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Randomized multimodel multiple hypothesis tracking

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We consider the multiple maneuvering target tracking (MMTT) problem in the framework of multiple hypothesis tracking (MHT) using randomized data association and random coefficient matrix Kalman filtering (RCMKF). MMTT is an issue of importance in target tracking [1–5]. The multiscan data association (MSDA) problem that arose in MHT is a $\{0,1\}$ linear integer programming (LIP) problem [6], which is nondeterministic polynominal-hard (NP-hard). The complicating factor in MMTT is not only the data association problem but also the uncertainty of target dynamics.

In this study, we propose a randomized multimodel multiple hypothesis tracking (RMM-MHT) method, which has three distinct advantages. First, by transforming the NP-hard LIP with $\{0,1\}$ integer constraints into linear programming (LP) with the convex interval $[0,1]$ constraints, the given system information yields a randomized data association solution that maximizes the expectation of the logarithm of the posterior probability, has low computational complexity, and is suitable for tracking a large number of targets. Second, the state estimation is derived by the RCMKF [7,8], where the random coefficient matrix dynamic system is constructed based on the randomized data association probability solution. Third, the probability that the target follows a specific dynamic model, is derived by jointly optimizing the multiple possible models and data association hypotheses, and does not require prior mode transition probabilities. Thus, the proposed method is more robust for tracking multiple maneuvering targets.

Randomized multimodel multiscan data association. At time k, we consider a known number T of targets and define the joint state as vector $\boldsymbol{X}_k = ((\boldsymbol{x}_k^1)', \ldots, (\boldsymbol{x}_k^T)')'$. For tracking maneuvering targets, a set of S_n dynamic models is applied to describe the target motion at time $k+n$. For target τ, the state equation of the s_n-th model is expressed as $\boldsymbol{x}_{k+n+1}^{\tau} = F_{k+n}^{(s_{n+1})}\boldsymbol{x}_{k+n}^{\tau} + v_{k+n}^{(s_{n+1})}$, where $F_{k+n}^{(\cdot)}$ and $v_{k+n}^{(\cdot)}$ are the state transition matrix and process noise, respectively. The measurements obtained from $k+1$ to $k+N$ are defined as $\boldsymbol{Z} = (\boldsymbol{z}'_{k+1}, \ldots, \boldsymbol{z}'_{k+N})'$, where $\boldsymbol{z}_{k+n} = ((\boldsymbol{z}_{k+n}^1)', \ldots, (\boldsymbol{z}_{k+n}^{R_n})')'$, and R_n is the number of measurements at time $k+n$, $n = 1, \ldots, N$. The measurement model is assumed to be linear Gaussian; i.e., if measurement r_n originates from target τ, then $\boldsymbol{z}_{k+n}^{r_n} = H_{k+n}\boldsymbol{x}_{k+n}^{\tau} + w_{k+n}^{r_n}$, where H_{k+n} and $w_{k+n}^{r_n}$ are the measurement matrix and noise, respectively.

A track hypothesis comprises a target, N models, and N measurements for a total of N scans, and it can be represented by the corresponding index $(\tau, \boldsymbol{s}, \boldsymbol{r})$, where τ is the index of a target, and $\boldsymbol{s} = (s_1, \ldots, s_N)$ and $\boldsymbol{r} = (r_1, \ldots, r_N)$ are the index vectors of the models and measurements in N scans, respectively.

The probability $P_{(\tau,\boldsymbol{s},\boldsymbol{r})}$ that the track hypothesis $(\tau, \boldsymbol{s}, \boldsymbol{r})$ is selected is expressed as follows:

$$\delta_{(\tau,\boldsymbol{s},\boldsymbol{r})} := \begin{cases} 1, & \text{with probability } P_{(\tau,\boldsymbol{s},\boldsymbol{r})}, \\ 0, & \text{with probability } 1 - P_{(\tau,\boldsymbol{s},\boldsymbol{r})}. \end{cases} \tag{1}$$

