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Event-triggered consensus control of heterogeneous multi-agent systems: model- and data-based approaches

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Abstract This article addresses the model- and data-based event-triggered consensus of heterogeneous leader/follower multi-agent systems (MASs). A dynamic periodic transmission protocol is developed to alleviate the communication and computational burden, where the followers can interact locally with neighbors to approach the dynamics of the leader. Capitalizing on a discrete-time looped-functional, a model-based consensus condition for the closed-loop MASs is derived as linear matrix inequalities (LMIs), along with a design method for obtaining distributed event-triggered controllers and the associated triggering parameters. Upon collecting noise-corrupted state-input measurements in offline open-loop experiments, a data-based leader/follower MAS representation is derived and employed to address the data-driven consensus control problem without explicit MAS models. This result is subsequently generalized to guarantee an $\mathcal{H}_\infty$-consensus control performance. Finally, a simulation example is given to corroborate the efficiency of the proposed distributed triggering scheme and the data-driven consensus controller.

Keywords data-driven control, multi-agent system, consensus, looped-functional, LMI

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1 Introduction

The consensus of multi-agent systems (MASs) has gained enormous attention over the last two decades because of the widespread applications of MASs in, e.g., mobile robots, sensor networks, energy systems, and unmanned air vehicles [1–12]. Consensus control problems can be classified into leader/follower and leaderless ones, depending on whether there is a leader system. Thus far, both cases have been widely studied, e.g., [13, 14]. This paper focuses on the leader/follower control of heterogeneous MASs, where agents have different types of dynamics.

To achieve this goal, information interaction is required between agents through a shared network. Considering the limited network resources (e.g., the bandwidth and energy of wireless transmission nodes), an intermittent transmission strategy applies to a digital network. One effective approach is the event-triggering scheme (ETS), whose remarkable feature is that the times of transmission actions and control updates are determined by predesigned triggering conditions [15, 16]. Fruitful theoretical achievements on the event-triggered consensus control of MASs are referred to a survey [17]. Recently, a class of ETSs known as dynamic ETSs was proposed in [18]. Compared to static ETSs [19] involving constant thresholds, dynamic ETSs are generally more effective in saving communications by introducing a positive state-dependent dynamic threshold in a static ETS's triggering condition. Because of this superiority, dynamic ETSs have recently been incorporated in MASs, e.g., for continuous-time [20] and discrete-time [21] MASs. Avoiding continuous state detection, a dynamic periodic distributed ETS was proposed

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in [22] that only needs to execute the trigger generator at periodically sampled times. Particularly, in the case of [22], the distributed dynamic variables need not to continuously evolve. However, these contributions are restricted to continuous-time systems. In terms of digital systems, it is of practical importance to generalize the dynamic ETS to discrete-time systems. Lately, some discrete-time dynamic distributed ETSs have been developed, e.g., [11, 21]. In these schemes, system state detection, as well as dynamic variable update, still needs to be performed successively at discrete instants, which undermines the efficacy of ETSs in saving communication resources. This situation has prompted us to devise a dynamic periodic distributed ETS for discrete-time MASs, where event triggers are only executed at predetermined points to further save communications.

On the one hand, the abovementioned event-triggered consensus control designs are model-based in that they require complete knowledge of all agents for controller design and implementation. Nevertheless, obtaining an accurate model of a real-world system can be computationally expensive, and the obtained models may be too complex for classic control methods to be employed [23]. Eliminating the requirement of explicit models of MASs for consensus control, data-driven control directly learns control laws from data [24–29]. For example, data-driven distributed protocols for MAS synchronization were developed based on reinforcement learning techniques in [30, 31]. However, these methods usually require many data and incur high computational overhead. Recently, Ref. [32] suggested an alternative based on the so-called Fundamental Lemma [24] for output regulation of leader/follower systems. Nevertheless, there is a limitation with [32], in which the disturbance is assumed to be measurable to implement the distributed data-driven protocols. On the other hand, a robust data-based event-triggered controller was proposed in [33], which can address unknown but bounded disturbances. It is, therefore, natural to develop data-based event-triggered consensus controllers for leader/follower MASs with unknown dynamics and disturbance.

These recent advances have motivated this work to focus on the data-driven leader/follower consensus control of event-triggered discrete-time MASs with unknown heterogeneous dynamics and noise. First, we develop a distributed dynamic periodic ETS for discrete-time MASs. By virtue of a discrete-time looped-functional (DLF) in [33], a model-based consensus condition is established for leader/follower MASs, which meanwhile provides a model-based method for codesigning the distributed controllers and ETS parameters. On top of [33], a simpler DLF without any integral terms of the system states is designed. On the basis of [33, 34] for single agents, we derive a data-based system parameterization as quadratic matrix inequalities to account for the measurement noise/disturbance in the offline data-collecting phase. By joining the data-based representation and the model-based criterion, a data-based solution for obtaining the consensus controller and the triggering parameters is developed. The results are further extended to the $\mathcal{H}_\infty$-synthesis of closed-loop MASs with noise in an online setting.

The main contributions of this paper are concisely listed as follows.

(c1) We develop a novel discrete-time distributed ETS on the basis of periodic sampling for leader/follower MASs, where event generators and dynamic variables are only executed after a predetermined time interval to mitigate the computation frequency.

(c2) We derive a model-based consensus criterion for the event-triggered MASs by capitalizing on a tailored discrete-time DLF, along with a model-based method for obtaining the distributed controllers and triggering matrices.

(c3) We derive distributed data-driven event-triggered consensus controllers to achieve a prescribed $\mathcal{L}_2$-gain performance.

The remainder of this article is structured as follows. In Section 2, we formulate the data-driven consensus problem for leader/follower MASs, along with a data-driven MAS representation and a novel dynamic distributed ETS. Model-based and data-driven methods for obtaining the distributed controller gains and the triggering matrices are presented in Section 3, along with an extension for achieving a prescribed $\mathcal{L}_2$-gain performance. Section 4 verifies the practicality of our methods using one practical example. Finally, Section 5 concludes the article.

Notation. Throughout this paper, let $\mathbb{N}$, $\mathbb{R}^n$, and $\mathbb{R}^{n\times m}$ denote the sets of all nonnegative integers, n-dimensional real vectors, and $n \times m$ real matrices, respectively. For any integers $a, b \in \mathbb{N}$, let $\mathbb{N}_0 := \mathbb{N}\cup\{0\}$ and $\mathbb{N}_{[a,b]} := \mathbb{N} \cap [a, b]$. The superscripts '$-1$' and 'T' represent the inverse and transpose of a matrix. Furthermore, we write $P \succ 0$ ($\succeq 0$) if P is a symmetric positive (semi)definite matrix. We use 0 (I) to denote zero (identity) matrices of appropriate dimensions. Symbol $\mathrm{diag}\{q_i\}_{i=1}^N$ represents a (block) diagonal matrix with $q_1, \ldots, q_N$ on its main diagonal. $\mathbf{1}_N$ ($\boldsymbol{I}_N$) denotes a column vector whose elements are 1 (I), $\otimes$ denotes the Kronecker product, and '$*$' represents the symmetric term in (block) symmetric matrices. Sym$\{P\}$ is the sum of P^{T} and P. The space of square-integrable vector functions over $[0,\ \infty]$ is given by $\mathcal{L}_2[0,\ \infty]$, and for $\varpi(t) \in \mathcal{L}_2[0,\ \infty]$, its norm is given by $\|\varpi(t)\|_{\mathcal{L}_2} = [\int_0^{\infty} \varpi^{\mathrm{T}}(t)\varpi(t)\mathrm{d}t]^{1/2}$. The symbol $\arg\min_x f(x)$ returns the optimal variable x^* at which the function $f(x)$ is minimized. Finally, $\|\cdot\|$ denotes the Euclidean norm of a vector.

2 Problem formulation

2.1 Description of MASs

This article considers MASs consisting of one leader and N followers. A directed graph $\mathcal{G} := \{\mathcal{V}, \mathcal{E}, \mathcal{C}\}$ is used to represent the communication topology among the agents, where $\mathcal{V} := \{v_0, v_1, v_2, \ldots, v_N\}$ is the set of nodes and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ represents the set of edges. The matrix $\mathcal{C} := [c_{ij}] \in \mathbb{R}^{(N+1)\times(N+1)}$ is the adjacency of $\mathcal{G}$, constructed by setting $c_{ij} = 1$ if node v_i can receive information from node v_j via communication channels and $c_{ij} = 0$, otherwise. Self-loops are not taken into consideration, i.e., $c_{ii} = 0$ for all $i \in \mathbb{N}_{[0,N]}$. The graph $\mathcal{G}$ is said to have a spanning tree, if there is a root node, and there exists a directed path from the root node to each other node. The neighbor set of node i is defined as $\mathcal{N} := \{j \in \mathbb{N}_{[0,N]} | j \neq i, c_{ij} = 1\}$.

