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Adaptive dynamic programming-based fault-tolerant attitude control for flexible spacecraft with limited wireless resources

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Abstract This paper investigates the attitude control for flexible spacecraft subject to actuator faults and limited onboard communication resources. The control torque quantization scheme is considered between the controller and actuator to reduce the communication burden on the spacecraft. An integral sliding mode control method is designed to stabilize the closed-loop attitude control system and ensure the near-optimal performance of the sliding motion. First, an iterative learning observer scheme is employed to reconstruct the actuator faults and the unknown nonlinear flexible dynamics. Second, an integral sliding surface is combined with the backstepping control method to resolve the time-varying characterization of the closed system. Third, a combination of the adaptive dynamic programming technique and the adaptive single critic neural network approximation is employed to examine the optimal control policy. Finally, the efficacy of the proposed spacecraft attitude control method is demonstrated via a simulation.

Keywords flexible spacecraft, fault-tolerant attitude control, signal quantization, adaptive dynamic programming, neural network learning

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1 Introduction

The attitude control system is crucial for conducting advanced aerospace projects [1, 2] such as surface navigation and positioning, rendezvous, and earth surveillance, all of which are expected to be carried out with a high degree of precision and reliability [3, 4]. However, the actuators may suffer from several physical constraints such as component faults, input saturation, and dead zone as a result of being subjected to a severe working environment [5, 6] that deteriorates the control performance and causes system instability. Therefore, spacecraft attitude control issues with faults and other actuator constraints have been extensively investigated over the past decade. In [7], a novel and effective periodic event-triggered adaptive control was proposed with new adaptive update laws for the robust attitude stabilization of a rigid spacecraft. In [8], a practical event-triggered sliding mode control was effectively designed for the attitude stabilization of a rigid spacecraft. Furthermore, it was validated that both the attitude quaternion and angular velocity can be ultimately bounded in the presence of external disturbances and model uncertainties. In [9], an extended state observer-based feed-forward compensation controller has proven to be effective in compensating for the external disturbances. In [10], the author reconstructed the actuator faults using a disturbance observer and proposed an active fault-tolerant control method. Yang et al. [11, 12] proposed a novel adaptive, resilient sliding mode control scheme to resist false data injection attacks in actuator channels. Ref. [13–16] detailed relevant fault-tolerant control (FTC) results for spacecraft.

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The plug-and-play spacecraft has attracted considerable research interest in recent years. It employs a novel architectural design concept wherein data exchange and power transfer take place via wireless technology [17]. Event-triggered and signal quantization communication mechanisms are effective solutions that can be implemented to reduce the data transmission burden between the controller and actuator module. Some pioneering studies regarding event-triggered control were published in [7, 18–20]. Furthermore, signal quantization techniques in spacecraft are discussed in [21, 22]. In practice, the on-orbit plug-and-play spacecraft is always equipped with limited fuel or electrical energy reserves owing to its cost. Hence, one of the most crucial and challenging problems faced by the plug-and-play spacecraft is the development of optimal control policies for attitude control systems with negligible energy costs, where unknown time-varying actuator faults and control torque quantization are simultaneously taken into consideration. Our current research is motivated by the need to close this gap by creating a novel and efficient digital attitude control design approach. Nevertheless, such a research challenge is still open and difficult for spacecraft research at this time.

In summary, this paper examines the optimal control problem for plug-and-play flexible spacecraft with an emphasis on time-varying actuator faults, unknown external disturbances, and rigid-flexible nonlinear dynamics. First, a hysteresis-type quantizer with a remarkable precision and a low communication rate is utilized for data transmission between the controller and actuator side. Second, a complex adaptive design iterative learning observer (ILO) is developed to estimate the unknown disturbance vector. Following the estimation, an integral-type sliding mode control law is designed to stabilize the closed-loop attitude control systems and subsequently resolve the optimal control policy for the designed sliding motion dynamics via the adaptive dynamic programming (ADP) and neural network (NN) learning approximation schemes. Finally, a simulation is conducted to illustrate the validity of the developed spacecraft attitude control procedures.

2 Problem formulation

2.1 Spacecraft attitude kinematics and dynamics

The modified Rodrigues parameters (MRPs)-based attitude kinematics and dynamics for flexible spacecraft are depicted as follows [23, 24]:

$$\dot{\sigma} = G(\sigma)\omega, \tag{1}$$
$$J\dot{\omega} + \delta^{\mathrm{T}}\ddot{\eta} = -\omega^{\times}\left(J\omega + \delta^{\mathrm{T}}\dot{\eta}\right) + \tau + \tau_d, \tag{2}$$
$$\ddot{\eta} + K_s\dot{\eta} + K_d\eta + \delta\dot{\omega} = 0, \tag{3}$$

where $\sigma \in \mathbb{R}^3$ is the MRPs variable denoting the spacecraft orientation, $G(\sigma) = (1/4)[(1-\sigma^{\mathrm{T}}\sigma)I_3+2\sigma\sigma^{\mathrm{T}}+2\sigma^{\times}]$ with I_3 meaning a 3×3 identity matrix, ω represents the angular velocity of spacecraft, $J \in \mathbb{R}^{3\times 3}$ denotes the inertia matrix, $\delta \in \mathbb{R}^{4\times 3}$ denotes the coupling parameter matrix between the rigid and elastic components, $\eta \in \mathbb{R}^4$ is the modal displacement coordinate vector, $K_s = \mathrm{diag}\{2\zeta_{n1}\omega_{n1}, \ldots, 2\zeta_{n4}\omega_{n4}\}$ is the damping matrices with ζ_{ni} and ω_{ni}, $i = 1, \ldots, 4$ denoting the damping ratio and natural frequency of the ith-order mode, respectively, $K_d = \mathrm{diag}\{\Omega_{n1}^2, \ldots, \omega_{n4}^2\}$ is the stiffness matrix, and τ and τ_d denote the actuator torque and disturbance torque applied to the spacecraft, respectively. In addition, the operator '$(\cdot)^{\times}$' is employed to perform the cross product, i.e., $a \times b = a^{\times}b$.

Remark 1. The singularity problem encountered in the attitude representation of σ can be solved by introducing the shadow MRPs and corresponding switching shadow sets, which have been widely employed in [25, 26]. To be specific, the raw representation σ is used when $\|\sigma\| \leqslant 1$, and its shadow function $\sigma_s = -\sigma/(\sigma^{\mathrm{T}}\sigma)$ is used when $\|\sigma\| > 1$. The above switching method can realize the non-singular description of the attitude globally, and the MRPs vector can always be kept in the unit ball.

Property 1. The properties for $G(\sigma)$ is provided by the following equalities:

$$G(\sigma)\sigma = G^{\mathrm{T}}(\sigma)\sigma = \frac{1}{4}(1+\|\sigma\|^2)\sigma, \tag{4}$$
$$G^{\mathrm{T}}(\sigma)G(\sigma) = \frac{(1+\|\sigma\|^2)^2}{16}I_3, \tag{5}$$
$$G^{-1}(\sigma) = \frac{16}{(1+\|\sigma\|^2)^2}G^{\mathrm{T}}(\sigma), \tag{6}$$

and let $m(\sigma) = \frac{(1+\|\sigma\|^2)^2}{16}$ for simplicity.

2.2 Hysteresis quantizer

In this paper, we consider the case that the data transmitted between the controller and actuator with wireless communication, and signal quantization behavior are thus taken into account for the attitude control system design. Moreover, since the hysteretic quantizer requires a low communication rate and excludes the chattering aroused by the traditional logarithmic quantizer, we employ it in this paper to quantize the control command.

In this study, the original control command generated in the controller side is defined as $u_c(t) = [u_{c1}(t)\ u_{c2}(t)\ u_{c3}(t)]^{\mathrm{T}}$, and the quantization behavior of $u_c(t)$ is defined as $Q(u_c)$, which is formulated as follows [27]:

$$Q(u_i(t)) = \begin{cases} \mu_i \mathrm{sgn}(u_i), & \dfrac{\mu_i}{1+\kappa_q} < |u_i| \leqslant \mu_i, \dot{u}_i < 0, \text{ or } \mu_i < |u_i| \leqslant \dfrac{\mu_i}{1-\kappa_q}, \dot{u}_i > 0, \\ \mu_i(1+\kappa_q)\mathrm{sgn}(u_i), & \mu_i < |u_i| \leqslant \dfrac{\mu_i}{1-\kappa_q}, \dot{u}_i < 0, \text{ or } \dfrac{\mu_i}{1-\kappa_q} < |u_i| \leqslant \dfrac{\mu_i(1+\kappa_q)}{1-\kappa_q}, \dot{u}_i > 0, \\ 0, & 0 \leqslant |u_i| < \dfrac{\mu_{\min}}{1+\kappa_q}, \dot{u}_i < 0, \text{ or } \dfrac{\mu_{\min}}{1+\kappa_q} < |u_i| \leqslant \mu_{\min}, \dot{u}_i > 0, \\ q(u_i(t^-)), & \dot{u}_i = 0, \end{cases} \tag{7}$$

where κ_q is defined as $\kappa_q = (1-\varkappa)/(1+\varkappa)$, $\mu_i = \varkappa^{(1-i)}\mu_{\min}$ for $i = 1,2,3$ with $\mu_{\min} > 0$, $0 < \varkappa < 1$ being basic parameters of the hysteresis quantizer. Specifically, $\mu_{\min}$ signifies the size of the quantizer dead-zone and $\varkappa$ denotes a measure of quantization density.

In order to compensate for the quantization error, the output of the hysteresis quantizer can be decomposed into a linear part and a nonlinear part as follows:

$$Q(u_{ci}) = u_{ci} + \varsigma(u_{ci}). \tag{8}$$

Lemma 1 ([28])**.** The nonlinear part satisfies that

$$\varsigma^2(u_{ci}) \leqslant \kappa_q^2 u_{ci}^2, \text{ for } |u_{ci}| \geqslant \mu_{\min},$$
$$\varsigma^2(u_{ci}) \leqslant \mu_{\min}^2, \text{ for } |u_{ci}| \leqslant \mu_{\min}.$$

According to Lemma 1, Eq. (8) can be rewritten as

$$Q(u_c) = u_c + \Delta_u u_c + \varrho \tag{9}$$

with the following constraints:

$$\begin{cases} \varrho_i = 0, \ -\kappa_q \leqslant \Delta_{ui} \leqslant \kappa_q, & \text{if } |u_i| \geqslant \mu_{\min}, \\ -\mu_{\min} \leqslant \varrho_i \leqslant \mu_{\min}, \ \Delta_{ui} = 0, & \text{if } |u_i| \leqslant \mu_{\min}. \end{cases} \tag{10}$$

It can be seen that $\|\varrho\| \leqslant \sqrt{3}\mu_{\min}$ holds whatever u values.