A randomized global hypothesis comprises track hypotheses whose probabilities satisfy the following constraints:

$$\sum_{\boldsymbol{s}}\sum_{\boldsymbol{r}} P_{(\tau,\boldsymbol{s},\boldsymbol{r})} = 1, \text{ for } \tau = 1, \ldots, T, \tag{2}$$

$$\sum_{\tau}\sum_{\boldsymbol{s}}\sum_{\boldsymbol{r}\setminus\{r_n\}} P_{(\tau,\boldsymbol{s},\boldsymbol{r})} = 1, \quad \text{for } r_n = 1, \ldots, R_n \text{ and } n = 1, \ldots, N, \tag{3}$$

$$0 \leqslant P_{(\tau,\boldsymbol{s},\boldsymbol{r})} \leqslant 1 \text{ for all } (\tau, \boldsymbol{s}, \boldsymbol{r}), \tag{4}$$

where $\boldsymbol{r} \setminus \{r_n\} := (r_1, \ldots, r_{n-1}, r_{n+1}, \ldots, r_N)$. For each track hypothesis $(\tau, \boldsymbol{s}, \boldsymbol{r})$, let $L_{(\tau,\boldsymbol{s},\boldsymbol{r})}$ denote its score or likelihood ratio. With the randomized decision (1), the objective function of randomized multimodel multiscan data association (RMM-MSDA) is used to maximize the expectation of the logarithm of the posterior probability with respect to the randomized decision $\delta_{(\tau,\boldsymbol{s},\boldsymbol{r})}$ as follows:

$$\max_{P_{(\tau,\boldsymbol{s},\boldsymbol{r})}} E_{\delta_{(\tau,\boldsymbol{s},\boldsymbol{r})}}\left(\log\left[\prod_{\tau}\prod_{\boldsymbol{s}}\prod_{\boldsymbol{r}}\left[L_{(\tau,\boldsymbol{s},\boldsymbol{r})}\right]^{\delta_{(\tau,\boldsymbol{s},\boldsymbol{r})}}\right]\right) = \min_{P_{(\tau,\boldsymbol{s},\boldsymbol{r})}} \sum_{\tau}\sum_{\boldsymbol{s}}\sum_{\boldsymbol{r}}\left[-\log L_{(\tau,\boldsymbol{s},\boldsymbol{r})}\right] P_{(\tau,\boldsymbol{s},\boldsymbol{r})}. \tag{5}$$

Here, the equality in (5) holds with $E(\delta_{(\tau,\boldsymbol{s},\boldsymbol{r})}) = P_{(\tau,\boldsymbol{s},\boldsymbol{r})}$. Then, the RMM-MSDA problem is an LP formulation that

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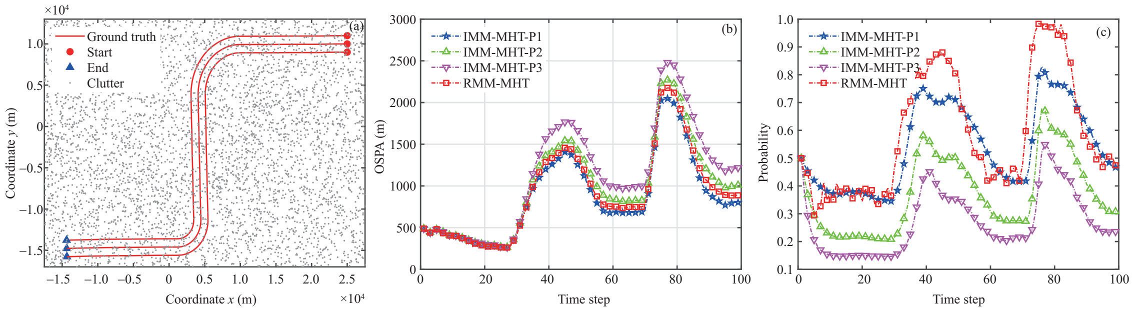

Figure 1 (Color online) (a) An MMTT scenario; (b) OSPA distances for IMM-MHT and RMM-MHT with the sampling period of 5 s; (c) probability of the target operating in the maneuvering mode ($q = 4$ m^2/s^3) with the sampling period of 5 s.

minimizes the following expression:

$$\sum_{\tau}\sum_{\boldsymbol{s}}\sum_{\boldsymbol{r}} C_{(\tau,\boldsymbol{s},\boldsymbol{r})}P_{(\tau,\boldsymbol{s},\boldsymbol{r})} \tag{6}$$

based on constraints (2)–(4). Here, $C_{(\tau,\boldsymbol{s},\boldsymbol{r})} = -\log L_{(\tau,\boldsymbol{s},\boldsymbol{r})}$ is the cost of the track hypothesis $(\tau, \boldsymbol{s}, \boldsymbol{r})$. The detailed derivations of $L_{(\tau,\boldsymbol{s},\boldsymbol{r})}$ are given in Appendix A.

