac{1}{4})T_0 \leqslant t \leqslant (s+\frac{1}{2})T_0, \\ 0, & if \ \ (s+\frac{1}{2})T_0 \leqslant t \leqslant (s+\frac{3}{4})T_0, \\ 0, & if \ \ (s+\frac{3}{4})T_0 \leqslant t \leqslant (s+1)T_0, \end{cases}$$

$$a_{02}(t) = \begin{cases} 0, & if \ \ sT_0 \leqslant t \leqslant (s+\frac{1}{4})T_0, \\ 1+\delta, & if \ \ (s+\frac{1}{4})T_0 \leqslant t \leqslant (s+\frac{1}{2})T_0, \\ 0, & if \ \ (s+\frac{1}{2})T_0 \leqslant t \leqslant (s+\frac{3}{4})T_0, \\ 0, & if \ \ (s+\frac{3}{4})T_0 \leqslant t \leqslant (s+1)T_0, \end{cases}$$

$$a_{23}(t) = \begin{cases} 0, & if \ \ sT_0 \leqslant t \leqslant (s+\frac{1}{4})T_0, \\ 1+\delta, & if \ \ (s+\frac{1}{4})T_0 \leqslant t \leqslant (s+\frac{1}{2})T_0, \\ 0, & if \ \ (s+\frac{1}{2})T_0 \leqslant t \leqslant (s+\frac{3}{4})T_0, \\ 0, & if \ \ (s+\frac{3}{4})T_0 \leqslant t \leqslant (s+1)T_0, \end{cases}$$

$$a_{14}(t) = \begin{cases} 0, & if \ \ sT_0 \leqslant t \leqslant (s+\frac{1}{4})T_0, \\ 0, & if \ \ (s+\frac{1}{4})T_0 \leqslant t \leqslant (s+\frac{1}{2})T_0, \\ 1+\delta, & if \ \ (s+\frac{1}{2})T_0 \leqslant t \leqslant (s+\frac{3}{4})T_0, \\ 0, & if \ \ (s+\frac{3}{4})T_0 \leqslant t \leqslant (s+1)T_0, \end{cases}$$

$$a_{34}(t) = \begin{cases} 0, & if \ \ sT_0 \leqslant t \leqslant (s+\frac{1}{4})T_0, \\ 0, & if \ \ (s+\frac{1}{4})T_0 \leqslant t \leqslant (s+\frac{1}{2})T_0, \\ 0, & if \ \ (s+\frac{1}{2})T_0 \leqslant t \leqslant (s+\frac{3}{4})T_0, \\ 1+\delta, & if \ \ (s+\frac{3}{4})T_0 \leqslant t \leqslant (s+1)T_0, \end{cases}$$

where $T_0 = 0.2$ and $s = 0, 1, 2, \ldots$. According to Remark 1 in [3], we can obtain that δ can describe capacity constraints, and transmission noises. Furthermore, we assume that $a_{ij}(t)$ can be zero over a certain period of time. In practice, this condition could be referred to as packet dropout. The dynamics of the four followers described by

$$\dot{q}_i = \frac{1}{2} q_i \odot q(\omega_i), \tag{D1a}$$

$$j_i \dot{\omega}_i = -\omega_i^{\times} j_i \omega_i + \tau_i, \quad i = 1, ..., N, \tag{D1b}$$

with the inertial matrices are given by $J_m = diag\{m, 2m, m+2\}, m = 1, 2, 3, 4$. We define the matrix S_0 as