Consider the control torque $\tau(t)$ in the actuator side with partial loss of efficiency, which can be modeled by

$$\tau(t) = \Lambda(\rho)Q(u_c), \tag{11}$$

where $\rho(t) = [\rho_1(t), \rho_2(t), \rho_3(t)]$ denotes the actuator faults efficiency factor with time-varying $\rho_i(t)$ belonging to the interval (0, 1], and $\Lambda(\rho) = \mathrm{diag}\{\rho_1(t),\ \rho_2(t),\ \rho_3(t)\}$ represents the diagonal fault matrix.

An auxiliary state is introduced as $\xi = \delta\omega + \dot{\eta}$ with

$$\dot{\xi} = \delta\dot{\omega} + \ddot{\eta} = -K_s\xi - K_s\delta\omega - K_d\eta. \tag{12}$$

By substituting (12) into (2), one yields

$$(J - \delta^{\mathrm{T}}\delta)\dot{\omega} = -\omega^{\times}(J - \delta^{\mathrm{T}}\delta)\omega + \tau + \psi + \tau_d, \tag{13}$$

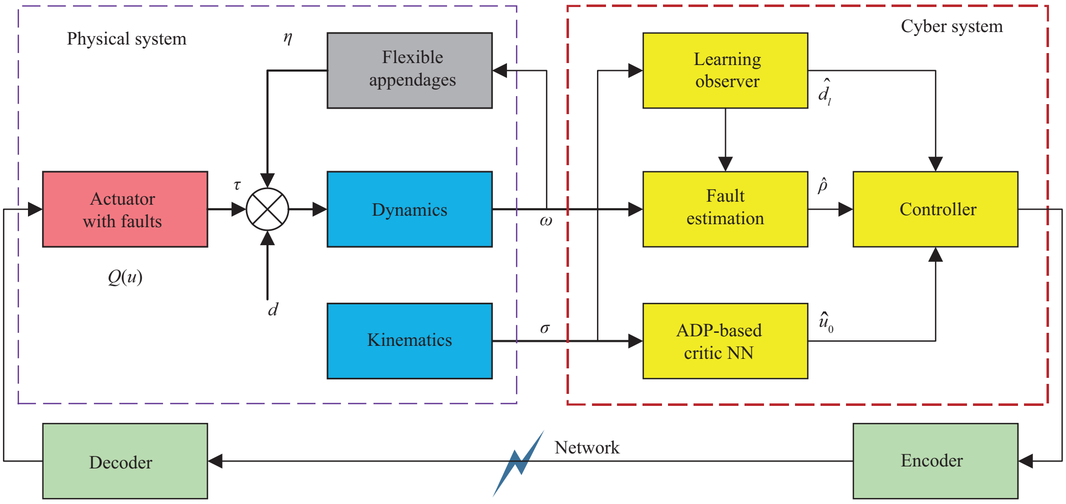

Figure 1 (Color online) The attitude control system architecture.

where

$$\psi = \delta^{\mathrm{T}} \begin{bmatrix} K_d & K_s \end{bmatrix} \begin{bmatrix} \eta \\ \xi \end{bmatrix} - \delta^{\mathrm{T}} K_s \delta\omega - \omega^{\times} \delta^{\mathrm{T}} \xi.$$

For brevity, let $J_0 = J - \delta^{\mathrm{T}}\delta$, and the attitude control problem with actuator faults and signal quantization can be formulated as

$$\dot{\sigma} = G(\sigma)\omega, \tag{14}$$

$$J_0\dot{\omega} = -\omega^{\times} J_0\omega + \Lambda(\rho)Q(u_c) + d_l, \tag{15}$$

where $d_l = \psi + \tau_d$ is treated as the lumped disturbances composed of external disturbance torque and rigid-flexible nonlinear dynamics.

Further, the following assumptions are made for the subsequent development.

Assumption 1. The efficiency and its derivative of the actuator are bounded with $0 < \underline{\rho}_m \leqslant \rho_i(t) \leqslant \bar{\rho}_M$ and $|\dot{\rho}_i| < \bar{\rho}_d$.

Assumption 2. The external disturbance torque τ_d is bounded, and the modal vibration variables involved with elastic appendages η and its time derivatives $\dot{\eta}$, $\ddot{\eta}$ are bounded.

The objective of this paper is to design a fault-tolerant control scheme for (14) and (15) with the energy cost being taken into consideration. First, an adaptive ILO design is presented to reconstruct the actuator faults and the rigid-flexible nonlinearity coupled with disturbances. Then, based on observer estimations, an integral-type sliding mode control law with ADP and neural networks schemes is developed which can stabilize the attitude control systems with nearly optimal performance of the nominal dynamics. The attitude control system architecture is illustrated in Figure 1.

Remark 2. Note that in the past few years there has been some interesting work focusing on attitude control of plug-and-play spacecraft with wireless communication among components [29, 30]. Similar to this paper, in these studies, the wireless data communication among control module and actuator module is also considered, and attitude control laws are proposed with the consideration of input (attitude control torques) quantization. On the other hand, with the development of spacecraft technology, it is desirable to design the attitude control systems involved with the optimal performance and minimal energy cost, which may be a feasible way to reduce design costs for practical spacecraft. However, this design constraint has not been taken into account in [29, 30], which seems to remain as an open problem. To this end, in this paper we will adopt the ADP approach to investigate and solve this research obstacle.

3 ADP-based sliding mode fault-tolerant control under input quantization

In this section, an adaptive ILO method will be proposed. Based on the estimation, an integral quantized sliding mode control law is developed. Moreover, in this design the nearly optimal control policy for the sliding motion dynamics is also considered simultaneously.

3.1 Fault reconstruction observer design

To reconstruct the actuator faults and estimate the lumped disturbances, we consider the following fault observer:

$$\begin{aligned} J_0\dot{\hat{\omega}} &= -\hat{\omega}^{\times} J_0\hat{\omega} + \Lambda(\hat{\rho})Q(u_c) + \hat{d}_l(t) - L_f\hat{\omega} + L_f\omega, \\ \hat{d}_l(t) &= K_{f1}\hat{d}_l(t-T_f) + K_{f2}\omega - K_{f2}\hat{\omega}, \end{aligned} \tag{16}$$

where $T_f > 0$ denotes the updating interval of the iterative learning observer (16), which is always set as the sampling time interval of the considered control systems. Moreover, the adaptive law for $\hat{\rho}(t)$ is designed as

$$\dot{\hat{\rho}}(t) = \Theta\Lambda(Q(u_c))P\tilde{\omega}, \tag{17}$$

where $\hat{\rho}(t)$ is the estimate of $\rho(t)$, $\Theta > 0$ is the diagonal adaptive gain matrix, $\Lambda(Q(u_c)) = \mathrm{diag}\{Q_1(u_c)(t),$ $Q_2(u_c)(t),\ Q_3(u_c)(t)\}$, and P, L_f, K_{f1}, K_{f2}, P are observer matrices to be designed. Before the subsequent analysis, we make the following assumptions.

Assumption 3. The lumped disturbance d_l is bounded, and $\|d_l(t) - d_l(t-T_f)\| \leqslant \bar{d}_l$ with $\bar{d}_l$ being an unknown positive constant.

Assumption 4. The nonlinear term "$\omega^{\times} J_0\omega - \hat{\omega}^{\times} J_0\hat{\omega}$" satisfies $\|\omega^{\times} J_0\omega - \hat{\omega}^{\times} J_0\hat{\omega}\| \leqslant \delta_0\tilde{\omega}$.

Theorem 1. If the gain matrices for ILO (16) are designed to satisfy the following constraints:

$$L_f > 0, \tag{18}$$

$$0 < \delta_1 K_{f1}^{\mathrm{T}} K_{f1} < I_3, \tag{19}$$

$$P = \bar{\gamma}_1 K_{f2} > 0, \tag{20}$$

$$2PL_f + \bar{\gamma}_1 K_{f2}^{\mathrm{T}} K_{f2} - 2\delta_0\lambda_{\max}(P)I > 0, \tag{21}$$

where $\delta_1 > \bar{\gamma}_1 > 1$. Then the estimation errors for $\rho(t)$ and d_l are bounded.

Proof. See Appendix A.