Indexing the leader by 0 and the followers by $1, 2, \ldots, N$, we model their dynamics using the following linear discrete-time recursions:

$$x_i(t+1) = A_i x_i(t) + B_i u_i(t), \quad t \in \mathbb{N}, \tag{1}$$

where $x_i(t) \in \mathbb{R}^n$ denotes the state vector of agent i, $u_i(t) \in \mathbb{R}^m$ is its control input, and $A_i \in \mathbb{R}^{n\times n}$ and $B_i \in \mathbb{R}^{n\times m}$ are constant system matrices. The MAS in (1) is heterogeneous, because the $N+1$ agents exhibit different dynamics (A_i, B_i).

Let $\varepsilon_i(t) := x_i(t) - x_0(t)$ denote the leader/follower errors. Upon collecting all the errors along with the state of the leader to form $\varepsilon(t) := [\varepsilon_1^{\mathrm{T}}(t)\ \cdots\ \varepsilon_N^{\mathrm{T}}(t)\ x_0^{\mathrm{T}}(t)]^{\mathrm{T}}$, and similarly for the inputs $u(t) := [u_1^{\mathrm{T}}(t)\ \cdots\ u_N^{\mathrm{T}}(t)\ u_0^{\mathrm{T}}(t)]^{\mathrm{T}}$, we have the following entire system expression:

$$\varepsilon(t+1) = A\varepsilon(t) + Bu(t), \quad t \in \mathbb{N}, \tag{2}$$

where

$$A := \left[\begin{array}{c|c} \mathrm{diag}\{A_i\}_{i=1}^N & \boldsymbol{I}_N \cdot \mathrm{diag}\{A_i - A_0\}_{i=1}^N \\ \hline 0 & A_0 \end{array}\right],\ B := \left[\begin{array}{c|c} \mathrm{diag}\{B_i\}_{i=1}^N & \boldsymbol{I}_N \otimes (-B_0) \\ \hline 0 & B_0 \end{array}\right].$$

In contrast to existing studies [20–22], this paper focuses on a more challenging situation, where the system matrices A_i and B_i are all assumed unknown. The objective is to design a distributed control strategy for unknown MASs (1) with intermittent communications to ensure the leader/follower consensus asymptotically, i.e., $\lim_{t\to\infty}(x_i(t) - x_0(t)) = 0$, $\forall i \in \mathbb{N}_{[0,N]}$. Note that achieving consensus of the MAS (1) can be reduced to show the stability of the error system (2), i.e., $\lim_{t\to\infty}\varepsilon(t) = 0$.

2.2 Data-driven representation for MASs

The lack of the system matrices poses a challenge in the design of consensus controllers for the MAS. Building on [34] that addresses single systems, we provide a data-driven parameterization for linear discrete-time MASs. Suppose one has collected a set of data $\{\{x_i(T)\}_{T=0}^{\rho}, \{u_i(T)\}_{T=0}^{\rho-1}\}$ (where $\rho \in \mathbb{N}_{[1,\infty)}$ represents the number of data) by running the following systems:

$$x_i(T+1) = A_i x_i(T) + B_i u_i(T) + D_i w_i(T), \tag{3}$$

where matrix $D_i \in \mathbb{R}^{n\times n_w}$ is assumed to be known and has full column rank, which models the influence of some unknown disturbance $w_i(T)$.

Imitating the expression in (2), we can rewrite the system (3) as

$$\varepsilon(T+1) = A\varepsilon(T) + Bu(T) + Dw(T), \tag{4}$$

where $w(T) := [w_1^{\rm T}(T)\ \cdots\ w_N^{\rm T}(T)\ w_0^{\rm T}(T)]^{\rm T} \in \mathbb{R}^{(N+1)n_w}$ and

$$D := \left[\begin{array}{c|c} \mathrm{diag}\{D_i\}_{i=1}^{N} & \boldsymbol{I}_N \otimes (-D_0) \\ \hline 0 & D_0 \end{array}\right].$$

We remark that the assumption on the measurements $\{\{x_i(T)\}_{T=0}^{\rho}, \{u_i(T)\}_{T=0}^{\rho-1}\}$ is mild, which can be easily fulfilled by running the each individual system offline in open-loop experiments. It is practical to assume the disturbance $\{w_i(T)\}$ is unknown, which accounts for the measurement noise in the data-collecting process. Certainly, the noise/disturbance is always bounded, which prompts us to pose a bound on the noise $W := [\, w(0)\ w(1)\ \cdots\ w(\rho-1)\,]$ as follows.

Assumption 1 (Bounded noise)**.** The noise sequence $\{w_i(T)\}_{T=0}^{\rho-1}$ $(i \in \mathbb{N}_{[0,N]})$ belongs to the set

$$\mathcal{W} = \left\{ W \in \mathbb{R}^{(N+1)n_w \times \rho} \middle| \begin{bmatrix} W^{\rm T} \\ I \end{bmatrix}^{\rm T} Q_d \begin{bmatrix} W^{\rm T} \\ I \end{bmatrix} \succeq 0 \right\},$$

where Q_d is a known symmetric matrix obeying $[\begin{smallmatrix} I \\ 0 \end{smallmatrix}]^{\rm T} Q_d [\begin{smallmatrix} I \\ 0 \end{smallmatrix}] \prec 0$.

Remark 1. Assumption 1 is general for modeling noisy MASs. For example, if the disturbance sequence $\{w(T)\}_{T=0}^{\rho-1}$ is norm-bounded by $\|w(T)\| \leqslant \bar{w}$ for all $T \in \mathbb{N}$, then Q_d can be given by

$$Q_d = \begin{bmatrix} -\mathrm{diag}\{q_i\}_{i=1}^{\rho} & 0 \\ 0 & \sum_{i=1}^{\rho} q_i \bar{w}^2 I \end{bmatrix},\ q_i \geqslant 0, \tag{5}$$

where q_is are free scalars. Choosing Q_d in (5) is less conservative compared to the ones in [33,35], where only one free scalar is allowed.

Based on the available data $\{x_i(T)\}_{T=0}^{\rho}$ and $\{u_i(T)\}_{T=0}^{\rho-1}\}$, we can compute and collect the leader/follower errors $\varepsilon_i(T) = x_i(T) - x_0(T)$ that are consistent with the system expression in (2) for all times $T \in \mathbb{N}_{[0,\rho]}$. Let us rearrange these data vectors to construct the following matrices:

$$E_+ := [\varepsilon(1)\ \varepsilon(2)\ \cdots\ \varepsilon(\rho)],$$
$$E := [\varepsilon(0)\ \varepsilon(1)\ \cdots\ \varepsilon(\rho-1)],$$
$$U := [u(0)\ u(1)\ \cdots\ u(\rho-1)].$$

Since the true noise sequence is unknown, the collected input-state data can belong to a set of systems $[A\ B]$ with the residual compensating for the bounded noise. To reflect this fact, we define the set of all systems $[A\ B]$ that are consistent with the data and the noise bound, as per (4) and Assumption 1, as follows:

$$\Sigma_{AB} := \left\{ [A\ B] \middle| E_+ = AE + BU + B_w W,\ W \in \mathcal{W} \right\}.$$

In order to ensure the stability of system (2) without knowing the system matrices, it suffices to derive a stability criterion for all $[A\ B] \in \Sigma_{AB}$. For this purpose, a data-based representation of $[A\ B]$ expressed as a QMI is introduced as follows.

