SCIENCE CHINA
Information Sciences

• **Supplementary File** •

A New SINS/RCNS Integrated Navigation Method Using Star Pixel Coordinates

Guangxin SONG[1], Yuqing YANG[1], Xiaolin NING[2*], Jiancheng FANG[1] & Weiren WU[3]

[1]*School of Instrumentation Science and Opto-electronics Engineering, Beihang University, Beijing* 100191*, China;*
[2]*School of Research Institute for Frontier Science, Beihang University, Beijing* 100191*, China;*
[3]*Lunar Exploration and Space Program Center, China National Space Administration, Beijing* 100037*, China*

Appendix A Measurement model

The measurement model of the star pixel coordinates cannot be expressed by a simple equation. According to (2) and (3), we can obtain the implict equation about the starlight refraction angle

$$\sqrt{r^2-u^2}+u\tan R-R_e=69.21177057R^{0.9805}-6.441326\ln R-21.74089877 \tag{A1}$$

where R_e is the Earth's radius, $r=|\boldsymbol{r}|$, $u=|\boldsymbol{u}|=|\boldsymbol{r}\cdot\boldsymbol{S}_i|$, $\boldsymbol{S}_i$ represents the unit starlight vector before refraction in the geocentric inertial frame, $\boldsymbol{r}=\boldsymbol{C}_e^i\boldsymbol{r}^e$ represents the position vector of the spacecraft in the geocentric inertial frame, $\boldsymbol{C}_e^i$ represents the transformation matrix between the earth fixed frame and the geocentric inertial frame, and $\boldsymbol{r}^e$ can be expressed as

$$\boldsymbol{r}^e=\begin{bmatrix}(R_e+H)\cos(L+\delta L)\cos(\lambda+\delta\lambda)\\(R_e+H)\cos(L+\delta L)\sin(\lambda+\delta\lambda)\\(R_e+H)\sin(L+\delta L)\end{bmatrix} \tag{A2}$$

where λ, L and H are the longitude, latitude and altitude calculated by SINS.

Sovling the implact equation, we can get the starlight refraction angle $\widehat{R}$. According to the starlighr vector $\boldsymbol{S}_i$ and position vector $\boldsymbol{r}$, the normal vector $\boldsymbol{n}$ of the refraction plan can be expressed as

$$\boldsymbol{n}=-\frac{\boldsymbol{r}\times\boldsymbol{S}_i}{|\boldsymbol{r}\times\boldsymbol{S}_i|} \tag{A3}$$

Taking the normal vector $\boldsymbol{n}$ of the refraction plan as the rotation axis and rotating $\boldsymbol{S}_i$ for $\widehat{R}$ in the refraction plan, the unit starlight vector $\boldsymbol{S}_r$ of refracted star in the geocentric inertial frame can be expressed as

$$\boldsymbol{S}_c=\begin{bmatrix}q_0^2+q_1^2-q_2^2-q_3^2 & 2(q_1q_2+q_0q_3) & 2(q_1q_3-q_0q_2)\\2(q_1q_2-q_0q_3) & q_0^2-q_1^2+q_2^2-q_3^2 & 2(q_2q_3+q_0q_1)\\2(q_1q_3+q_0q_2) & 2(q_2q_3-q_0q_1) & q_0^2-q_1^2-q_2^2+q_3^2\end{bmatrix}\boldsymbol{S}_i \tag{A4}$$

where $q_0=\cos(\frac{\widehat{R}}{2})$, $q_1=n(1)\sin(\frac{\widehat{R}}{2})$, $q_2=n(2)\sin(\frac{\widehat{R}}{2})$, $q_3=n(3)\sin(\frac{\widehat{R}}{2})$, where $n(1)$ $n(2)$ and $n(3)$ represent the weight of $\boldsymbol{n}$ in three directions.

According to equation $S_s=\boldsymbol{C}_b^s\boldsymbol{C}_n^b\boldsymbol{C}_n^{n'}\boldsymbol{C}_e^n\boldsymbol{C}_i^e\boldsymbol{S}_c$, the unit starlight vector S_s in star sensor fram can be obtained, and the $\boldsymbol{C}_n^{n'}$ and $\boldsymbol{C}_e^n$ can be expressed as

$$\boldsymbol{C}_n^{n'}=\begin{bmatrix}0 & -\phi_U & \phi_N\\\phi_U & 0 & -\phi_E\\-\phi_N & \phi_E & 0\end{bmatrix} \tag{A5}$$

$$\boldsymbol{C}_e^n=\begin{bmatrix}-\sin(\lambda+\delta\lambda) & \cos(\lambda+\delta\lambda) & 0\\-\sin(L+\delta L)\cos(\lambda+\delta\lambda) & -\sin(L+\delta L)\sin(\lambda+\delta\lambda) & \cos(L+\delta L)\\\cos(L+\delta L)\cos(\lambda+\delta\lambda) & \cos(L+\delta L)\sin(\lambda+\delta\lambda) & \sin(L+\delta L)\end{bmatrix} \tag{A6}$$

Then the star pixel coodinates can be expressed as

$$(P_x,P_y)=\left(-f\frac{S_s(1)}{S_s(3)},-f\frac{S_s(2)}{S_s(3)}\right) \tag{A7}$$

where $S_s(1)$, $S_s(2)$ and $S_s(3)$ represent the weight of $\boldsymbol{S}_s$ in three directions, and f represents the focus of star sensor.

* Corresponding author (email: ningxiaolin@buaa.edu.cn)

Appendix B Simulation results

The Mento-Carlo simulation results of the for navigation methods at the final moment is shown in Table B1.

Table B1 Mento-Carlo simulation results of different navigation methods at the final moment

Navigation Method	Position error[m]			Velocity error[m/s]			Posture error["]		
	Longitude	Latitude	Altitude	East	North	Zenith	Yaw	Pitch	Row
SINS	428.80	561.47	970.36	0.84	0.52	1.36	81.53	7.62	11.76
Refraction apparent height	161.96	148.24	8.16	0.28	0.29	0.14	82.58	11.44	35.56
Starlight refraction angle	161.33	144.59	8.32	0.27	0.26	0.14	82.54	11.49	35.87
Star pixel coordinates	154.96	133.89	11.87	0.25	0.25	0.13	1.75	2.08	6.94