3.2 Sliding surface design and reachability analysis

Let $z = [z_1\ z_2]^{\mathrm{T}}$ with $z_1 = \sigma(t)$ and $z_2 = \omega(t) - \omega^*(t)$ with virtual control $\omega^*(t)$ designed as

$$\omega^*(t) = -k_\sigma\frac{G^{\mathrm{T}}(\sigma)}{m(\sigma)}\sigma = -\frac{4k_\sigma}{1+\|\sigma\|^2}\sigma, \tag{22}$$

where k_σ is a positive scalar. Then the following coordinate transformation system is presented:

$$\dot{z}_1 = G(z_1)z_2 + G(z_1)\omega^*, \tag{23}$$

$$\dot{z}_2 = f(z_2) + r(z_2)\Lambda(\rho)Q(u_c) + r(z_2)d_l, \tag{24}$$

where $f(z) = -J_0^{-1}(\omega^* + z_2)^{\times} J_0(\omega^* + z_2) - \dot{\omega}^*,\ r(z) = J_0^{-1}$ with

$$\dot{\omega}^* = -k_\sigma\frac{4G(\sigma) - 2\sigma\sigma^{\mathrm{T}}}{1+\|\sigma\|^2}(\omega^* + z). \tag{25}$$

The above equation can be written in a compact version:

$$\dot{z}(t) = F(z) + R(z)\Lambda(\rho)Q(u_c) + R(z)d_l, \tag{26}$$

where

$$F(z)=\begin{bmatrix} G(z_1)z_2+G(z_1)\omega^* \\ -J_0^{-1}(\omega^*+z_2)^{\times}J_0(\omega^*+z_2)-\dot{\omega}^* \end{bmatrix},$$
$$R(z)=\begin{bmatrix} 0_{3\times 3} \\ J_0^{-1} \end{bmatrix}. \tag{27}$$

Remark 3. Under the virtual control $\omega^*(t)$, if we choose the Lyapunov function of the attitude kinematics subsystem as

$$V_\sigma=\sigma^{\mathrm{T}}\sigma,$$

it is easy to derive that

$$\dot{V}_\sigma=-k_\sigma\sigma^{\mathrm{T}}\sigma+k_\sigma\frac{1+\sigma^{\mathrm{T}}\sigma}{4}\sigma^{\mathrm{T}}z_2.$$

Recalling that $\|\sigma\|\leqslant 1$ as discussed in Remark 1, we can further derive that

$$\dot{V}_\sigma\leqslant -k_\sigma\sigma^{\mathrm{T}}\sigma+\frac{k_\sigma}{2}\sigma^{\mathrm{T}}z_2\leqslant -\frac{k_\sigma}{2}\sigma^{\mathrm{T}}\sigma+\frac{k_\sigma}{8}z_2^{\mathrm{T}}z_2,$$

which implies that $\sigma(t)$ is bounded by $\|\sigma(t)\|\leqslant\frac{\|z_2\|}{2}$. Hence, if z_2 is proven to be bounded, we can conclude σ is bounded. Since $\omega=\omega^*+z_2$, it can be further concluded that $\omega(t)$ is bounded. This is just the advantage that we introduce the virtual law and backstepping control method in this design.

We now design the control command in (26) as $u_c=u_0+u_1+u_2$, where u_0 is the term to be designed for the nearly optimal sliding motion dynamic in the subsequent discussion, and u_1 and u_2 are the discontinuous controller components to reject unknown faults, lumped nonlinear/disturbance, and the effect of signal quantization error.

In this paper, the following integral-type sliding surface is considered for system (26):

$$s(t)=z(t)-z(t_0)-\int_{t_0}^{t}[F(z(v))+R(z(v))u_0(v)]\mathrm{d}v. \tag{28}$$

Considering the integral term $F_R(t)=\int_{t_0}^{t}[F(z(v))+R(z(v))u_0(v)]\mathrm{d}v$ in the sliding mode manifold (28), it can be derived mathematically that $\dot{F}_R(t)=F(z(v))+R(z(v))u_0(v)$. Subsequently, the derivative of $s(t)$ can be calculated as

$$\begin{aligned}\dot{s}(t)=&\dot{z}(t)-F(z)-R(z)u_0\\=&R(z)\Lambda(\rho)Q(u_c)+R(z)d_l-R(z)u_0\\=&R(z)\Lambda(\rho)[(I+\Delta_u)u_c+\varrho]+R(z)d_l-R(z)u_0\\=&R(z)[\Lambda(\rho)(I+\Delta_u)(u_0+u_1+u_2)+\Lambda(\rho)\varrho+d_l-u_0].\end{aligned} \tag{29}$$

We define

$$K_b=\frac{1}{\rho_m}[(\Lambda(\rho)\Lambda(\hat{\rho})^{-1}-I)u_0+d_l-\Lambda(\rho)\Lambda(\hat{\rho})^{-1}\hat{d}_l]. \tag{30}$$

Then it can be seen that there exists an upper bound value $\bar{K}_b>0$ such that

$$s^{\mathrm{T}}R(z)K_b\leqslant\bar{K}_b\|s^{\mathrm{T}}R(z)\| \tag{31}$$

holds.

Theorem 2. If the control components $u_1(t)$ and $u_2(t)$ are designed as

$$u_1=-(I-\Lambda(\hat{\rho})^{-1})u_0-\Lambda(\hat{\rho})^{-1}\hat{K}_b\mathrm{sgn}(R^{\mathrm{T}}(z)s)-\Lambda(\hat{\rho})^{-1}\hat{d}_l$$

$$- \frac{1}{\rho_m} k_s R^{\mathrm{T}}(z)s - \sqrt{3}\mu_{\min}\mathrm{sgn}(R^{\mathrm{T}}(z)s), \tag{32}$$

$$u_2 = -\frac{\kappa_q}{\rho_m(1-\kappa_q)}(\|u_0\| + \|u_1\|)\mathrm{sgn}(R^{\mathrm{T}}(z)s), \tag{33}$$

where $\hat{K}_b$ is the estimation of K_b, the adaptive law of which is designed as

$$\dot{\hat{K}}_b = \varsigma_0 \|s^{\mathrm{T}}(t)R(z)\|, \ \varsigma_0 > 0, \tag{34}$$

and $k_s > 0$ and $\varsigma_0 > 0$ are the controller parameters to be designed. Then the sliding motion can be guaranteed since the initial time.

Proof. Let $\tilde{K}_b = \bar{K}_b - \hat{K}_b$, and the Lyapunov function for the sliding surface (28) is selected as

$$V_s(t) = \frac{1}{2}s(t)^{\mathrm{T}}s(t) + \frac{\rho_m}{2\varsigma_0}\tilde{K}_b^2. \tag{35}$$

Then the derivative of $V_s(t)$ is obtained as

$$\begin{aligned}
\dot{V}_s =& s^{\mathrm{T}}\dot{s} - \rho_m \varsigma_0^{-1}\tilde{K}_b \dot{\hat{K}}_b \\
=& s^{\mathrm{T}}R(z)[\Lambda(\rho)(I+\Delta_u)(u_0+u_1+u_2) + \Lambda(\rho)\varrho + d_l - u_0] - \rho_m\tilde{K}_b\|s^{\mathrm{T}}(t)R(z)\| \\
=& s^{\mathrm{T}}R(z)(\Lambda(\rho)-I)u_0 + s^{\mathrm{T}}R(z)\Lambda(\rho)u_1 + s^{\mathrm{T}}R(z)d_l + s^{\mathrm{T}}R(z)\Lambda(\rho)\varrho + s^{\mathrm{T}}R(z)\Lambda(\rho)\Delta_u(u_0+u_1) \\
&+ s^{\mathrm{T}}R(z)\Lambda(\rho)(I+\Delta_u)u_2 - \rho_m\tilde{K}_b\|s^{\mathrm{T}}(t)R(z)\|.
\end{aligned} \tag{36}$$

First, under the control law (32), one can obtain

$$\begin{aligned}
\dot{V}_s =& s^{\mathrm{T}}R(z)(\Lambda(\rho)\Lambda(\hat{\rho})^{-1} - I)u_0 - \rho_m\tilde{K}_b\|s^{\mathrm{T}}(t)R(z)\| + s^{\mathrm{T}}R(z)(d_l - \Lambda(\rho)\Lambda(\hat{\rho})^{-1}\hat{d}_l) \\
&- s^{\mathrm{T}}R(z)\Lambda(\rho)\Lambda(\hat{\rho})^{-1}\hat{K}_b\mathrm{sgn}(R^{\mathrm{T}}(z)s) + s^{\mathrm{T}}R(z)\Lambda(\rho)\varrho - \sqrt{3}\mu_{\min}\|s^{\mathrm{T}}R(z)\| \\
&- k_s s^{\mathrm{T}}R(z)R^{\mathrm{T}}(z)s + s^{\mathrm{T}}R(z)\Lambda(\rho)\Delta_u(u_0+u_1) + s^{\mathrm{T}}R(z)\Lambda(\rho)(I+\Delta_u)u_2.
\end{aligned} \tag{37}$$

Recalling that $0 < \rho_m \leqslant \rho_M \leqslant 1$ and $0 < \rho_m \leqslant \hat{\rho} \leqslant 1$, it yields that $I \leqslant \Lambda(\rho)\rho_m^{-1}$. Then, it is derived

$$\begin{aligned}
\dot{V}_s \leqslant& - k_s s^{\mathrm{T}}R(z)R^{\mathrm{T}}(z)s + s^{\mathrm{T}}R(z)\Lambda(\rho)\varrho - \sqrt{3}\mu_{\min}\|s^{\mathrm{T}}R(z)\| + \rho_m K_b\|s^{\mathrm{T}}R(z)\| \\
&- \rho_m\hat{K}_b\|s^{\mathrm{T}}R(z)\| - \rho_m\tilde{K}_b\|s^{\mathrm{T}}(t)R(z)\| + s^{\mathrm{T}}R(z)\Lambda(\rho)\Delta_u(u_0+u_1) \\
&+ s^{\mathrm{T}}R(z)\Lambda(\rho)(I+\Delta_u)u_2.
\end{aligned} \tag{38}$$

Noting that the quantization error satisfies $\|\varrho\| \leqslant \sqrt{3}\mu_{\min}$, it is obvious that the following inequality holds:

$$s^{\mathrm{T}}R(z)\Lambda(\rho)\varrho - \sqrt{3}\mu_{\min}\|s^{\mathrm{T}}R(z)\| \leqslant 0. \tag{39}$$

Recalling that $\|\Delta_u\| \leqslant \kappa_q$, it is derived that

$$\begin{aligned}
\dot{V}_s \leqslant& - k_s s^{\mathrm{T}}R(z)R^{\mathrm{T}}(z)s + \rho_m K_b\|s^{\mathrm{T}}R(z)\| - \rho_m\hat{K}_b\|s^{\mathrm{T}}R(z)\| - \rho_m\tilde{K}_b\|s^{\mathrm{T}}(t)R(z)\| \\
&+ \kappa_q\|s^{\mathrm{T}}R(z)\|(\|u_0\| + \|u_1\|) + s^{\mathrm{T}}R(z)\Lambda(\rho)(I+\Delta_u)u_2 \\
\leqslant& - k_s s^{\mathrm{T}}R(z)R^{\mathrm{T}}(z)s + \rho_m K_b\|s^{\mathrm{T}}R(z)\| - \rho_m\hat{K}_b\|s^{\mathrm{T}}R(z)\| - \rho_m\tilde{K}_b\|s^{\mathrm{T}}(t)R(z)\| \\
&- \kappa_q\|s^{\mathrm{T}}R(z)\|(\|u_0\| + \|u_1\|) - \frac{\kappa_q(\|u_0\| + \|u_1\|)}{\rho_m(1-\kappa_q)}s^{\mathrm{T}}R(z)\Lambda(\rho)(I+\Delta_u)\mathrm{sgn}(R^{\mathrm{T}}(z)s).
\end{aligned} \tag{40}$$

Since $\tilde{K}_b = \bar{K}_b - \hat{K}_b$, it is shown that $\dot{V}_s \leqslant -k_s\|R(z)s(t)\|^2$ holds, which implies that the reachability condition can be guaranteed for all $s(t) \neq 0$. From (28), it can be known that $s(0) = 0$. As a result, one can conclude that both $s(t) = 0$ and $\dot{s}(t) = 0$ hold for all $t \geqslant 0$. Therefore, the sliding motion dynamics can be ensured from the initial time.