Lemma 1 (Data-driven system representation)**.** The set Σ_{AB} is equivalent to

$$\Sigma_{AB} = \left\{ [A\ B] \middle| \begin{bmatrix} [A\ B]^{\rm T} \\ I \end{bmatrix}^{\rm T} \Theta_{AB} \begin{bmatrix} [A\ B]^{\rm T} \\ I \end{bmatrix} \succeq 0 \right\},$$

where

$$\Theta_{AB} := \left[\begin{array}{cc} -E & 0 \\ -U & 0 \\ \hline E_+ & B_w \end{array}\right] Q_d \left[\begin{array}{cc} -E & 0 \\ -U & 0 \\ \hline E_+ & B_w \end{array}\right]^{\rm T}.$$

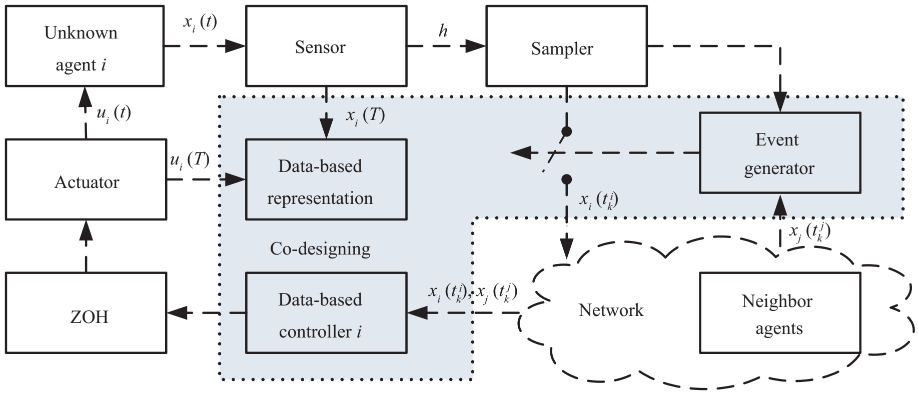

Figure 1 (Color online) Data-driven consensus control of MASs under an ETS.

Lemma 1 provides a purely data-driven parameterization of the unknown system (2) using only data E, E_+, and U. Note that such data are collected from the perturbed system (4), while we analyze the stability of the unperturbed system (2). Introducing the disturbance here is to capture possible noise in the data-collecting process offline with sensors, rather than to explain the disturbance affecting the system dynamics. We present the results on obtaining a performance guarantee on the closed-loop $\mathcal{L}_2$-gain in Subsection 3.3. Note also that, in Figure 1, the data are collected offline in an open-loop experiment, while the leader/follower consensus is realized in closed-loop online, which is separated from the open-loop sampling. In Subsection 2.3, a distributed control strategy with event-triggered transmissions is proposed for MAS consensus.

2.3 Distributed event-triggered control

Letting t_k^i denote the kth transmitted instant of agent i, we consider the following linear feedback law for system (1) during $t \in \mathbb{N}_{[t_k^i, t_{k+1}^i - 1]}$:

$$u_i(t) = \begin{cases} K_i x_i(t_k^i), & i = 0, \\ \sum_{j \in \mathcal{N}} K_{ij} e_{ij}(t_k^i), & i > 0, \end{cases} \tag{6}$$

where $e_{ij}(t_k^i) := x_i(t_k^i) - x_j(t_{k'(t)}^j)$ is the state measurement error of agents i and j; $k'(t) := \arg\min_{l \in \mathbb{N}: t \geqslant t_l^j} \{t - t_l^j\}$, where t_l^j is the transmitted instant of agent j; therefore, for each $t \in \mathbb{N}_{[t_k^i, t_{k+1}^i - 1]}$, $t_{k'(t)}^j$ is the last transmitted time of agent j; and the consensus controller gain matrices K_0, $K_{ij} \in \mathbb{R}^{m \times n}$ are designed in Section 3. Note that in (6) the leader has only access to its own sampled states, and the control inputs of the followers only contain errors with respect to their neighbors. Specifically, a nonzero coupling matrix K_{ij} implies that there exists a communication channel through which controller i can utilize $e_{ij}(t_k^i)$, otherwise $K_{ij} = 0$.

In our distributed control, each follower agent updates its own control input at transmitted times by capitalizing on all information locally available as well as received from its neighboring agents. Specifically, for all followers $i > 0$, the error of the local sampled state $x_i(t_k^i)$ and the state $x_j(t_{k'(t)}^j)$ of the neighboring agent are employed, and its control input is obtained using the law in (6).

Up to date, there are some studies exploring event-triggered control for discrete-time multi-agent systems [21]. However, in these schemes, sensors are required to operate frequently at every discrete instant, so are the extra dynamic variables. This results in considerable energy consumption, especially when the discretized interval of the system is small. This paper introduces a distributed dynamic ETS with an adjustable sampling interval. It is assumed that during the closed-loop operation, the state of each agent is periodically sampled with a common period h in a synchronous manner, where h is an adjustable parameter satisfying $1 \leqslant \underline{h} \leqslant h \leqslant \bar{h}$ with given bounds $\underline{h}$, $\bar{h} \in \mathbb{N}$. The event generator determines whether each sampled state is transmitted to local or neighbors' controllers. In this case, transmission events are only executed at preset sampling points $\{0h, 1h, 2h, \ldots\}$ instead of each discrete instant in $\{0, 1, 2, \ldots\}$. Clearly, the transmitted instants t_k^i take values from $\{0h, 1h, 2h, \ldots\}$.

Motivated by [20–22], a distributed periodic ETS is introduced to determine the transmitted instants $\{t_k^i\}$, capitalizing on the following criterion:

$$\eta_i(\tau_{k,v}^i) + \theta_i \rho_i(\tau_{k,v}^i) < 0 \tag{7}$$

with $\tau_{k,v}^i := t_k^i + vh$ for all $v \in \mathbb{N}_{[0,m_k^i]}$, where $\theta_i > 0$ is to be designed, $m_k^i = \frac{t_{k+1}^i - t_k^i}{h} - 1$, and

$$\rho_0(\tau_v^0) := \sigma_0 x_0^{\mathrm{T}}(t_k^0)\Omega_0 x_0(t_k^0) - e_0^{\mathrm{T}}(\tau_v^0)\Omega_0 e_0(\tau_v^0),$$

$$\rho_i(\tau_{k,v}^i) := \sum_{j\neq i}^{N} \sigma_{ij} e_{ij}^{\mathrm{T}}(t_k^i)\Omega_i e_{ij}(t_k^i) - e_i^{\mathrm{T}}(\tau_{k,v}^i)\Omega_i e_i(\tau_{k,v}^i), \quad i > 0,$$

where $\Omega_i \succ 0$ is a weight matrix, and σ_0, $\{\sigma_{ij}\}_{j\in\mathcal{N}}$, θ_i are parameters, both to be designed ($\sigma_{ij} = 0$ when there is no transmission path from agents i to j); $e_i(\tau_{k,v}^i) := x_i(\tau_{k,v}^i) - x_i(t_k^i)$ denotes the error of agent i between the latest transmitted signal $x_i(t_k^i)$ and the current sampled signal $x_i(\tau_{k,v}^i)$; and, $\eta_i(\tau_{k,v}^i)$ in the condition (7) is a discrete-time variable satisfying

$$\eta_i(\tau_{k,v+1}^i) - \eta_i(\tau_{k,v}^i) = -\lambda_i \eta_i(\tau_{k,v}^i) + \rho_i(\tau_{k,v}^i), \tag{8}$$

where $\eta_i(0) \geqslant 0$ and $\lambda_i > 0$ are given parameters. To sum up, the event-triggering policy is described as

$$t_{k+1}^i = t_k^i + h \min_{v\in\mathbb{N}} \Big\{ v > 0 \Big| \eta_i(\tau_{k,v}^i) + \theta_i \rho_i(\tau_{k,v}^i) < 0 \Big\}. \tag{9}$$

In our distributed control strategy, the sampled state $x_i(\tau_{k,v}^i)$ is transmitted to the local controller and neighbors for agent $i > 0$, as soon as the condition (7) is met. According to the control law (6), a new control input is computed using the state $x_i(t_{k+1}^i)$ and the received neighbor's state $x_j(t_{k'(t)+1}^j)$ (only for the followers), and kept by a zero-order holder (ZOH) during $[t_{k+1}^i,\ t_{k+2}^i - 1]$. Meanwhile, the event-triggering function is renewed using the latest transmitted measurements, and then one periodically evaluates the function at $\{t_{k+1}^i + vh\}_{v\in\mathbb{N}}$ to determine the next transmission time t_{k+2}^i based on the policy (9). The following lemma is useful for deriving our results in Section 3.

Lemma 2 (Nonnegativity). For any positive definite matrix $\Omega_i \succ 0$, nonnegative scalar $\eta_i(0) \geqslant 0$, and positive constants $\lambda_i > 0$, $\theta_i > 0$ satisfying $1 - \lambda_i - \frac{1}{\theta_i} \geqslant 0$. Then, for all $v \in \mathbb{N}_{[0,m_k^i]}$, it holds that $\eta_i(\tau_{k,v}^i) \geqslant 0$ under the triggering condition (9).

The proof of Lemma 2 is similar to [33, Lemma 3], which is omitted here.