State estimation via RCMKF. The optimal solution of RMM-MSDA yields the posterior probabilities of track hypotheses. Specifically, let P^h_{τ,r_1} denote the association probability that measurement $\boldsymbol{z}^{r_1}_{k+1}$ is associated with target τ, i.e., $P^h_{\tau,r_1} := \sum_{\boldsymbol{s}}\sum_{(r_2,\ldots,r_N)} P_{(\tau,\boldsymbol{s},\boldsymbol{r})}$. Then, based on the association probability P^h_{τ,r_1}, the random coefficient matrix measurement equation for $\boldsymbol{z}^{r_1}_{k+1}$, where $r_1 = 1,\ldots,R_1$, is expressed as follows:

$$\boldsymbol{z}^{r_1}_{k+1} := \begin{cases} H_{k+1}\boldsymbol{x}^1_{k+1} + w^{r_1}_{k+1}, & \text{with probability } P^h_{1,r_1},\\ \cdots & \\ H_{k+1}\boldsymbol{x}^T_{k+1} + w^{r_1}_{k+1}, & \text{with probability } P^h_{T,r_1}. \end{cases} \tag{7}$$

Similarly, the probability that target τ follows the s_1-th model is obtained using $P^f_{\tau,s_1} := \sum_{(s_2,\ldots,s_N)}\sum_{\boldsymbol{r}} P_{(\tau,\boldsymbol{s},\boldsymbol{r})}$. With the probability P^f_{τ,s_1}, the random coefficient matrix state equation for target τ, where $\tau = 1,\ldots,T$, is expressed as follows:

$$\boldsymbol{x}^{\tau}_{k+1} := \begin{cases} F^{(1)}_k\boldsymbol{x}^{\tau}_k + v^{(1)}_k, & \text{with probability } P^f_{\tau,1},\\ \cdots & \\ F^{(S_1)}_k\boldsymbol{x}^{\tau}_k + v^{(S_1)}_k, & \text{with probability } P^f_{\tau,S_1}. \end{cases} \tag{8}$$

Once Eqs. (7) and (8) are established, we use RCMKF to obtain the state estimate of the joint state vector $\boldsymbol{X}_{k+1}$ (see Appendix B). The track management and the RMM-MHT algorithm are given in Appendixes C and D, respectively.

Simulation results. We consider a typical MMTT scenario that includes both closely spaced and maneuvering targets described in [9]. The ground truths of the scenario are shown in Figure 1(a). To demonstrate the performance of RMM-MHT, we compare it with interacting multimodel MHT (IMM-MHT) with three different mode probability transition matrices (MPTM). The hypothesis depths of RMM-MHT and IMM-MHT are two scans. In Figures 1(b) and (c), the optimal subpattern assignment (OSPA) distances and model switching probabilities of RMM-MHT and IMM-MHT are plotted as functions of time steps, respectively. The plots show that the overall performance of IMM-MHT is sensitive to the MPTM, and the OSPA performance of RMM-MHT is closer to that of the best IMM-MHT-P1, even though RMM-MHT does not require a prior MPTM. The reason is that an improper MPTM may result in an incorrect data association in IMM-MHT. Meanwhile, the model switching probability of RMM-MHT is derived by jointly optimizing the multiple possible models and data association hypotheses. More details are given in Appendix E.

Conclusion. We have proposed a novel RMM-MHT method for MMTT. The proposed method avoided the NP-hard $\{0, 1\}$ LIP problem inherent in MSDA and jointly optimized model selection and data association. The proposed method also yielded the probabilities of model selection and data association by solving the LP. Moreover, the state estimation was obtained by the RCMKF. The simulation results showed that RMM-MHT is suitable for MMTT when the MPTM is unknown.

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Supporting information Appendixes A–E. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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