3.3 Nearly optimal sliding mode control based on ADP

Note that under $\dot{s}(t)=0$ the equivalent control can be derived as

$$u_{\rm eq}=(I+\Delta_u)^{-1}\Lambda(\rho)^{-1}u_0-(I+\Delta_u)^{-1}\Lambda(\rho)^{-1}d_l. \tag{41}$$

By substituting (41) into (26), the sliding motion is obtained as

$$\dot{z}=F(z)+R(z)u_0. \tag{42}$$

For system (42), we consider the following performance function:

$$\Gamma(z(t_0),u_0)=\int_{t_0}^{t}[M(z)+u_0^{\rm T}Nu_0]{\rm d}\sigma. \tag{43}$$

The Hamiltonian function is written as

$$H(z,u_0,\Gamma_z)=M(z)+u_0^{\rm T}Nu_0+\Gamma_z(z(t_0),u_0)\dot{z}(t), \tag{44}$$

where $\Gamma_z(z(t_0),u_0)=\partial\Gamma(z(t_0),u_0)/\partial z$, and $\dot{z}(t)$ defers to (26).

Our objective now is to solve the admissible optimal control law u_0^* to stabilize the sliding motion dynamics (42) and minimize the performance index (43) simultaneously. Define the optimal function as

$$\Gamma^*(z)=\min_{u_0\in\Omega_u}\left(\int_{t_0}^{t}[M(z)+u_0^{\rm T}Nu_0]{\rm d}\sigma\right). \tag{45}$$

Based on the optimal control theory, the optimal control policy u_0^* and the Hamilton-Jacobi-Bellman (HJB) equation with respect to Γ_z^* are obtained as

$$u_0^*(z)=-\frac{1}{2}N^{-1}R^{\rm T}(z)\Gamma_z^*, \tag{46}$$

$$0=M(z)-\frac{1}{4}\Gamma_z^{*\rm T}R(z)N^{-1}R^{\rm T}(z)\Gamma_z^*+\Gamma_z^{*\rm T}F(z), \tag{47}$$

where $\Gamma_z^*(z)=\partial\Gamma_z^*/\partial z$ denotes the solution of the HJB equation (47).

To obtain the optimal control policy u_0^*, one should solve an analytical solution for $\Gamma_z^*(z)$ of the HJB equation (47). However, it is difficult to solve such a nonlinear differential equation directly. To this end, the adaptive NN learning scheme is employed in the subsequent analysis to approximate $\Gamma_z^*(z)$.

Based on the universal approximation property of NN [31], a standard NN can be adopted as

$$\Gamma^*(z)=W_c^{*\rm T}h(z)+\varepsilon(z), \tag{48}$$

whose derivative with respect to z refers to

$$\Gamma_z^*(z)=\nabla h^{\rm T}W_c^{*\rm T}+\nabla\varepsilon, \tag{49}$$

where $W_c^*\in\mathbb{R}^n$, $\nabla h=\partial h(z)/\partial z$, and $\nabla\varepsilon=\partial\varepsilon(z)/\partial z$.

By substituting (49) into (46) and (47), it yields

$$\begin{aligned}
&u^*(z)=-\frac{1}{2}N^{-1}R^{\rm T}(z)(\nabla h^{\rm T}W_c^{*\rm T}+\nabla\varepsilon),\\
&0=\varepsilon_{\rm HJB}+M(z)+W_c^{*\rm T}\nabla h(z)F(z)-\frac{1}{4}W_c^{*\rm T}\nabla h(z)R(z)N^{-1}R^{\rm T}(z)\nabla h^{\rm T}(z)W_c^*,\\
&\quad\triangleq H(z,u_0^*,\Gamma_z^*),
\end{aligned} \tag{50}$$

where $H(z,u_0^*,\Gamma_z^*)$ denotes the HJB function [32], and $\varepsilon_{\rm HJB}=-\frac{1}{2}\nabla\varepsilon^{\rm T}R(z)N^{-1}R^{\rm T}(z)\nabla h^{\rm T}(z)W_c^*-\frac{1}{4}\nabla\varepsilon^{\rm T}R(z)N^{-1}R^{\rm T}(z)\nabla\varepsilon+\nabla\varepsilon^{\rm T}F(z)$.

Since the ideal weights W_c^* are unknown, we employ a critic NN to approximate the cost function:

$$\hat{\Gamma}(z)=\hat{W}_c^{\rm T}(t)h(z), \tag{51}$$

where $\hat{W}_c(t)$ is the estimation of the ideal weights W_c^*. Further, we can obtain the derivative of $\hat{\Gamma}(z)$ with respect to z as

$$\hat{\Gamma}_z(z) = \nabla h^{\mathrm{T}}(z)\hat{W}_c. \tag{52}$$

Considering (46), the approximated optimal control is obtained as

$$\hat{u}_0(z) = -\frac{1}{2}N^{-1}R^{\mathrm{T}}(z)\nabla h^{\mathrm{T}}\hat{W}_c. \tag{53}$$

By applying the control law (53) into (42), the closed-loop sliding motion dynamics is obtained:

$$\dot{z} = F(z) - \frac{1}{2}R(z)N^{-1}R^{\mathrm{T}}(z)\nabla h^{\mathrm{T}}\hat{W}_c. \tag{54}$$

The approximation for the HJB function $H(z, u_0^*, \Gamma_z^*)$ is defined as

$$\hat{H}(z, \hat{u}_0, \hat{W}_c) = M(z) + \hat{W}_c^{\mathrm{T}}\nabla h(z)F(z) - \frac{1}{4}\hat{W}_c^{\mathrm{T}}\nabla h(z)R(z)N^{-1}R^{\mathrm{T}}(z)\nabla h^{\mathrm{T}}(z)\hat{W}_c. \tag{55}$$

Then the Hamiltonian error is derived as (note $H(z, u_0^*, \Gamma_z^*) = 0$)

$$e_H(t) = \hat{H}(z, \hat{u}_0, \hat{W}_c) - H(z, u_0^*, \Gamma_z^*) = \hat{H}(z, \hat{u}_0, \hat{W}_c). \tag{56}$$

The objective now becomes designing the NN scheme to minimize $e_H(t)$. Motivated by [32], to obtain the weights of the critic NN, the gradient-descent-based adaptive tuning law is designed as

$$\dot{\hat{W}}_c = -\alpha\frac{\pi}{(\pi^{\mathrm{T}}\pi + 1)^2}e_H(t) + \frac{\beta}{2}\chi(z, \hat{u}_0)\nabla h(z)R(z)N^{-1}R^{\mathrm{T}}(z)V_{Hz}, \tag{57}$$

with

$$\chi(z, \hat{u}_0) = \begin{cases} 0, \text{ if } \dot{V}_H(z) < 0, \\ 1, \text{ otherwise}, \end{cases} \tag{58}$$

where $\alpha > 0$ and $\beta > 0$ are constants to be designed for learning rate, $\pi = \nabla h\big(F(z) - R(z)N^{-1}R^{\mathrm{T}}(z)\nabla h^{\mathrm{T}} \cdot\hat{W}_c/2\big)$, $V_H(z) = V_{Hz}^{\mathrm{T}}(F(z) - \frac{1}{2}N^{-1}R^{\mathrm{T}}(z)\nabla h^{\mathrm{T}}\hat{W}_c)$, which refers to the Lyapunov function candidate to be selected later, and V_{Hz} represents its partial derivative to z. Based on [32], the first term in (57) seeks to minimize $e_H(t)$ which was derived by the normalized gradient descent scheme, while the second term $\chi(z, \hat{u}_0)$ is an adjusting term which ensures the states of the closed loop system remain bounded while the NN scheme learns $\Gamma^*(z)$.

Remark 4. In general, the trajectory of the optimal closed-loop sliding motion dynamics $F(z)+R(z)u_0^*$ is always bounded by a positive constant, while here it is bounded as $\|F(z) + R(z)u_0^*\| \leqslant \phi(z)$, where $\phi(z) = \sqrt[4]{g_0\|V_{Hz}\|}$ and $V_H(z)$ is selected as a continuously differentiable and radially unbounded Lyapunov function candidate for (42). In addition, it satisfies $\dot{V}_H(z) = V_{Hz}^{\mathrm{T}}(F(z)+R(z)u_0^*) < 0$ with $V_{Hz} = \partial V_H/\partial z$. According to [32], $V_H(z)$ can be deliberately selected to satisfy the above requirements. Also, note that one can always find a positive definite matrix function $\Psi(z)$ such that the following holds:

$$V_{Hz}^{\mathrm{T}}(F(z) + R(z)u_0^*) = -V_{Hz}^{\mathrm{T}}\Psi(z)V_{Hz}. \tag{59}$$

We now define the NN weight approximation error:

$$\tilde{W}_c(t) = W_c^* - \hat{W}_c(t), \tag{60}$$

and in the following discussion, the convergence of $\tilde{W}_c(t)$ will be considered along with the stability of system (26).