Remark 2. The transmission scheme (9) can be seen as a discrete-time counterpart of the continuous dynamic ETS in [22]. Besides, our ETS subsumes the dynamic ETS proposed in [21] (cf. (9) with $h = 1$) as special cases. In (9), because every event-generator is only executed at sampling times $\tau_{k,v}^i$, the variable does not need to successively evolve at every discrete time in the event-generator. Our triggering scheme is expected to further reduce data transmissions and computational burden when compared to [21]. Note that the dynamic thresholds $\eta_i(\tau_{k,v}^i)$ remain positive definite according to Lemma 2, thus to provide less transmissions compared to the static ones (cf. (9) with $\theta_i \to \infty$).

2.4 Problem statement

Having introduced MASs with an ETS and the data-driven system representation, the problem considered is described in this subsection. At the beginning, we put forward the following closed-loop system expression combining the feedback control law (6) and the open-loop system in (2):

$$\varepsilon(t+1) = A\varepsilon(t) + BK\varepsilon(t_k),\ t \in \mathbb{N}_{[t_k^i, t_{k+1}^i - 1]}, \tag{10}$$

where $\varepsilon(t_k) := [\varepsilon_1^{\mathrm{T}}(t_k^1)\ \cdots\ \varepsilon_N^{\mathrm{T}}(t_k^N)\ x_0^{\mathrm{T}}(t_{k'(t)}^0)]^{\mathrm{T}}$, $\varepsilon_i(t_k^i) := x_i(t_k^i) - x_0(t_{k'(t)}^0)$,

$$K := \begin{bmatrix} \sum_{j\in\mathcal{N}} K_{1j} & -K_{12} & \cdots & -K_{1N} & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ -K_{N1} & -K_{N2} & \cdots & \sum_{j\in\mathcal{N}} K_{Nj} & 0 \\ 0 & 0 & \cdots & 0 & K_0 \end{bmatrix},$$

and the transmission instant t_k^i is determined by the event-triggering law (9). Based on (10), the problem is given as follows.

Problem 1 (Data-driven consensus). Given state-input measurements $\{\{x_i(T)\}_{T=0}^{\rho}, \{u_i(T)\}_{T=0}^{\rho-1}\}$ of the MASs (1) and a directed graph $\mathcal{G}$, design a control law of the form (6) as well as a triggering strategy in the form of (9) (cf. system (10)), such that, for any initial states $x_i(0)$, $\lim_{t\to\infty}(x_i(t) - x_0(t)) = 0$, $\forall i \in \mathbb{N}_{[0,N]}$.

3 Main results

This section provides a data-driven consensus control strategy for MASs (1), which solves Problem 1. In particular, two steps are taken into consideration. A model-based analysis of the MASs under the ETS is performed in Subsection 3.1, based on the DLF approach in [33]. Next, a data-driven design strategy for obtaining the control gains and the ETS matrices is studied in Subsection 3.2. In Subsection 3.3, we further extend the data-driven results in Subsection 3.2 to the case of achieving $\mathcal{L}_2$-gain performance. Before moving on, the following lemma is required to obtain our results.

Lemma 3. For any matrix $R \in \mathbb{R}^{n\times n} \succ 0$, $N \in \mathbb{R}^{m\times n}$, vector $\vartheta \in \mathbb{R}^m$, and a sequence $\{x(s)\}_{s=\alpha}^{\beta-1}$, it holds for $\alpha \leqslant \beta \in \mathbb{N}$ that

$$-\sum_{i=\alpha}^{\beta-1} y^{\mathrm{T}}(i)Ry(i) \leqslant (\beta-\alpha)\vartheta^{\mathrm{T}}MR^{-1}M^{\mathrm{T}}\vartheta + \mathrm{Sym}\left\{\vartheta^{\mathrm{T}}M[x(\beta)-x(\alpha)]\right\}$$

with $y(i) = x(i+1) - x(i)$.

Lemma 3 is a simpler version of the results in [36, Lemma 2] without considering summation terms on the right side of the inequality. The proof is similar to [36, Lemma 2], which is not omitted here. Lemma 3 provides the basis for the following results.

3.1 Model-based consensus and controller design

Theorem 1 (Model-based consensus). Consider the system (1) under the triggering condition (9) and the control law (6). Given positive scalars σ_0, σ_{ij}, $\bar{h}$, $\underline{h}$, and λ_i, θ_i satisfying $1 - \lambda_i - \frac{1}{\theta_i} \geqslant 0$ for all $i \in \mathbb{N}_{[0,N]}$ and $j \in \mathcal{N}$, asymptotic consensus of the system is achieved, and dynamic values $\eta_i(\tau_{k,v}^i)$ converge to the origin for any $\eta_i(0) \geqslant 0$, if there exist matrices $R_1 \succ 0$, $R_2 \succ 0$, $P \succ 0$, $S = S^{\mathrm{T}}$, M_1, M_2, F, and $\Omega_i \succ 0$ for all $i \in \mathbb{N}_{[0,N]}$, satisfying the following linear matrix inequalities (LMIs) $\forall h \in \{\underline{h}, \bar{h}\}$:

$$\begin{bmatrix} \Xi_0 + h\Xi_\varsigma + \Psi + \mathcal{Q} & hM_\varsigma \\ * & -hR_\varsigma \end{bmatrix} \prec 0, \ \varsigma = 1, 2, \tag{11}$$

where

$$\begin{aligned}
\Xi_0 &:= \mathrm{Sym}\big\{M_1(H_1 - H_3) + M_2(H_4 - H_1)\big\} + H_2^{\mathrm{T}}PH_2 - H_1^{\mathrm{T}}PH_1 \\
&\quad + (H_2 - H_1)^{\mathrm{T}}(R_2 - R_1)(H_2 - H_1) - \left[H_3^{\mathrm{T}}, H_4^{\mathrm{T}}\right] S \left[H_3^{\mathrm{T}}, H_4^{\mathrm{T}}\right]^{\mathrm{T}}, \\
\Xi_1 &:= (H_2 - H_1)^{\mathrm{T}}R_2(H_2 - H_1) - \left[H_3^{\mathrm{T}}, H_4^{\mathrm{T}}\right] S \left[H_3^{\mathrm{T}}, H_4^{\mathrm{T}}\right]^{\mathrm{T}}, \\
\Xi_2 &:= (H_2 - H_1)^{\mathrm{T}}R_1(H_2 - H_1) + \left[H_3^{\mathrm{T}}, H_4^{\mathrm{T}}\right] S \left[H_3^{\mathrm{T}}, H_4^{\mathrm{T}}\right]^{\mathrm{T}}, \\
\Psi &:= \mathrm{Sym}\big\{F(AH_1 + BKH_5 - H_2)\big\}, \\
\mathcal{Q} &:= H_5^{\mathrm{T}}\Omega_a H_5 - \begin{bmatrix} H_3 \\ H_5 \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} \Omega_b & -\Omega_b \\ * & \Omega_b \end{bmatrix} \begin{bmatrix} H_3 \\ H_5 \end{bmatrix}, \\
H_\iota &:= \left[0_{n\times(\iota-1)n}, \ I_n, \ 0_{n\times(5-\iota)n}\right], \ \iota = 1, \ldots, 5, \ H_0 := 0_{n\times 7n}, \\
\Omega_{ai} &:= \sigma_{i0}\Omega_i + \sum_{j\in\mathcal{N}} \sigma_{ij}\Omega_i + \sigma_{ji}\Omega_j,
\end{aligned}$$

$$\Omega_a := \begin{bmatrix} \Omega_{a1} & \cdots & -\sigma_{1N}\Omega_1 - \sigma_{N1}\Omega_N & 0 \\ * & \ddots & \vdots & \vdots \\ * & * & \Omega_{aN} & 0 \\ * & * & * & \Omega_0 \end{bmatrix},\ \Omega_b := \left[\begin{array}{c|c} \mathrm{diag}\{\Omega_i\}_{i=1}^N & \boldsymbol{I}_N \cdot \mathrm{diag}\{\Omega_i\}_{i=1}^N \\ \hline * & \sum_{i=0}^N \Omega_i \end{array}\right].$$

Proof. Considering the intervals $\mathbb{N}_{[\tau_{k,v}^i, \tau_{k,v+1}^i - 1]}$ for all $v \in \mathbb{N}_{[0, m_k^i]}$, we choose a functional candidate for system (10) as follows:

$$V(t) = V_a(t) + V_d(t) + t\sum_{i=1}^N [\eta_i(\tau_{k,v+1}^i) - \eta_i(\tau_{k,v}^i)], \tag{12}$$

where Lyapunov functional $V_a(t) = \varepsilon^{\mathrm{T}}(t)P\varepsilon(t),\ P \succ 0$; the dynamic variable $\eta_i(\tau_{k,v}^i)$ is provided as in (8); and, the DLF $V_d(t)$ is designed as

$$\begin{aligned} V_d(t) = {} & (t - \tau_{k,v}^i)(\tau_{k,v+1}^i - t)\big[x^{\mathrm{T}}(\tau_{k,v}^i),\ x^{\mathrm{T}}(\tau_{k,v+1}^i)\big] S \big[x^{\mathrm{T}}(\tau_{k,v}^i),\ x^{\mathrm{T}}(\tau_{k,v+1}^i)\big]^{\mathrm{T}} \\ & + (\tau_{k,v+1}^i - t)\left[\sum_{s=\tau_{k,v}^i}^{t} y^{\mathrm{T}}(s)R_1 y(s) - y^{\mathrm{T}}(t)R_1 y(t)\right] \\ & + (t - \tau_{k,v}^i)\left[\sum_{s=t}^{\tau_{k,v+1}^i} y^{\mathrm{T}}(s)R_2 y(s) - y^{\mathrm{T}}(t)R_2 y(t)\right], \end{aligned} \tag{13}$$

where $y(s) := x(s+1) - x(s)$, and $S = S^{\mathrm{T}},\ R_1 \succ 0,\ R_2 \succ 0$.