$$S_0 = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{D2}$$

Then we choose $\alpha = \beta = 10, k_{i1} = k_{i2} = 20$. All initial adaptive parameter values are designed as $a_{\xi i}(0) = a_{\eta i}(0) = 1$ to satisfy the requirement in Theorem 1. The initial attitudes are set to $q_0(0) = [1, 0, 0, 0]^{\mathrm{T}}, q_1(0) = [0, 1, 0, 0]^{\mathrm{T}}, q_2(0) = [0, 0, 1, 0]^{\mathrm{T}}, q_4(0) = [1/2, \sqrt{3}/2, 0, 0]^{\mathrm{T}}$ to satisfy the requirement that $q_0(t) \in \mathbb{Q}_u$ ($\|q_0\| = 1$). The initial angular velocity of the leader is set to $\omega_0(0) = [1, 2, 3]^{\mathrm{T}}$. Other parameters randomly generated initial conditions. The estimation errors of $\xi_i, \eta_i, \ i = 1, 2, 3, 4$ obtained by observers are shown in Fig. D2. Furthermore, Fig. D3 shows consensus errors of attitude and angular velocities, respectively. Adaptive parameter trajectories are shown in Fig. D4. For attitude and angular velocities, it is observed that the leader-following consensus has satisfactory resilience to CLFs.

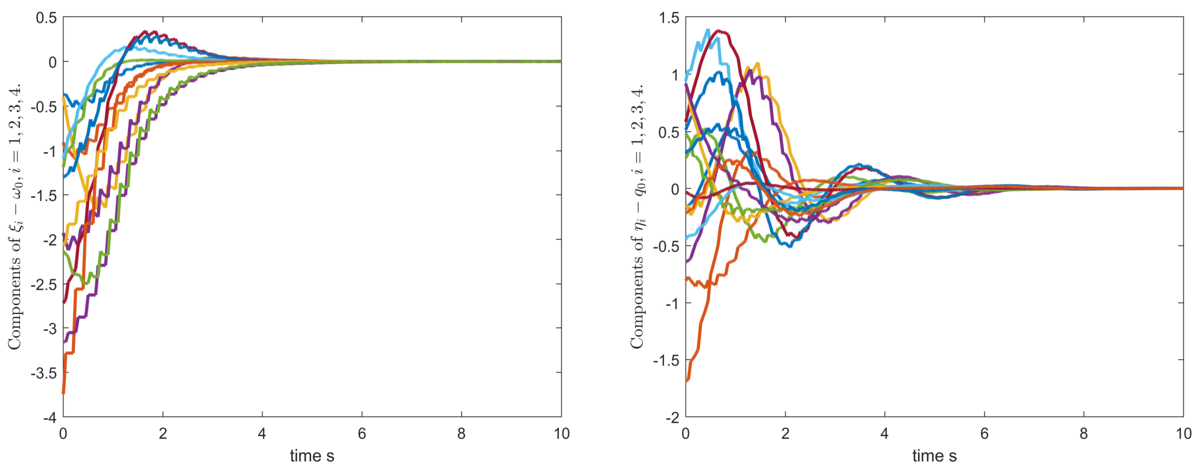

Figure D2 Estimate errors of followers.

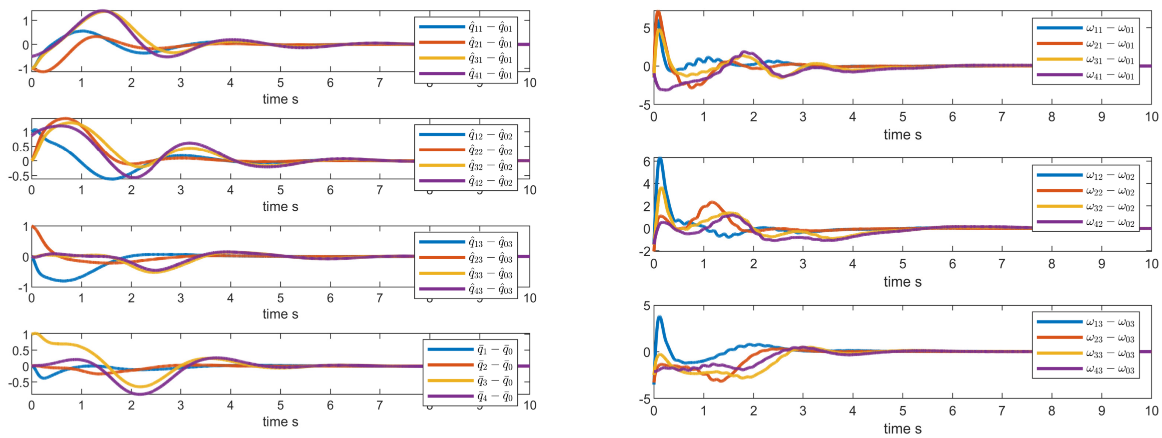

Figure D3 (a) Consensus of attitudes; (b) Consensus of angular velocities.