Note that $\dot{\tilde{W}}_c(t) = -\dot{\hat{W}}_c(t)$, and $\dot{\hat{W}}_c(t)$ is involved with $e_H(t)$ as in (57). Thus we should find the relationship between $e_H(t)$ and $\tilde{W}_c(t)$. To this end, based on (55) and (56), it is shown that

$$e_H(t) = -\tilde{W}_c^{\mathrm{T}}\nabla h(z)(F(z) + R(z)u_0^* + \frac{1}{2}R(z)N^{-1}R^{\mathrm{T}}(z)\nabla\varepsilon(z))$$

$$
\begin{aligned}
&-\frac{1}{4}\tilde{W}_c^{\mathrm{T}}\nabla h(z)R(z)N^{-1}R^{\mathrm{T}}(z)\nabla h^{\mathrm{T}}(z)\tilde{W}_c-\varepsilon_{\mathrm{HJB}} \\
=&-\tilde{W}_c^{\mathrm{T}}\nabla h(z)T(z)-\frac{1}{4}\tilde{W}_c^{\mathrm{T}}B(z)\tilde{W}_c-\varepsilon_{\mathrm{HJB}},
\end{aligned}
\tag{61}
$$

with $B(z)=\nabla h(z)R(z)N^{-1}R^{\mathrm{T}}(z)\nabla h^{\mathrm{T}}(z)$.

As a result, it is derived that

$$
\begin{aligned}
\dot{\tilde{W}}_c(t)=&-\frac{\alpha}{\pi_c^2}(\nabla h(z)T(z)+\frac{1}{2}B(z)\tilde{W}_c(t))\left(\tilde{W}_c^{\mathrm{T}}\nabla h(z)T(z)+\frac{1}{4}\tilde{W}_c^{\mathrm{T}}B(z)\tilde{W}_c+\varepsilon_{\mathrm{HJB}}(t)\right) \\
&-\frac{\beta}{2}\chi(z,\hat{u}_0)\nabla h(z)R(z)N^{-1}R^{\mathrm{T}}(z)V_{Hz}.
\end{aligned}
\tag{62}
$$

We present the following Assumption 5, and the stability analysis and convergence of $\tilde{W}_c(t)$ are presented in the following Theorem 3.

Assumption 5. For the critic NN, the activation function $h(z)$ with its derivative $\nabla h(z)$, the ideal NN weight W_c^*, and the approximation error ε with its derivative $\nabla\varepsilon$ are bounded on a compact set, specifically, $\|h(z)\|\leqslant h_M$, $\|\nabla h(z)\|\leqslant h_{dM}$, $\|W_c^*\|\leqslant W_M$, $\|\varepsilon\|\leqslant\varepsilon_M$, and $\|\nabla\varepsilon\|\leqslant\varepsilon_{dM}$ with $h_M,h_{dM},W_M,\varepsilon_M.$ and $\varepsilon_{dM}>0$.

Theorem 3. Considering the sliding motion dynamics (42) with the cost function $\Gamma(z)$, under the control input u_0 in (53) and the adaptive tuning law for critic NNs in (57), the attitude state $z(t)$ and the approximation error for $\widetilde{W}_c(t)$ can be ensured to be uniformly ultimately bounded (UUB). Moreover, the obtained control input $\hat{u}_0$ will converge to the optimal control u_0^* approximately with an upper bound.

Proof. The Lyapunov function candidate is chosen as

$$
V_{\mathrm{HJB}}(t)=\beta V_H(z)+\frac{1}{2}\tilde{W}_c^{\mathrm{T}}\tilde{W}_c. \tag{63}
$$

Then it is derived that

$$
\dot{V}_{\mathrm{HJB}}(t)=\beta V_{Hz}^{\mathrm{T}}\dot{z}+\tilde{W}_c^{\mathrm{T}}\dot{\tilde{W}}_c. \tag{64}
$$

Substituting (62) into (64) yields

$$
\dot{V}_{\mathrm{HJB}}(t)=Y_1(z,t)-\frac{\beta}{2}\tilde{W}_c^{\mathrm{T}}\chi(z,\hat{u}_0)\nabla h(z)R(z)N^{-1}R^{\mathrm{T}}(z)V_H(z)+\beta V_{Hz}^{\mathrm{T}}\dot{z}, \tag{65}
$$

where

$$
\begin{aligned}
Y_1(z,t)\triangleq&-\frac{\alpha}{\pi_c^2}\big((\tilde{W}_c^{\mathrm{T}}\nabla h(z)T(z))^2+\frac{3}{4}\tilde{W}_c^{\mathrm{T}}\nabla h(z)T(z)\tilde{W}_c^{\mathrm{T}}B(z)\tilde{W}_c \\
&+\frac{1}{8}(\tilde{W}_c^{\mathrm{T}}B(z)\tilde{W}_c)^2+\tilde{W}_c^{\mathrm{T}}\nabla h(z)T(z)\varepsilon_{\mathrm{HJB}}+\frac{1}{2}\tilde{W}_c^{\mathrm{T}}B(z)\tilde{W}_c\varepsilon_{\mathrm{HJB}}(t)\big).
\end{aligned}
\tag{66}
$$

Note that the following holds:

$$
\begin{aligned}
Y_1(z,t)\leqslant&-\frac{\alpha}{16\pi_c^2}\|\tilde{W}_c^{\mathrm{T}}B(z)\tilde{W}_c\|^2+\frac{4\alpha}{\pi_c^2}\|\tilde{W}_c^{\mathrm{T}}\nabla h(z)T(z)\|^2+\frac{5\alpha}{2\pi_c^2}\varepsilon_{\mathrm{HJB}}^2(t) \\
\leqslant&-\frac{\alpha m_0}{16\pi_c^2}\|\tilde{W}_c^{\mathrm{T}}\nabla h(z)\|^4+\frac{4\alpha}{\pi_c^2}\|\tilde{W}_c^{\mathrm{T}}\nabla h(z)\|^2\|T(z)\|^2+\frac{5\alpha}{2\pi_c^2}\varepsilon_{\mathrm{HJB}}^2(t),
\end{aligned}
\tag{67}
$$

where $m_0=r_m^2/\lambda_{\max}(N)$, $\lambda_m\leqslant\|R(z)\|\leqslant\lambda_M$.

Moreover, for the term $\|\tilde{W}_c^{\mathrm{T}}\nabla h(z)\|^2\|T(z)\|^2$ in (67), note that the following inequality holds:

$$
\|\tilde{W}_c^{\mathrm{T}}\nabla h(z)\|^2\|T(z)\|^2\leqslant\frac{1}{2m_1^2}\|\tilde{W}_c^{\mathrm{T}}\nabla h(z)\|^4+\frac{m_1^2}{2}\|T(z)\|^4, \tag{68}
$$

for any given constant $m_1>0$. Considering the fact that $1/\pi_c^2\leqslant 1$, and supposing that $\|\pi_c\|\leqslant\pi_M$ with $\pi_M>0$, it yields that

$$
Y_1(z,t)\leqslant-\left(\frac{m_0}{16\pi_M^2}-\frac{2}{m_1^2}\right)\alpha\|\tilde{W}_c^{\mathrm{T}}\nabla h(z)\|^4+2m_1^2\alpha\|T(z)\|^4+\frac{5\alpha}{2\pi_c^2}\varepsilon_{\mathrm{HJB}}^2(t), \tag{69}
$$

where m_1 satisfies $m_0/(16\pi_M^2) - 2/m_1^2 > 0$. The term $\|T(z)\|$ satisfies the property $\|T(z)\| \leqslant \phi(z) + m_2$ according to Remark 4 with $m_2 = r_M^2 \varepsilon_{dM}/(2\lambda_{\min}(N))$. Considering the fact $\phi(z) = \sqrt[4]{g_0 V_{Hz}}$, based on Cauchy-Schwarz inequality, it is derived that

$$\begin{aligned} Y_1(z,t) \leqslant & -\left(\frac{m_0}{16\pi_M^2} - \frac{2}{m_1^2}\right)\alpha\|\tilde{W}_c^{\mathrm{T}}\nabla h(z)\|^4 + 16m_1^2\alpha\|V_{Hz}\| + 16m_1^2\alpha m_2^4 + \frac{5\alpha}{2\pi_c^2}\varepsilon_M^2 \\ \leqslant & -\left(\frac{m_0}{16\pi_M^2} - \frac{2}{m_1^2}\right)\alpha h_M^4\|\tilde{W}_c^{\mathrm{T}}\|^4 + 16\alpha m_1^2\|V_{Hz}\| + \alpha\Xi_m, \end{aligned} \tag{70}$$

where $\Xi_m = 16m_1^2 m_2^4 + 5\varepsilon_M^2/(2\pi_c^2)$.

Let $\bar{\ell}_0 = (\frac{m_0}{16\pi_M^2} - \frac{2}{m_1^2})$. Then the derivative of (63) satisfies

$$\begin{aligned} \dot{V}_{\mathrm{HJB}} \leqslant & -\alpha\bar{\ell}_0 h_M^4\|\tilde{W}_c^{\mathrm{T}}\|^4 + 16\alpha m_1^2\|V_{Hz}\| + \alpha\Xi_m + \beta V_{Hz}^{\mathrm{T}}\dot{z} \\ & -\frac{\beta}{2}\tilde{W}_c^{\mathrm{T}}\chi(z,\hat{u}_0)\nabla h(z)R(z)N^{-1}R^{\mathrm{T}}(z)V_{Hz}. \end{aligned} \tag{71}$$

We now consider the adjusting term $\chi(z,\hat{u}_0)$ in (71) in terms of the following two cases (i) and (ii).