The forward difference of the functional $V(t)$ is given as

$$\Delta V(t) = \Delta V_a(t) + \Delta V_d(t) + \sum_{i=1}^N [\eta_i(\tau_{k,v+1}^i) - \eta_i(\tau_{k,v}^i)], \tag{14}$$

where

$$\begin{aligned} \Delta V_a(t) &= \xi^{\mathrm{T}}(t)\Big(H_2^{\mathrm{T}} P H_2 - H_1^{\mathrm{T}} P H_1\Big)\xi(t), \\ \Delta V_d(t) &= \xi^{\mathrm{T}}(t)\Big[(H_2 - H_1)^{\mathrm{T}}(R_2 - R_1)(H_2 - H_1) - \big[H_3^{\mathrm{T}}, H_4^{\mathrm{T}}\big]\, S\, \big[H_3^{\mathrm{T}}, H_4^{\mathrm{T}}\big]^{\mathrm{T}}\Big]\xi(t) \\ & \quad - \sum_{s=\tau_{k,v}^i}^{t-1} y^{\mathrm{T}}(s)R_1 y(s) - \sum_{s=t}^{\tau_{k,v+1}^i - 1} y^{\mathrm{T}}(s)R_2 y(s), \end{aligned}$$

with $\xi(t) := [\varepsilon^{\mathrm{T}}(t),\ \varepsilon^{\mathrm{T}}(t+1),\ \varepsilon^{\mathrm{T}}(\tau_{k,v}^i),\ \varepsilon^{\mathrm{T}}(\tau_{k,v+1}^i),\ \varepsilon^{\mathrm{T}}(t_k)]^{\mathrm{T}}$. By Lemma 3, we have that

$$-\sum_{s=\tau_{k,v}^i}^{t-1} y^{\mathrm{T}}(i)Ry(i) \leqslant \xi^{\mathrm{T}}(t)\left[(t - \tau_{k,v}^i)M_1 R_1^{-1} M_1^{\mathrm{T}} + M_1(H_1 - H_3)\right]\xi(t), \tag{15}$$

$$-\sum_{s=t}^{\tau_{k,v+1}^i - 1} y^{\mathrm{T}}(s)R_2 y(s) \leqslant \xi^{\mathrm{T}}(t)\left[(\tau_{k,v+1}^i - t)M_2 R_2^{-1} M_2^{\mathrm{T}} + M_2(H_4 - H_1)\right]\xi(t). \tag{16}$$

Through the descriptor method [37], we have the following equation according to the system representation (10):

$$\begin{aligned} 0 &= 2\xi^{\mathrm{T}}(t)F\big[A\varepsilon(t) + BK\varepsilon(t_k) - \varepsilon(t+1)\big] \\ &= 2\xi^{\mathrm{T}}(t)F\big(AH_1 + BKH_7 - H_2\big)\xi(t). \end{aligned} \tag{17}$$

Summing up (14)–(17) gives rise to

$$\Delta V(t) \leqslant \xi^{\mathrm{T}}(t)\left[(t - \tau_{k,v}^i)(\Xi_1 + M_1 R_1^{-1} M_1^{\mathrm{T}}) + (\tau_{k,v+1}^i - t)(\Xi_2 + M_2 R_2^{-1} M_2^{\mathrm{T}}) + \Xi_0 + \Psi\right]\xi(t)$$

$$+\sum_{i=1}^{N}[\eta_i(\tau^i_{k,v+1})-\eta_i(\tau^i_{k,v})]. \tag{18}$$

In light of the triggering condition (9), Lemma 2 asserts that $\eta_i(\tau^i_{k,v})\geqslant 0$ for $\eta_i(0)\geqslant 0$, $\Omega_i\succ 0$, and $\lambda_i>0$, $\theta_i>0$ satisfying $1-\lambda_i-\frac{1}{\theta_i}\geqslant 0$. Then, according to (8), it holds that

$$\sum_{i=1}^{N}[\eta_i(\tau^i_{k,v+1})-\eta_i(\tau^i_{k,v})]\leqslant \xi^{\mathrm{T}}(t)\mathcal{Q}\xi(t). \tag{19}$$

From (18) and (19), the difference $\Delta V(t)$ satisfies

$$\Delta V(t)\leqslant \xi^{\mathrm{T}}(t)\left[\frac{t-\tau^i_{k,v}}{h}\Upsilon_1(h)+\frac{\tau^i_{k,v+1}-t}{h}\Upsilon_2(h)\right]\xi(t), \tag{20}$$

where $\Upsilon_\varsigma(h)=\Xi_0+h\Xi_\varsigma+\Psi+\mathcal{Q}+hM_\varsigma R_\varsigma^{-1}M_\varsigma^{\mathrm{T}}$, $\varsigma=1,2$.

According to the Schur Complement Lemma, inequalities $\Upsilon_1(h)\prec 0$ and $\Upsilon_2(h)\prec 0$ are equivalent to the LMIs in (11), which are convex with respect to h. Therefore, the LMIs in (11) at the vertices of $[\underline{h},\bar{h}]$ certificate $\Delta V(t)<0$ $\forall h\in[\underline{h},\bar{h}]$. By the DLF approach in [33], it holds that $\forall\varepsilon(\tau^i_{k,v})\neq 0$,

$$\sum_{s=\tau^i_{k,v}}^{\tau^i_{k,v+1}-1}\Delta V(s)=V_a(\tau^i_{k,v+1})+(h-1)\sum_{i=1}^{N}\eta_i(\tau^i_{k,v+1})-V_a(\tau^i_{k,v})-(h-1)\sum_{i=1}^{N}\eta_i(\tau^i_{k,v})<0, \tag{21}$$

which implies

$$V_a(\tau^i_{k,v+1})+(h-1)\sum_{i=1}^{N}\eta_i(\tau^i_{k,v+1})<V_a(\tau^i_{k,v})+(h-1)\sum_{i=1}^{N}\eta_i(\tau^i_{k,v}). \tag{22}$$

Finally, similar to [38], there exists some $\delta<\infty$ satisfying $||\varepsilon(t)||\leqslant\delta||\varepsilon(\tau^i_{k,v})||$ for all $t\in\mathbb{N}_{[\tau^i_{k,v},\tau^i_{k,v+1}-1]}$ from the fact $||A^h+\underline{B}^h\underline{K}^h||<\infty$, where $\underline{B}^h:=[A^{h-1}B\ A^{h-2}B\ \cdots\ B]$ and $\underline{K}^h:=[K\ K\ \cdots\ K]$. Thus, we conclude that, on the basis of $V_a(t)>0$ and $\eta_i(\tau^i_{k,v})>0$, the errors of system (10) and $\eta_i(\tau^i_{k,v})$ converge to the origin under the triggering condition (9) and the feedback control law (6), which also implies that MASs (1) achieve asymptotic consensus. This completes the proof.

Remark 3 (DLF). Looped-functional approach proposed in [39,40] has been shown to yield less conservative stability results and recently been employed for sampled-data control of discrete-time systems [33]. We extend this method to address the multi-agent system consensus here. Compared to [33], a simper DLF that only contains sampled states of the agents is constructed in (25), whose aim is to reduce the matrices in the resulting consensus condition at the expense of the conservatism. It can be easily proven that $V_d(\tau^i_{k,v})=V_d(\tau^i_{k,v+1})=0$, asserting $V_d(t)$ is a DLF in [33]. Besides, obtaining a sampling-dependent condition (cf. Theorem 1) is another reason for introducing the DLF (25). An allowable sampling interval can be searched for using LMIs in (11), which is beneficial for designing sampling-based triggering schemes and feedback controllers.

