• Supplementary File •

Collapse mechanism and structural optimization based on fractional core in complex systems

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Appendix A Experimental setup

Our experiment is conducted on the Think Station which has an NVIDIA Quadro P600, 2.50 GHz Intel(R) Xeon(R) CPU E3-1230 v6 with 4 logical cores, 16 GB RAM and 128 GB local NVMe SSD, running Windows 7 Ultimate with Service Pack 1 (x64). Our models and simulations are tested on MATLAB R2016b.

Appendix B Results

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Figure B1 a-c We teste the influence between the average activity $\langle x \rangle$ and K_{γ} under different network sizes of SF networks. a Dots and solid lines are used for indicating the simulation results and the analytical solutions, respectively when N = 100. b, c Similar to a, here the network size is 500 and 5000. d, e, and f, the power exponents of SF networks is fitted based on N = 100, 500, 5000, respectively.



Figure B2 a-f We test the influence between the average activity $\langle x \rangle$ and K_{γ} under different power exponents of SF networks. a Dots and solid lines are used for indicating the simulation results and the analytical solutions when $\lambda = 6$, respectively. b-f Similar to a, here the power exponents of SF networks is $\lambda = 3, 2.7, 2.5, 2.3, 1.5$, respectively.



Figure B3 We test the relation between the average activity $\langle x \rangle$ and K_{γ} against (1) extinction of a fraction f_n of species; (2) extinction of a fraction f_l of interactions; (3) decreasing all weights on average to a fraction f_w of their original value, simulating a global change in the environmental conditions on Net 2-5, where similar fraction is considered in three perturbations, f = 0.01, f = 0.2, f = 0.4, f = 0.6, respectively.



Figure B4 We test the relation between the average activity $\langle x \rangle$ and K_{γ} against (1) extinction of a fraction f_n of species; (2) extinction of a fraction f_l of interactions; (3) decreasing all weights on average to a fraction f_w of their original value, simulating a global change in the environmental conditions on Net 6-8, where similar fraction is considered in three perturbations, f = 0.01, f = 0.2, f = 0.4, f = 0.6, respectively.



Figure B5 We test the relation between the average activity $\langle x \rangle$ and K_{γ} against (1) extinction of a fraction f_n of species; (2) extinction of a fraction f_l of interactions; (3) decreasing all weights on average to a fraction f_w of their original value, simulating a global change in the environmental conditions on Net 9-11, where similar fraction is considered in three perturbations, f = 0.01, f = 0.2, f = 0.4, f = 0.6, respectively.



Figure B6 We test the relation between the average activity $\langle x \rangle$ and K_{γ} when d = 0.5 against (1) extinction of a fraction f_n of species; (2) extinction of a fraction f_l of interactions; (3) decreasing all weights on average to a fraction f_w of their original value, simulating a global change in the environmental conditions on Net 1, where similar fraction is considered in three perturbations, f = 0.01, f = 0.2, f = 0.4, f = 0.6, respectively. Here the lines represent analytical results by the F-core, dotted lines represent analytical results by the k-core, and error bars represent simulation results.