(i) For the case $\chi(z,\hat{u}_0) = 0$, it can be concluded that $\dot{V}_H < 0$, i.e. $V_{Hz}^{\mathrm{T}}\dot{z} < 0$. Considering that $\|z\| > 0$ and the strict inequality $V_{Hz}^{\mathrm{T}}\dot{z} < 0$ holds, there must exist a constant $\dot{z}_{\min}$ satisfying $0 < \dot{z}_{\min} < \|\dot{z}\|$. Then the following inequality is derived:

$$\dot{V}_{\mathrm{HJB}} \leqslant -\alpha\bar{\ell}_0 h_M^4\|\tilde{W}_c^{\mathrm{T}}\|^4 - (\beta\dot{z}_{\min} - 16\alpha m_1^2)\|V_{Hz}\| + \alpha\Xi_m. \tag{72}$$

For (72), given that α and β are selected to guarantee $\beta\dot{z}_{\min} - 16\alpha m_1^2 > 0$, since $\chi(z,\hat{u}_0) = 0$, $\dot{V}_{\mathrm{HJB}} < 0$ holds if the following inequalities are satisfied:

$$\|V_{Hz}\| > \frac{\alpha\Xi_m}{\beta\dot{z}_{\min} - 16\alpha m_1^2} = \mathcal{A}_1 \tag{73}$$

or

$$\|\tilde{W}_c\| > \frac{1}{h_M}\sqrt[4]{\frac{\Xi_m}{\alpha\bar{\ell}_0}} = \mathcal{B}_1. \tag{74}$$

(ii) Considering the case $\chi(z,\hat{u}_0) = 1$, which means the closed-loop sliding motion exhibits unstable behavoir. Adding and substracting the term $\beta V_{Hz}^{\mathrm{T}}R(z)N^{-1}R^{\mathrm{T}}(z)(\nabla h^{\mathrm{T}}(z)W_c^* + \nabla\varepsilon)/2$ to (71), it yields

$$\begin{aligned} \dot{V}_{\mathrm{HJB}} \leqslant & -\alpha\bar{\ell}_0 h_M^4\|\tilde{W}_c^{\mathrm{T}}\|^4 + \beta V_{Hz}^{\mathrm{T}}(F(z) + R(z)u_0^*) \\ & + \frac{\beta}{2}V_{Hz}^{\mathrm{T}}R(z)N^{-1}R^{\mathrm{T}}(z)\nabla\varepsilon + 16\alpha m_1^2\|V_{Hz}\| + \alpha\Xi_m. \end{aligned} \tag{75}$$

According to (59), it yields that

$$\dot{V}_{\mathrm{HJB}} \leqslant -\alpha\bar{\ell}_0 h_M^4\|\tilde{W}_c^{\mathrm{T}}\|^4 - \frac{\beta}{2}\lambda_{\min}(\Psi)\|V_{Hz}\|^2 + \frac{m_2^2}{2\beta\lambda_{\min}(\Psi)} + \alpha\Xi_m, \tag{76}$$

where $m_2 = 16\alpha m_1^2 + \beta r_M^2\varepsilon_{dM}^2/(2\lambda_{\min}(N))$ and $\lambda_{\min}(\Psi)$ represents the minimum eigenvalue of $\Psi(z)$. Therefore, $\dot{V}_{\mathrm{HJB}} < 0$ holds in the case $\chi(z,\hat{u}_0) = 1$ when

$$\|V_{Hz}\| > \frac{\sqrt{m_2^2 + 2\alpha\beta\lambda_{\min}(\Psi)\Xi_m}}{\beta\lambda_{\min}(\Psi)} = \mathcal{A}_2 \tag{77}$$

or

$$\|\tilde{W}_c\| > \frac{1}{h_M}\sqrt[4]{\frac{m_2^2 + 2\alpha\beta\lambda_{\min}(\Psi)\Xi_m}{\alpha\beta\lambda_{\min}(\Psi)\bar{\ell}_0}} = \mathcal{B}_2. \tag{78}$$

We now summarize the aforementioned two cases (i) and (ii) as follows. If α and β are selected to satisfy $\beta\dot{z}_{\min} - 16\alpha m_1^2 > 0$, then $\dot{V}_{\mathrm{HJB}} < 0$ holds for $\|V_{Hz}\| > \max\{\mathcal{A}_1, \mathcal{A}_2\} = \bar{\mathcal{A}}$, or $\|\tilde{W}_c\| > \max\{\mathcal{B}_1, \mathcal{B}_2\} = \bar{\mathcal{B}}$.

Table 1 Flexible spacecraft parameters

Symbol	Value
J	$\begin{bmatrix} 350 & 3 & 4 \\ 3 & 280 & 10 \\ 4 & 10 & 190 \end{bmatrix} (\mathrm{kg}\cdot\mathrm{m}^2)$
δ	$\begin{bmatrix} 6.45637 & 1.27814 & 2.15629 \\ -1.25619 & 0.91756 & -1.67264 \\ 1.11687 & 2.48901 & -0.83674 \\ 1.23637 & -2.6581 & -1.12503 \end{bmatrix} (\mathrm{kg}^{\frac{1}{2}}\cdot\mathrm{m/s}^2)$
Ω_n	$[0.7681, 1.1038, 1.8733, 2.5496]^{\mathrm{T}}$
ζ	$[0.005607, 0.008620, 0.01283, 0.02516]^{\mathrm{T}}$

That means $\|V_{Hz}\|$ and $\tilde{W}_c$ are UUB. Recalling the fact that $V_H(z)$ is a Lyapunov function candidate, the boundedness of $\|V_{Hz}\|$ thus implies that of $z(t)$.

In the following, we prove that the approximated control law $\hat{u}_0$ converges to a bounded neighborhood of the optimal control policy u_0^*. Note that the following:

$$\hat{u}_0 - u_0^* = -\frac{1}{2}N^{-1}R^{\mathrm{T}}(z)\nabla h^{\mathrm{T}}\tilde{W}_c - \frac{1}{2}N^{-1}R^{\mathrm{T}}(z)\nabla\varepsilon \tag{79}$$

holds. According to Assumption 5, it can be obtained that

$$\|\hat{u}_0 - u_0^*\| \leqslant \frac{1}{2}\lambda_{\max}(N^{-1})r_M(h_{dM}\bar{\mathcal{B}} + \varepsilon_{dM}), \tag{80}$$

which implies that $\hat{u}_0$ is approximated to u_0^* with an upper bound.

4 Illustrative example

In this section, the simulation results are presented to show the efficacy of the proposed method. The flexible spacecraft parameter refers to [33], which is illustrated in Table 1. The initial attitude is set as $\sigma(0) = [0.2,\ -0.01,\ 0.01]^{\mathrm{T}}$, $\omega(0) = [-1, -2,\ -3]^{\mathrm{T}}$ °/s.

The actuator is assumed to be subjected to the time-varying partial loss of efficiency with the efficiency matrix given as

$$\Lambda(\rho) = \begin{bmatrix} 0.8 + 0.2\mathrm{e}^{-0.5t} & 0 & 0 \\ 0 & 0.7 & 0 \\ 0 & 0 & 0.64 + 0.3^{-0.3t} \end{bmatrix}.$$

Then the lower and upper bounds for the actuator efficiency factors are given as $\underline{\rho}_m = 0.3$, $\overline{\rho}_M = 1$. The external disturbances are given as

$$\begin{bmatrix} 5 + 2.5\sin(0.1t) \\ -4 + 2\cos(0.05t) \\ 3 - 8\sin(0.3t) \end{bmatrix} \times 10^{-3}\ \mathrm{N}\cdot\mathrm{m}.$$

The hysteresis quantizer is employed in the controller-actuator channel with $\mu_{\min} = 0.1$ and $\varkappa = 0.85$. Then κ_q can be calculated as $\kappa_q = (1-\varkappa)/(1+\varkappa)$.

For the observer (16), the gain parameters are set as $T_f = 0.01$ s,

$$L_f = \begin{bmatrix} 84 & 0 & 0 \\ 0 & 38 & 0 \\ 0 & 0 & 110 \end{bmatrix}, \quad K_{f_1} = \begin{bmatrix} 0.9 & 0 & 0 \\ 0 & 0.9 & 0 \\ 0 & 0 & 0.9 \end{bmatrix},$$

$$K_{f_2} = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix}, \quad \Theta = \begin{bmatrix} 0.4 & 0 & 0 \\ 0 & 0.42 & 1.6 \end{bmatrix},$$

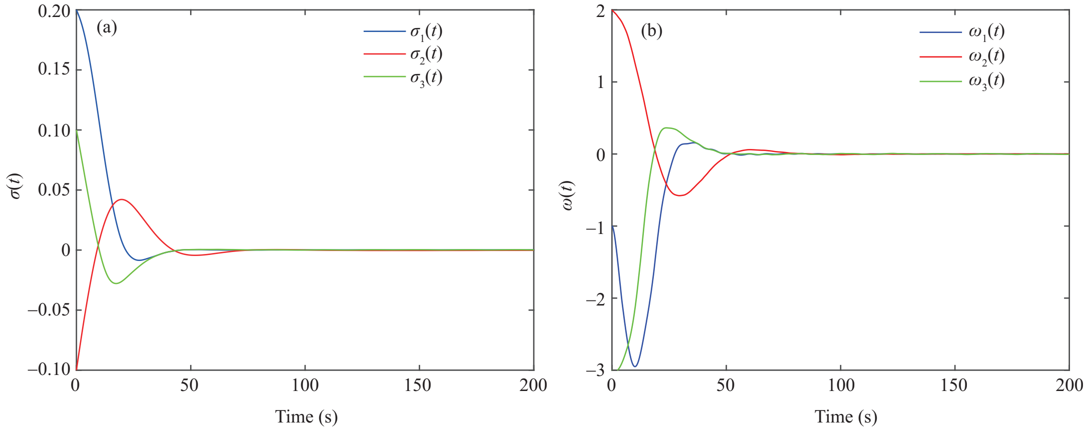

Figure 2 (Color online) The curves of (a) the attitude MRPs $\sigma(t)$ and (b) the angular velocity $\omega(t)$.