Theorem 1 provides a stability condition for a given ETS. A design method for obtaining the distributed controllers and the event-triggering parameters, can be derived based on Theorem 1, while guaranteeing the consensus. To this end, an algebraically equivalent system to system (10) is given as follows by defining $\varepsilon_i(t)=G_iz_i(t)$:

$$z(t+1)=G^{-1}AGz(t)+G^{-1}BK_cz(t_k),\ t\in\mathbb{N}_{[t^i_k,t^i_{k+1}-1]}, \tag{23}$$

where $G_i\in\mathbb{R}^{n\times n}$ is a nonsingular matrix, $z(t):=[z_1^{\mathrm{T}}(t)\ \cdots\ z_N^{\mathrm{T}}(t)\ z_0^{\mathrm{T}}(t)]^{\mathrm{T}}$, $K_c:=KG$, and $G:=\mathrm{diag}\{G_i\}_{i=1}^N$. Imitating Theorem 1, the following theoretical result is proposed.

Theorem 2 (Model-based design). Consider the system (1) under the triggering condition (9) and the control law (6). Given the same scalars as in Theorem 1, there exists a block controller gain K such that asymptotic consensus of the system is achieved, and $\eta_i(\tau^i_{k,v})$ tends to zero for any $\eta_i(0)\geqslant 0$, if there exist matrices $R_1 \succ 0$, $R_2 \succ 0$, $P \succ 0$, $S = S^{\mathrm{T}}$, M_1, M_2, G, K_c, and $\bar{\Omega}_i \succ 0$ for all $i \in \mathbb{N}_{[0,N]}$, satisfying the following LMIs $\forall h \in \{\underline{h}, \bar{h}\}$:

$$\begin{bmatrix} \Xi_0 + h\Xi_\varsigma + \bar{\Psi} + \bar{\mathcal{Q}} & hM_\varsigma \\ * & -hR_\varsigma \end{bmatrix} \prec 0, \ \varsigma = 1, 2, \tag{24}$$

where

$$\bar{\Psi} := \mathrm{Sym}\big\{\mathcal{D}(AGH_1 + BK_cH_5 - GH_2)\big\}, \ \mathcal{D} := (H_1 + 2H_2)^{\mathrm{T}},$$
$$\bar{\mathcal{Q}} := H_5^{\mathrm{T}}\bar{\Omega}_a H_5 - \begin{bmatrix} H_3 \\ H_5 \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} \bar{\Omega}_b & -\bar{\Omega}_b \\ * & \bar{\Omega}_b \end{bmatrix} \begin{bmatrix} H_3 \\ H_5 \end{bmatrix},$$

and $\bar{\Omega}_a$ and $\bar{\Omega}_b$ are defined similar to Ω_a and Ω_b in Theorem 1 by replacing Ω_i with $\bar{\Omega}_i$. Moreover, the desired block controller and triggering matrices are co-designed as $K = K_cG^{-1}$, $\Omega_a = {G^{-1}}^{\mathrm{T}}\bar{\Omega}_aG^{-1}$, and $\Omega_b = {G^{-1}}^{\mathrm{T}}\bar{\Omega}_bG^{-1}$.

Proof. Choose the following functional for the system (23) by replacing ε in (12) with z:

$$\begin{aligned} V_z(t) = {} & z^{\mathrm{T}}(t)Pz(t) + (t - \tau^i_{k,v})(\tau^i_{k,v+1} - t)\big[z^{\mathrm{T}}(\tau^i_{k,v}), z^{\mathrm{T}}(\tau^i_{k,v+1})\big] S \big[z^{\mathrm{T}}(\tau^i_{k,v}), z^{\mathrm{T}}(\tau^i_{k,v+1})\big]^{\mathrm{T}} \\ & + (\tau^i_{k,v+1} - t)\left[\sum_{s=\tau^i_{k,v}}^{t} y_z^{\mathrm{T}}(s)R_1y_z(s) - y_z^{\mathrm{T}}(t)R_1y_z(t)\right] \\ & + (t - \tau^i_{k,v})\left[\sum_{s=t}^{\tau^i_{k,v+1}} y_z^{\mathrm{T}}(s)R_2y_z(s) - y_z^{\mathrm{T}}(t)R_2y_z(t)\right] \\ & + t\sum_{i=1}^{N}[\eta_i(\tau^i_{k,v+1}) - \eta_i(\tau^i_{k,v})], \end{aligned} \tag{25}$$

where $y_z(s) := z(s+1) - z(s)$.

Based on (19), the following inequality holds with $\varepsilon_i(t) = G_iz_i(t)$ and $\xi^{\mathrm{T}}(t)\mathcal{Q}\xi(t) = \xi_z^{\mathrm{T}}(t)\bar{\mathcal{Q}}\xi_z(t)$:

$$\sum_{i=1}^{N}[\eta_i(\tau^i_{k,v+1}) - \eta_i(\tau^i_{k,v})] \leqslant \xi_z^{\mathrm{T}}(t)\bar{\mathcal{Q}}\xi_z(t), \tag{26}$$

where $\xi_z(t) := [z^{\mathrm{T}}(t),\ z^{\mathrm{T}}(t+1),\ z^{\mathrm{T}}(\tau^i_{k,v}),\ z^{\mathrm{T}}(\tau^i_{k,v+1}),\ z^{\mathrm{T}}(t_k)]^{\mathrm{T}}$.

It can be deduced by imitating (20) that

$$\Delta V_z(t) \leqslant \xi_z^{\mathrm{T}}(t)\left[\frac{t-\tau_j}{h}\bar{\Upsilon}_1(h) + \frac{\tau_{j+1}-t}{h}\bar{\Upsilon}_2(h)\right]\xi_z(t), \tag{27}$$

where $\bar{\Upsilon}_\varsigma(h) := \Xi_0 + h\Xi_\varsigma + \bar{\Psi} + \bar{\mathcal{Q}} + hM_\varsigma R_\varsigma^{-1}M_\varsigma^{\mathrm{T}},\ \varsigma = 1, 2$. By Schur Complement Lemma, inequalities $\bar{\Upsilon}_1(h) \prec 0$ and $\bar{\Upsilon}_2(h) \prec 0$ are equivalent to the LMIs in (24). Similar to Theorem 1, MASs (1) achieve asymptotic consensus under the triggering condition (9) and the feedback control law (6), with the desired $K = K_cG^{-1}$, since system (23) exhibits the same dynamic behavior and stability properties as (10), which completes the proof.

3.2 Data-driven consensus and controller design

We are now ready to provide a data-driven solution for consensus and controller design of the system (1) with unknown matrix pair $[A\ B]$ under the triggering condition (9) and the feedback control law (6). The core idea, inspired by [35, 41], is to replace the matrix pair $[A\ B]$ in Theorem 2 with a data-driven system expression using the measurements $\{\{x_i(T)\}_{T=0}^{\rho}, \{u_i(T)\}_{T=0}^{\rho-1}\}$. Following this line, a data-based design method guaranteeing the consensus is obtained on the basis of Lemma 1 and Theorem 2.

Theorem 3 (Data-driven consensus and design)**.** Consider the system (1) under the triggering condition (9) and the control law (6). Given the same scalars as in Theorem 1, there exists a block controller gain K such that asymptotic consensus of the system is achieved for any $[A\ B] \in \Sigma_{AB}$, and $\eta_i(\tau^i_{k,v})$ tends to zero for any $\eta_i(0) \geqslant 0$, if there exist matrices $R_1 \succ 0$, $R_2 \succ 0$, $P \succ 0$, $S = S^{\mathrm{T}}$, M_1, M_2, G, K_c, and $\bar{\Omega}_i \succ 0$ for all $i \in \mathbb{N}_{[0,N]}$, satisfying LMIs $\forall h \in \{\underline{h}, \bar{h}\}$, $\varsigma = 1, 2$,

$$\begin{bmatrix} \mathcal{T}_1 & \mathcal{F} + \mathcal{T}_2 & 0 \\ * & \Xi_0 + h\Xi_\varsigma + \hat{\Psi} + \bar{\mathcal{Q}} + \mathcal{T}_3 & hM_\varsigma \\ * & * & -hR_\varsigma \end{bmatrix} \prec 0, \tag{28}$$

where

$$\hat{\Psi} := \mathrm{Sym}\{-\mathcal{D}GH_2\},\ \mathcal{F} := \left[H_1^{\mathrm{T}}G^{\mathrm{T}}, H_5^{\mathrm{T}}K_c^{\mathrm{T}}\right]^{\mathrm{T}},$$
$$\mathcal{D} := (H_1 + \epsilon H_2)^{\mathrm{T}},\ \mathcal{V}_1 := \begin{bmatrix} I & 0 \end{bmatrix},\ \mathcal{V}_2 := \begin{bmatrix} 0 & \mathcal{D} \end{bmatrix},$$
$$\mathcal{T}_1 := \mathcal{V}_1\Theta_{AB}\mathcal{V}_1^{\mathrm{T}},\ \mathcal{T}_2 := \mathcal{V}_1\Theta_{AB}\mathcal{V}_2^{\mathrm{T}},\ \mathcal{T}_3 := \mathcal{V}_2\Theta_{AB}\mathcal{V}_2^{\mathrm{T}}.$$

Moreover, $K = K_cG^{-1}$ is the desired block controller matrix, and the triggering matrices are co-designed as $\Omega_a = {G^{-1}}^{\mathrm{T}}\bar{\Omega}_aG^{-1}$, and $\Omega_b = {G^{-1}}^{\mathrm{T}}\bar{\Omega}_bG^{-1}$.