Network ID	Species	$\langle x \rangle$	K_{γ_c}	Network ID	Species	$\langle x \rangle$	K_{γ_c}
Net 1	Non-remove	0.628	3.63	Net 5	Non-remove	0.519	3.81
	Remove F-core<1	0.719	3.63		Remove F-core<1	0.699	3.81
	Remove F-core<2	0.739	3.55		Remove F-core<1.97	0.706	3.81
	Remove F-core<2.52	0.741	3.47		Remove F-core<2	0.746	3.72
	Remove F-core<2.66	0.740	3.39		Remove F-core<2.98	0.750	3.63
	Remove F-core<2.93	0.739	3.18		Remove F-core<3.05	0.752	3.18
	Remove F-core<3.05	0.702	2.47		Non-remove	0.669	8.05
Net 2	Non-remove	0.489	3.05	Net 6	Remove F-core<1	0.788	8.05
	Remove F-core<1	0.551	3.05		Remove F-core<2	0.808	7.73
	Remove F-core<2	0.510	2.10		Remove F-core<3.05	0.822	7.16
	Remove F-core<2.20	0.480	1.94		Remove F-core<3.55	0.822	7.16
Net 3 Net 4	Non-remove	0.683	5.80		Remove F-core<3.91	0.823	7.16
	Remove F-core<1	0.767	5.80		Remove F-core<4.33	0.830	6.90
	Remove F-core<2	0.814	5.80		Remove F-core<4.57	0.829	6.90
	Remove F-core<2.98	0.824	5.80		Remove F-core<4.57	0.828	6.00
	Remove F-core<3.05	0.842	5.80		Remove F-core<5.12	0.816	6.00
	Remove F-core<3.55	0.843	5.62		Remove F-core<5.44	0.801	6.00
	Remove F-core<3.72	0.844	5.62		Remove F-core<5.62	0.800	4.97
	Remove F-core<4.00	0.846	5.44		Remove F-core<5.81	0.753	4.97
	Remove F-core<4.56	0.847	5.44		Remove F-core<6	0.766	4.83
	Remove F-core<4.97	0.835	4.56		Remove F-core<6.65	0.764	4.83
	Remove F-core<5.12	0.833	4.44		Remove F-core<6.90	0.767	3.32
	Non-remove	0.584	7.73		Remove F-core<7.16	0.676	6.00
	Remove F-core<1	0.583	5.12	Net 7	Non-remove	0.485	5.12
	Remove F-core<1.94	0.585	5.12		Remove F-core<1	0.587	4.97
	Remove F-core<2	0.588	4.57		Remove F-core<2	0.609	4.00
	Remove F-core<2.04	0.598	4.33		Remove F-core<2.51	0.615	4.00
	Remove F-core<3.05	0.542	3.72		Remove F-core<2.98	0.613	4.00
	Remove F-core<4	0.536	3.05		Remove F-core<3.05	0.595	3.05
	Remove F-core<5.12	0.541	2.81		Remove F-core<3.32	0.581	3.05
	Remove F-core<5.28	0.541	2.81		Remove F-core<4.00	0.584	3.05
	Remove F-core<5.62	0.518	2.61	Net 8	Non-remove	0.632	4.70
	Remove F-core <6	0.513	2.56		Remove F-core<1	0.732	4.70
	Remove F-core<6.21	0.517	2.56		Remove F-core<2	0.798	4.70
	Remove F-core < 6.65	0.524	2.56		Remove F-core<2.75	0.800	4.70
	Remove F-core<7.16	0.227	1.00		Remove F-core<3.05	0.811	4.33
Net 9	Non-remove	0.568	3.00		Remove F-core<3.55	0.812	4.33
	Remove F-core<1	0.512	2.30		Remove F-core<4	0.809	3.91
	Remove F-core<1.26	0.541	2.30	Net 10	Non-remove	0.644	3.63
	Remove F-core<2	0.550	2.30		Remove F-core<1	0.639	3.63
Net 11	Non-remove	0.575	4.00		Remove F-core<2	0.630	2.98
	Remove F-core <1	0.693	4.00		Remove F-core<2.76	0.435	1.97
	Remove F-core <2	0.737	3.91		Remove F-core<3.05	0.432	1.71
	Remove F-core<3.05	0.713	2.81				

 Table B1
 Optimal network structure in Table 1 as F-core extinction



Figure B7 We test the relation between the average activity $\langle x \rangle$ and K_{γ} when d = 2 against (1) extinction of a fraction f_n of species; (2) extinction of a fraction f_l of interactions; (3) decreasing all weights on average to a fraction f_w of their original value, simulating a global change in the environmental conditions on Net 1, where similar fraction is considered in three perturbations, f = 0.01, f = 0.2, f = 0.4, f = 0.6, respectively. Here the lines represent analytical results by the F-core, dotted lines represent analytical results by the k-core, and error bars represent simulation results.



Figure B8 We test the relation between the tipping point K_{γ_c} as F-core extinction from low to high. In each panel, each square dot represents the network structure when species which belong to corresponding F-core go extinct vs the result of the tipping point by F-core, each triangular dot represents the network structure when species which belong to corresponding F-core go extinct vs the result of the tipping point by k-core, and each circular dot represents the network structure when species which belong to corresponding F-core go extinct vs the result of the tipping point by k-core, and each circular dot represents the network structure when species which belong to corresponding F-core go extinct vs the result of the tipping point by numerically integrating. Overall, the method by the F-core and numerically integrating show the same tendency which is fixed at first and then descends, but the tipping point by the method of the k-core is always fixed.