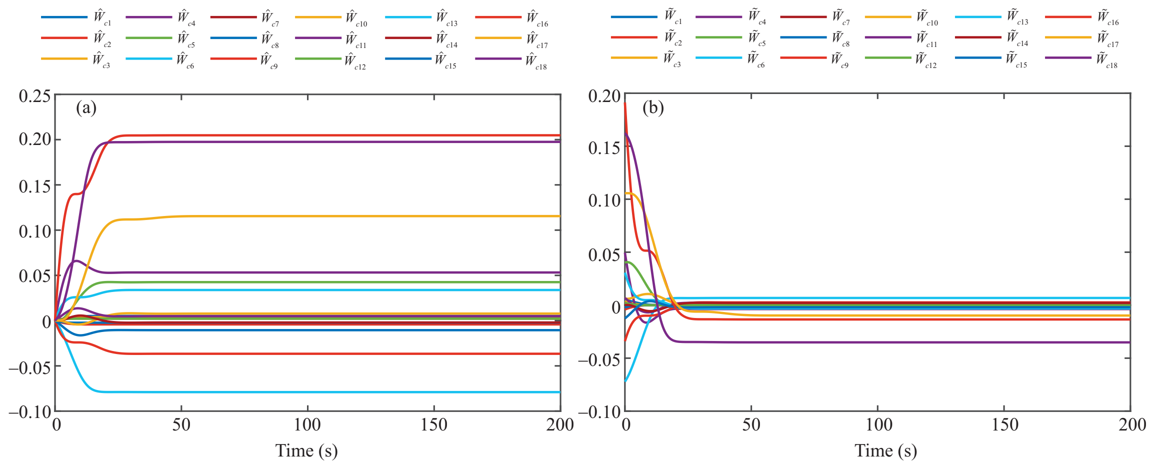

Figure 3 (Color online) (a) Estimation of NN weights $\hat{W}_c(t)$ and (b) NN weight approximation errors $\tilde{W}_c(t)$.

and we set $P = 1.01K_{f_2}$.

The parameter in the virtual control law (22) is set as $k_\sigma = 0.1$, and the controller parameters are set as $k_s = 0.1$, $\varsigma_0 = 50$. The performance function (43) is designed as

$$\Gamma(z(t_0), u_0) = \int_{t_0}^{t} [z^{\mathrm{T}}(t)Mz(t) + u_0^{\mathrm{T}} N u_0]\mathrm{d}\sigma \tag{81}$$

with $M = I_6$ and $N = 4J_0^{-\mathrm{T}}J_0^{-1}$.

The activation function for NN is selected as

$$\begin{aligned} h(z) = \big[&z_1^2,\ z_1z_2,\ z_1z_3,\ z_1z_4,\ z_1z_5,\ z_1z_6,\ z_2^2,\\ &z_2z_3,\ z_2z_4,\ z_2z_5,\ z_2z_6,\ z_3^2,\ z_3z_4,\ z_3z_5,\ z_3z_6,\\ &10z_4\arctan(10z_4) - 0.5\ln(1+100z_4^2),\\ &10z_5\arctan(10z_5) - 0.5\ln(1+100z_5^2),\\ &10z_6\arctan(10z_6) - 0.5\ln(1+100z_6^2)\big]. \end{aligned}$$

Figures 2(a) and (b) present the attitude MRPs $\sigma(t)$ and angular velocity $\omega(t)$, respectively. Figures 3(a) and (b) show that the NN weights and approximation errors are convergent. Figure 4(a) shows that the actuator faults are identified with small errors, and the estimated lumped disturbances with accepted errors are illustrated in Figure 4(b). Figures 5(a)–(c) illustrate the control command u_c with its quantization $Q(u_c)$.

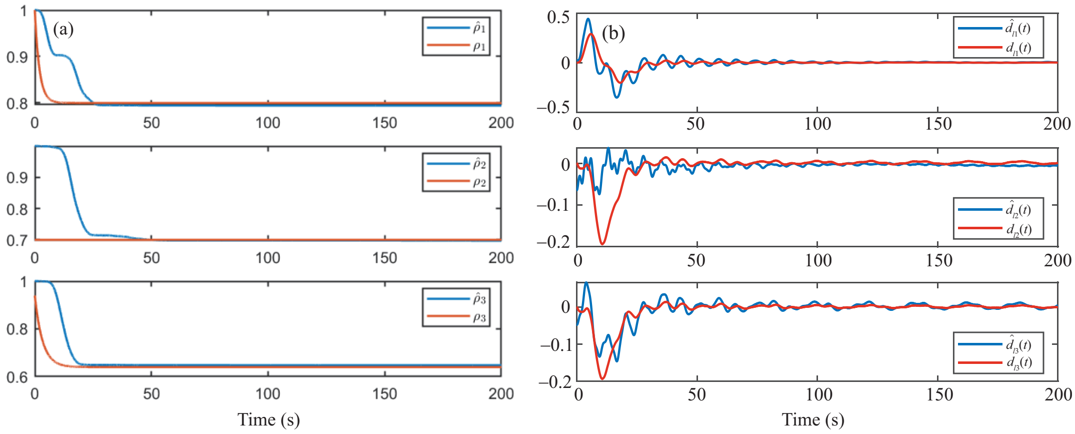

Figure 4 (Color online) Estimation of (a) the actuator efficiency factors $\hat{\rho}$ and (b) the lumped disturbances $\hat{d}_l$.

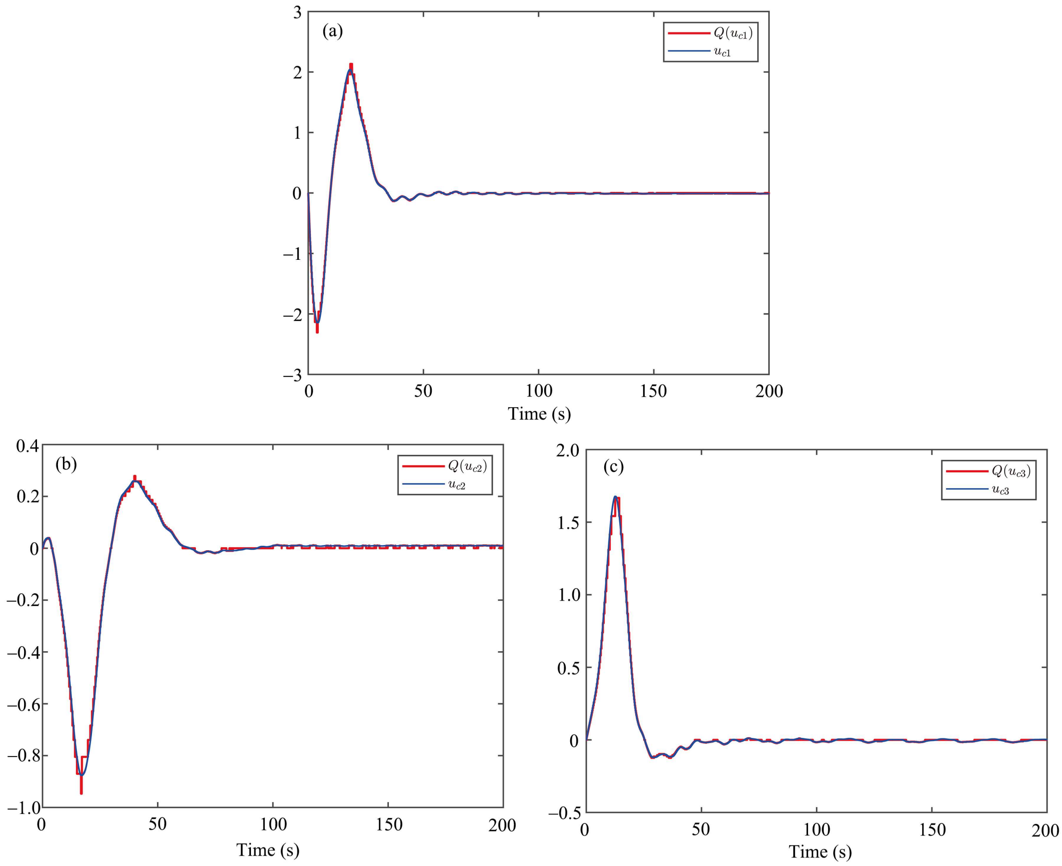

Figure 5 (Color online) Control commands (a) u_{c1}, (b) u_{c2}, and (c) u_{c3} and their quantization.

5 Conclusion

In this paper, the FTC problem for flexible spacecraft with actuator faults and input quantization has been investigated. An adaptive ILO procedure is first proposed to estimate the actuator faults and rigid-flexible nonlinear dynamics, and following the observer estimation and the ADP and NN methods, an integral sliding mode control law has been developed by considering optimal performance and minimal energy cost. Future work will consider extending the acquired design techniques to multi-spacecraft or spacecraft swarm problems.

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Appendix A Proof of Theorem 1

Let $\tilde{\omega} = \omega - \hat{\omega}$, $\tilde{d}_l = d_l - \hat{d}_l$, $\tilde{\rho} = \rho - \hat{\rho}$, and $\tilde{\theta} = d_l(t) - K_{f1} d_l(t - T_f)$. The estimation error dynamics

$$J_0 \dot{\tilde{\omega}} = -(\omega^{\times} J_0 \omega - \hat{\omega}^{\times} J_0 \hat{\omega}) - L_f \tilde{\omega} + \Lambda(\tilde{\rho}) Q(u_c) + \tilde{d}_l \tag{A1}$$

with $\tilde{d}_l(t) = K_{f1}\tilde{d}_l(t-T_f) - K_{f2}\tilde{\omega} + \tilde{\theta}(t)$. There exists

$$\begin{aligned}\tilde{d}_l^{\mathrm{T}}(t)\tilde{d}_l(t) =& \tilde{d}_l^{\mathrm{T}}(t-T_f)K_{f1}^{\mathrm{T}}K_{f1}\tilde{d}_l(t-T_f) + \tilde{\omega}^{\mathrm{T}}K_{f2}^{\mathrm{T}}K_{f2}\tilde{\omega} + \tilde{\theta}(t)^{\mathrm{T}}\tilde{\theta}(t) - 2\tilde{\omega}^{\mathrm{T}}K_{f2}^{\mathrm{T}}K_{f1}\tilde{d}_l(t-T_f)\\ &+ 2\tilde{d}_l^{\mathrm{T}}(t-T_f)K_{f1}\tilde{\theta}(t) - 2\tilde{\omega}^{\mathrm{T}}K_{f2}^{\mathrm{T}}\theta(t).\end{aligned} \tag{A2}$$

The Lyapunov function is selected as

$$V_0 = \tilde{\omega}PJ_0\tilde{\omega} + \int_{t-T_f}^{t}\tilde{d}_l^{\mathrm{T}}(s)\tilde{d}_l(s)ds + \tilde{\rho}^{\mathrm{T}}\Theta^{-1}\tilde{\rho}. \tag{A3}$$

The derivative of V_0 is calculated as

$$\begin{aligned}\dot{V}_0 =& 2\tilde{\omega}^{\mathrm{T}}P[-(\omega^{\times}J_0\omega - \hat{\omega}^{\times}J_0\hat{\omega}) - L_f\tilde{\omega} + \Lambda(\tilde{\rho})Q(u_c) + K_{f1}\tilde{d}_l(t-T_f) - K_{f2}\tilde{\omega} + \tilde{\theta}(t)]\\ &- \gamma_1\tilde{d}_l^{\mathrm{T}}(t)\tilde{d}_l(t) - \tilde{d}_l^{\mathrm{T}}(t-T_f)\tilde{d}_l(t-T_f) + \bar{\gamma}_1\tilde{d}_l^{\mathrm{T}}(t)\tilde{d}_l(t) + 2\tilde{\rho}^{\mathrm{T}}\Theta^{-1}\dot{\tilde{\rho}}\end{aligned} \tag{A4}$$

with $\bar{\gamma}_1 = \gamma + 1$.