Proof. Restructure $\bar{\Upsilon}_\varsigma(h)$ in (27) of Theorem 2 as follows:

$$\bar{\Upsilon}_\varsigma(h) := \begin{bmatrix} [\mathcal{D}A\ \mathcal{D}B]^{\mathrm{T}} \\ I \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} 0 & \mathcal{F} \\ * & \Xi_0 + h\Xi_\varsigma + \hat{\Psi} + \bar{\mathcal{Q}} + hM_\varsigma R_\varsigma^{-1}M_\varsigma^{\mathrm{T}} \end{bmatrix} \begin{bmatrix} [\mathcal{D}A\ \mathcal{D}B]^{\mathrm{T}} \\ I \end{bmatrix}.$$

According to Lemma 1, it is met for any $[A\ B] \in \Sigma_{AB}$,

$$\begin{bmatrix} [A\ B]^{\mathrm{T}} \\ I \end{bmatrix}^{\mathrm{T}} \Theta_{AB} \begin{bmatrix} [A\ B]^{\mathrm{T}} \\ I \end{bmatrix} \succeq 0. \tag{29}$$

Then, the full-block S-procedure [42] ensures $\bar{\Upsilon}_i(h) \prec 0$ for any $[A\ B] \in \Sigma_{AB}$ if the following LMIs are satisfied:

$$\begin{bmatrix} 0 & \mathcal{F} \\ * & \Xi_0 + h\Xi_\varsigma + \hat{\Psi} + \bar{\mathcal{Q}} + hM_\varsigma R_\varsigma^{-1}M_\varsigma^{\mathrm{T}} \end{bmatrix} + \begin{bmatrix} \mathcal{V}_1\Theta_{AB}\mathcal{V}_1^{\mathrm{T}} & \mathcal{V}_1\Theta_{AB}\mathcal{V}_2^{\mathrm{T}} \\ * & \mathcal{V}_2\Theta_{AB}\mathcal{V}_2^{\mathrm{T}} \end{bmatrix} \prec 0. \tag{30}$$

Through Schur Complement Lemma, the inequalities in (30) are equivalent to the LMIs in (28). Subsequently, we can draw the same conclusion as Theorem 2 that MASs (1) achieve asymptotic consensus under the triggering condition (9) and the feedback control law (6), with the desired $K = K_cG^{-1}$, for any $[A\ B] \in \Sigma_{AB}$.

Remark 4 (Data-driven design algorithm)**.** Note that Theorem 3 is a sufficient condition for achieving consensus of system (1). Any conclusion cannot be reached if the LMIs in (28) are not solvable. Here, we summarize the data-driven design procedure, assuming that Theorem 3 contains feasible solutions.

Step 1. Collect offline data $\{\{x_i(T)\}_{T=0}^{\rho}, \{u_i(T)\}_{T=0}^{\rho-1}\}$ from all agents i, and construct the data matrices E^+, E, and U;

Step 2. Suppose that noise $\{w(T)\}_{t=0}^{\rho-1}$ is bounded as $\|w(T)\|_2 \leqslant \bar{w}$ ($\bar{w} > 0$) satisfying Assumption 1;

Step 3. Build matrix Q_d to form Θ_{AB} of Lemma 1;

Step 4. Choose proper parameters σ_0, σ_{ij}, λ_i, h, θ_i, $\eta_i(0)$, and search for feasible matrices K_c, G, and $\bar{\Omega}$ for (28). If the eigenvalue $\mathrm{eig}(G) \neq 0$, go to Step 5; otherwise, repeat Step 4;

Step 5. Compute the controller gain $K = K_cG^{-1}$, and the required triggering matrices $\Omega_a = {G^{-1}}^{\mathrm{T}}\bar{\Omega}_aG^{-1}$, $\Omega_b = {G^{-1}}^{\mathrm{T}}\bar{\Omega}_bG^{-1}$.

Remark 5 (Parameter optimization)**.** There are some parameters embedded in the triggering condition (9) and the stability criteria (see model-based in (24) and data-driven in (28)). The selection of the parameters may influence the system performance. Without losing generality, we here provide a method for optimizing the parameter θ_i while guaranteeing a desired system performance and minimizing the number of transmissions. At the beginning, two kinds of system performance indexes are defined as follows:

$$\mathcal{P}_i(x) = \frac{1}{x_i^{\mathrm{T}}(0)x_i(0)} \sum_{t=0}^{T} x_i^{\mathrm{T}}(t)x_i(t), \qquad \mathcal{R}_i = \frac{\text{Number of transmitted data}}{\text{Number of sampled data}},\ i \in \mathbb{N}_{[0,N]},$$

where $\mathcal{P}_i(x)$ quantities the cost of system performance (see also [18]), $T \in \mathbb{N}$ is a given simulation time, and $\mathcal{R}_i$ is the ratio of transmitted and sampled data for each agent. Then, the parameter optimization can be defined as searching for a solution θ_i solving

$$\begin{array}{ll} \text{minimize} & \mathcal{R}_i \\ \text{subject to} & \theta_i > 0,\ \mathcal{P}_i(x) \leqslant \bar{\mathcal{P}}_i(x),\ (24) \text{ or } (28), \end{array} \tag{31}$$

where $\bar{\mathcal{P}}_i(x)$ specifies a desired cost of system performance. Similarly, other parameters, such as σ_0, σ_{ij}, λ_i, h, θ_i, and $\eta_i(0)$, can be optimized.

3.3 Data-driven $\mathcal{H}_\infty$ consensus and controller design

This subsection deals with data-driven $\mathcal{H}_\infty$ consensus of MASs subject to disturbances, whose dynamics is given as follows:

$$x_i(t+1) = A_i x_i(t) + B_i u_i(t) + B_d^i d_i(t),\ t \in \mathbb{N}, \tag{32}$$

where $d_i(t) \in \mathbb{R}^{n_w}$ is the external disturbance and belongs to $\mathcal{L}_2[0,\infty]$; $B_d^i \in \mathbb{R}^{n \times n_d}$ is a known constant matrix describing the disturbance. We assume that model matrices A_i and B_i are unknown, but the state-input data $\{\{x_i(T)\}_{T=0}^{\rho}, \{u_i(T)\}_{T=0}^{\rho-1}\}$ collected offline are available from (3), and noise sequence $\{w(T)\}_{t=0}^{\rho-1}$ satisfies Assumption 1.

Under the feedback control law (6), system (32) can be reformed as the following error equation:

$$\varepsilon(t+1) = A\varepsilon(t) + BK\varepsilon(t_k) + B_d d(t),\ t \in \mathbb{N}_{[t_k^i, t_{k+1}^i - 1]}, \tag{33}$$

where $d(t) := [d_1^{\mathrm{T}}(t)\ \cdots\ d_N^{\mathrm{T}}(t)\ d_0^{\mathrm{T}}(t)]^{\mathrm{T}} \in \mathbb{R}^{(N+1)n_d}$ and

$$B_d := \left[\begin{array}{c|c} \mathrm{diag}\{B_d^i\}_{i=1}^N & \boldsymbol{I}_N \otimes (-B_d^0) \\ \hline 0 & B_d^0 \end{array}\right].$$

The definition of $\mathcal{H}_\infty$ stabilization for system (33) is given as follows.

Definition 1. Given a scalar $\gamma > 0$, the MASs (33) achieve $\mathcal{H}_\infty$ consensus with the disturbance attenuation γ if the following conditions hold.