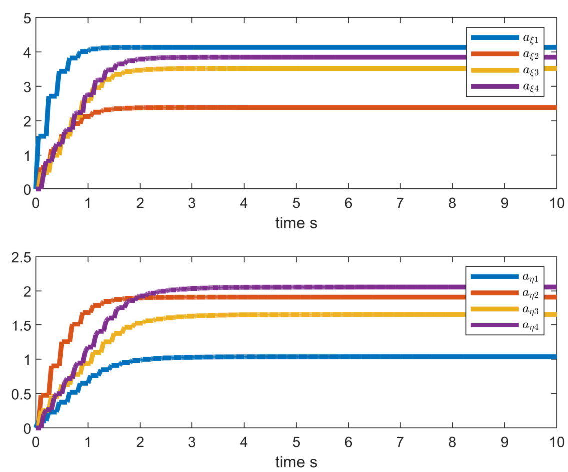

Figure D4 Trajectories of adaptive parameters.

Appendix E Proof of Lemmas

Lemma 1. If the digraph $\mathcal{G}$ contains a spanning tree with the leader as its root at time t, then there exists a positive definition diagonal matrix $Q(t)$ such that $Q(t)L_G(t) + L_G^{\mathrm{T}}(t)Q(t) = P(t)$, where $P(t)$ is positive-definite, and $L_G(t) = L(t) + G(t)$.

Proof. Lemma 1 can be extracted from the results of the structural analysis of the directed graph with CLFs in [4].

Lemma 2. If Assumptions 1-5 hold, Lemma 1 is satisfied by a positive definition diagonal matrix $Q(t)$. In the meanwhile, $Q(t)$ and its derivative are bounded.

Proof. Because the digrpah $\mathcal{G}$ contains a spanning tree with leader as its root, $L_G(t)$ must be a nonsingular M-matrix. This implies that $(L_G^T(t))^{-1}$ exists and is positive. Then we choose $Q(t)$ to be

$$Q(t) = diag(a_1(t), \dots, a_N(t)), \tag{E1}$$

and we choose its diagonal elements as

$$a(t) = [a_1(t), \dots, a_N(t)]^T = (L_G^T(t))^{-1}\mathbf{1}_N. \tag{E2}$$

According to the definition of $Q(t)$, it satisfies that $Q(t)L_G(t) + L_G^{\mathrm{T}}(t)Q(t) = P(t)$, where $P(t)$ is positive-definite. We can further obtain that $det(L_G^T(t))$ is nonzero and bounded due to the boundedness of $L_G(t)$ which is given by Assumption 3 and that $(L_G^T(t))^{-1}$exists. This implies that

$$\|(L_G^T(t))^{-1}\|_F = \|\frac{adj(L_G^T(t))}{det(L_G^T(t))}\|_F, \tag{E3}$$

which shows that all elements of $Q(t)$ are bounded. Then we take the derivative of (E2),

$$\dot{a}(t) = -(L_G^T(t))^{-1}\frac{d(L_G^T(t))}{dt}(L_G^T(t))^{-1}\mathbf{1}_{nN}. \tag{E4}$$

From Assumption 3, we can obtain that all the elements of $\frac{d(L_G^T(t))}{dt}$ are bounded due to the boundedness of $\delta_{ij}^a(t)$ and $\delta_i^g(t)$. This further implies that $\dot{a}(t)$ is bounded, which completes the proof.

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