In (A4), note that the following derivation holds:

$$\begin{aligned}\Lambda(\tilde{\rho})Q(u_c) =& \mathrm{diag}\{\tilde{\rho}_1(t),\ \tilde{\rho}_2(t),\ \tilde{\rho}_3(t)\}[Q_1(u_c),\ Q_2(u_c),\ Q_3(u_c)]^{\mathrm{T}}\\ =& [\tilde{\rho}_1(t)Q_1(u_c),\ \tilde{\rho}_2(t)Q_2(u_c),\ \tilde{\rho}_3(t)Q_3(u_c)]^{\mathrm{T}}\\ =& \mathrm{diag}\{Q_1(u_c),\ Q_2(u_c),\ Q_3(u_c)\}[\tilde{\rho}_1(t),\ \tilde{\rho}_2(t),\ \tilde{\rho}_3(t)]^{\mathrm{T}}\\ =& \Lambda(Q(u_c))\tilde{\rho}(t).\end{aligned}$$

Then, by substituting (A2) into (A4), one can obtain

$$\begin{aligned}\dot{V}_0 =& -2\tilde{\omega}^{\mathrm{T}}PL_f\tilde{\omega} - 2\tilde{\omega}^{\mathrm{T}}P(\omega^{\times}J_0\omega - \hat{\omega}^{\times}J_0\hat{\omega}) + 2\tilde{\omega}^{\mathrm{T}}P\Lambda(Q(u_c))\tilde{\rho} + 2\tilde{\omega}^{\mathrm{T}}PK_{f1}\tilde{d}_l(t-T_f)\\ &- 2\tilde{\omega}^{\mathrm{T}}PK_{f2}\tilde{\omega} + 2\tilde{\omega}^{\mathrm{T}}P\tilde{\theta} - \gamma_1\tilde{d}_l^{\mathrm{T}}(t)\tilde{d}_l(t) - \tilde{d}_l^{\mathrm{T}}(t-T_f)\tilde{d}_l(t-T_f) + \bar{\gamma}_1\tilde{\omega}^{\mathrm{T}}K_{f2}^{\mathrm{T}}K_{f2}\tilde{\omega}\\ &+ \bar{\gamma}_1\tilde{d}_l^{\mathrm{T}}(t-T_f)K_{f1}^{\mathrm{T}}K_{f1}\tilde{d}_l(t-T_f) + \bar{\gamma}_1\tilde{\theta}^{\mathrm{T}}\tilde{\theta} - 2\bar{\gamma}_1\tilde{\omega}^{\mathrm{T}}K_{f2}^{\mathrm{T}}K_{f1}\tilde{d}_l(t-T_f) + 2\bar{\gamma}_1\tilde{d}_l^{\mathrm{T}}(t-T_f)K_{f1}^{\mathrm{T}}\tilde{\theta}\\ &- 2\bar{\gamma}_1\tilde{\omega}^{\mathrm{T}}K_{f2}^{\mathrm{T}}\tilde{\theta} + 2\tilde{\rho}^{\mathrm{T}}\Theta^{-1}\dot{\tilde{\rho}}.\end{aligned} \tag{A5}$$

Noticing $P = \bar{\gamma}_1K_{f2}$, it yields that

$$\begin{aligned}\dot{V}_0 =& -2\tilde{\omega}^{\mathrm{T}}PL_f\tilde{\omega} - 2\tilde{\omega}^{\mathrm{T}}P(\omega^{\times}J_0\omega - \hat{\omega}^{\times}J_0\hat{\omega}) + 2\tilde{\omega}^{\mathrm{T}}P\Lambda(Q(u_c))\tilde{\rho} - \bar{\gamma}_1\tilde{\omega}^{\mathrm{T}}K_{f2}^{\mathrm{T}}K_{f2}\tilde{\omega}\\ &- \gamma_1\tilde{d}_l^{\mathrm{T}}(t)\tilde{d}_l(t) - \tilde{d}_l^{\mathrm{T}}(t-T_f)\tilde{d}_l(t-T_f) + \bar{\gamma}_1\tilde{\theta}^{\mathrm{T}}\tilde{\theta} + \bar{\gamma}_1\tilde{d}_l^{\mathrm{T}}(t-T_f)K_{f1}^{\mathrm{T}}K_{f1}\tilde{d}_l(t-T_f)\\ &+ 2\bar{\gamma}_1\tilde{d}_l^{\mathrm{T}}(t-T_f)K_{f1}^{\mathrm{T}}\tilde{\theta} + 2\tilde{\rho}^{\mathrm{T}}\Theta^{-1}\dot{\tilde{\rho}}.\end{aligned} \tag{A6}$$

Considering the term $2\bar{\gamma}_1\tilde{d}_l^{\mathrm{T}}(t-T_f)K_{f1}^{\mathrm{T}}\tilde{\theta}$, there exists

$$2\bar{\gamma}_1\tilde{d}_l^{\mathrm{T}}(t-T_f)K_{f1}^{\mathrm{T}}\tilde{\theta} \leqslant \gamma_2\tilde{d}_l^{\mathrm{T}}(t-T_f)K_{f1}^{\mathrm{T}}K_{f1}d_l(t-T_f) + \frac{\bar{\gamma}_1^2}{\gamma_2}\tilde{\theta}^{\mathrm{T}}\tilde{\theta}. \tag{A7}$$

Substituting (A7) into (A6) yields

$$\begin{aligned}\dot{V}_0 \leqslant& -2\tilde{\omega}^{\mathrm{T}}PL_f\tilde{\omega} - 2\tilde{\omega}^{\mathrm{T}}P(\omega^{\times}J_0\omega - \hat{\omega}^{\times}J_0\hat{\omega}) + 2\tilde{\omega}^{\mathrm{T}}P\Lambda(Q(u_c))\tilde{\rho} - \bar{\gamma}_1\tilde{\omega}^{\mathrm{T}}K_{f2}^{\mathrm{T}}K_{f2}\tilde{\omega}\\ &- \tilde{d}_l^{\mathrm{T}}(t-T_f)(I - \delta_1K_{f1}^{\mathrm{T}}K_{f1})\tilde{d}_l(t-T_f) - \gamma_1\tilde{d}_l^{\mathrm{T}}(t)\tilde{d}_l(t) + \delta_2\tilde{\theta}^{\mathrm{T}}\tilde{\theta} + 2\tilde{\rho}^{\mathrm{T}}\Theta^{-1}\dot{\tilde{\rho}}\end{aligned} \tag{A8}$$

with $\delta_1 = \gamma_1 + \gamma_2$ and $\delta_2 = \bar{\gamma}_1 + \bar{\gamma}_1^2/\gamma_2$.

According to Assumption 4 and applying the adaptive law for $\hat{\rho}$ to (A8), it can be obtained that

$$\begin{aligned}\dot{V}_0 \leqslant& -2\tilde{\omega}^{\mathrm{T}}PL_f\tilde{\omega} + 2\delta_0\lambda_{\max}(P)\tilde{\omega}^{\mathrm{T}}\tilde{\omega} - \bar{\gamma}_1\tilde{\omega}^{\mathrm{T}}K_{f2}^{\mathrm{T}}K_{f2}\tilde{\omega} - \gamma_1\tilde{d}_l^{\mathrm{T}}(t)\tilde{d}_l(t) + \delta_2\tilde{\theta}^{\mathrm{T}}\tilde{\theta}\\ &- \tilde{d}_l^{\mathrm{T}}(t-T_f)(I - \delta_1K_{f1}^{\mathrm{T}}K_{f1})\tilde{d}_l(t-T_f) + 2\tilde{\rho}^{\mathrm{T}}\Theta^{-1}\dot{\rho}.\end{aligned} \tag{A9}$$

According to Assumption 1, the term $\tilde{\rho}\Theta^{-1}\dot{\rho}$ can be unfolded as

$$\tilde{\rho}\Theta^{-1}\dot{\rho} = \tilde{\rho}\Theta^{-1}(-\tilde{\rho} + \tilde{\rho} + \dot{\rho}) = -\tilde{\rho}\Theta^{-1}\tilde{\rho} + \sum_{i=1}^{3}(\rho_M - \rho_m)\vartheta_i(\rho_M - \rho_m + \bar{\dot{\rho}}). \tag{A10}$$

It can be further derived that

$$\begin{aligned}\dot{V}_0 \leqslant& -\tilde{\omega}^{\mathrm{T}}(2PL_f + \bar{\gamma}_1K_{f2}^{\mathrm{T}}K_{f2} - 2\delta_0\lambda_{\max}(P)I)\tilde{\omega} - 2\tilde{\rho}\Theta^{-1}\tilde{\rho} - \gamma_1\tilde{d}_l^{\mathrm{T}}(t)\tilde{d}_l(t)\\ &- \tilde{d}_l^{\mathrm{T}}(t-T_f)(I - \delta_1K_{f1}^{\mathrm{T}}K_{f1})\tilde{d}_l(t-T_f) + \delta_2\tilde{\theta}^{\mathrm{T}}\tilde{\theta} + 2\sum_{i=1}^{3}(\rho_M - \rho_m)\vartheta_i(\rho_M - \rho_m + \bar{\dot{\rho}}).\end{aligned} \tag{A11}$$

It can be concluded that $\tilde{\omega}$, $\tilde{\rho}$, $\tilde{d}_l$ are bounded.