(1) The error system (33) with the controller (6) is asymptotically stable with zero disturbance $d(t) = 0$;

(2) The following bounded $\mathcal{L}_2$-gain condition is satisfied under zero initial condition for all nonzero $d_i(t) \in \mathcal{L}_2[0,\infty]$:

$$\sum_{t=0}^{+\infty} \varepsilon^{\mathrm{T}}(t)\varepsilon(t) \leqslant \sum_{t=0}^{+\infty} \gamma^2 d^{\mathrm{T}}(t)d(t). \tag{34}$$

Then, based on Theorem 2, we provide a data-driven co-design method for event-triggered MASs with external disturbance, such that system (32) achieves consensus stability and $\mathcal{H}_\infty$ performance.

Theorem 4 (Data-driven $\mathcal{H}_\infty$ consensus and design)**.** Consider the system (32) under the triggering condition (9) and the control law (6). Given the same scalars as in Theorem 1, there exists a block controller gain K such that $\mathcal{H}_\infty$ consensus of the system is achieved with a given disturbance attenuation $\gamma > 0$ for any $[A\ B] \in \Sigma_{AB}$, and $\eta_i(\tau_{k,v}^i)$ tends to zero for any $\eta_i(0) \geqslant 0$, if there exist matrices $R_1 \succ 0$, $R_2 \succ 0$, $P \succ 0$, $S = S^{\mathrm{T}}$, M_1, M_2, G, K_c, and $\bar{\Omega}_i \succ 0$ for all $i \in \mathbb{N}_{[0,N]}$, satisfying LMIs $\forall h \in \{\underline{h}, \bar{h}\}$, $\varsigma = 1, 2$,

$$\begin{bmatrix} \mathcal{T}_1 & \mathcal{F} + \mathcal{T}_2 & 0 & 0 \\ * & \Xi_0 + h\Xi_\varsigma + \tilde{\Psi} + \bar{\mathcal{Q}} + \mathcal{T}_3 & hM_\varsigma & \mathcal{D}B_d G \\ * & * & -hR_\varsigma & 0 \\ * & * & * & -\gamma^2 G^{\mathrm{T}} G \end{bmatrix} \prec 0, \tag{35}$$

where $\tilde{\Psi} = \hat{\Psi} + H_1^{\mathrm{T}} G^{\mathrm{T}} G H_1$. Moreover, $K = K_c G^{-1}$ is the desired block controller matrix, and the triggering matrices are co-designed as $\Omega_a = {G^{-1}}^{\mathrm{T}} \bar{\Omega}_a G^{-1}$, and $\Omega_b = {G^{-1}}^{\mathrm{T}} \bar{\Omega}_b G^{-1}$.

Proof. One can observe that Eq. (35) ensures (28) of Theorem 3, which leads to condition (1) of Definition 1 with $d(t) = 0$. Now, we consider the case of $d(t) \neq 0$. The disturbance system model is written as

$$z(t+1) = G^{-1}AGz(t) + G^{-1}BK_c z(t_k) + G^{-1}B_d G d_z(t)$$

with defining $\varepsilon(t) := Gz(t)$ and $d(t) := Gd_z(t)$. Then, it follows from Schur Complement Lemma and Theorem 2 that LMIs in (35) imply

$$\Delta V_z(t) + z^{\mathrm{T}}(t) G^{\mathrm{T}} G z(t) - \gamma^2 d_z^{\mathrm{T}}(t) G^{\mathrm{T}} G d_z(t) < 0. \tag{36}$$

Summing (36) from $t = 0$ to $+\infty$ yields that

$$\sum_{t=0}^{+\infty} z^{\mathrm{T}}(t) G^{\mathrm{T}} G z(t) < \sum_{t=0}^{+\infty} \gamma^2 d_z^{\mathrm{T}}(t) G^{\mathrm{T}} G d_z(t) - \sum_{t=0}^{+\infty} \Delta V_z(t). \tag{37}$$

Finally, with the zero initial condition and $V_z(+\infty) > 0$, it holds that $\sum_{t=0}^{+\infty} x^{\mathrm{T}}(t)x(t) < \sum_{t=0}^{+\infty} \gamma^2 d^{\mathrm{T}}(t) d(t)$, which meets the condition (1) of Definition 1 since G is a nonsingular matrix. This completes the proof.

Remark 6. The distributed control strategy (6) and ETS (9) only require local information of the MASs, i.e., the agent's and its neighbors' sampled states. However, our design procedures (cf. Theorems 1–4) rely on the global information of the network graph, e.g., when constructing the data-driven MAS representation in Lemma 1. In this sense, the presented data-driven control protocols are not fully distributed, which may restrict their applications. How to avoid using global information in data-driven consensus control design motivates our future research.

Remark 7 (Uncertainty)**.** To capture uncertainties in the system matrices, let the unknown system matrices A and B belong to the convex hull of a set of unknown but fixed system matrices as follows:

$$[A\ B] \in \mathcal{C}o([A^1\ A^2], \ldots, [A^l\ B^l]) \subseteq \Sigma_{AB},\ l \in \mathbb{N}_0,$$

where $\mathcal{C}o$ denotes the convex hull operator. That is, the set $\mathcal{W}$ of noise in Assumption 1 covers the above set of uncertainties. In this situation, Theorems 1, 3, and 4 that hold for all $[A\ B] \in \Sigma_{AB}$ are also robust to such certainties. However, we cannot conclude anything while $\mathcal{C}o([A^1\ A^2], \ldots, [A^l\ B^l]) \subsetneq \Sigma_{AB}$. Our future work will be devoted to devising data-driven control of MASs containing general forms of uncertainties.

4 Example and simulation

A set of four mass-spring-damper systems [43] is employed in this section to examine the proposed data-driven event-triggered control method. All numerical computations are performed using Matlab, together with the SeDuMi toolbox [44].

Example 1. The system dynamics is given as $\dot{x}_i(c) = \bar{A}_i x_i(c) + \bar{B}_i u_i(c), t \geqslant 0$ with

$$\bar{A}_i = \begin{bmatrix} 0 & 1 \\ -\frac{f_i}{\phi_i} & -\frac{\varphi_i}{\phi_i} \end{bmatrix},\ \bar{B}_i = \begin{bmatrix} 0 \\ \frac{1}{\phi_i} \end{bmatrix},\ i \in \mathbb{N}_{[0,3]},$$

where the state vector $x_i(c)$ comprises the displacement and velocity of the mass; $u_i(c)$ is the input force; φ_i, ϕ_i, and f_i are the mass, damping constant, and spring constant, respectively. Subsystems' parameters $(f_i, \phi_i, \varphi_i)^i$ are $(1,1,2)^0$, $(1,1.1,2)^1$, $(1,1.2,2)^2$, and $(1,0.8,2)^3$, respectively. The interaction topology is given in Figure 2 for a pictorial description, where the leader is indexed by 0 and the followers by $1,2,3$. The adjacency matrix $\mathcal{C}$ describes the communication graph. Upon discretization, we arrive at the discrete-time linear system as in (1) with the matrices $A_i = e^{\bar{A}_i T_k}$ and $B_i = \int_0^{T_k} e^{\bar{A}_i s} \bar{B}_i \mathrm{d}s$, where $T_k > 0$ is the discretization interval. The proposed data-driven ETS (9) and distributed control law (6) are then applied to this system. The codesigned results are displayed in the following part.

Testing the data-driven method. The matrices A_i and B_i are treated as unknown for our data-driven controller design. We generate $\rho = 40$ state-input data $\{x(T)\}_{T=0}^{\rho}$ and $\{u(T)\}_{T=0}^{\rho-1}$ from the

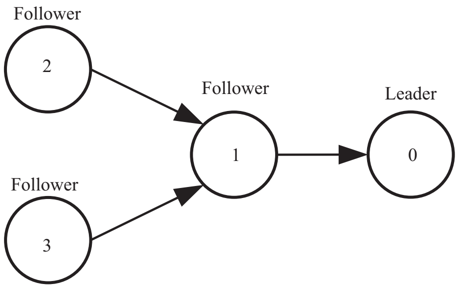

Figure 2 Communication graph.

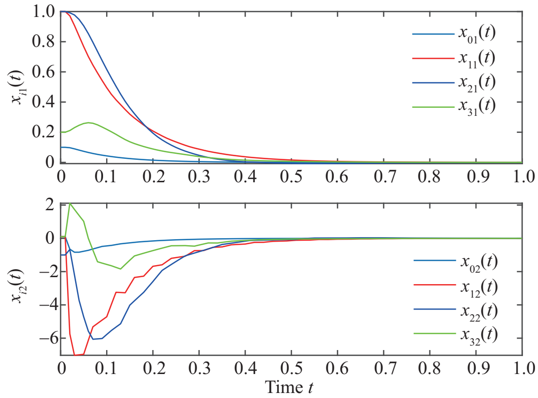

Figure 3 (Color online) Trajectories of agents under the data-driven ETS.

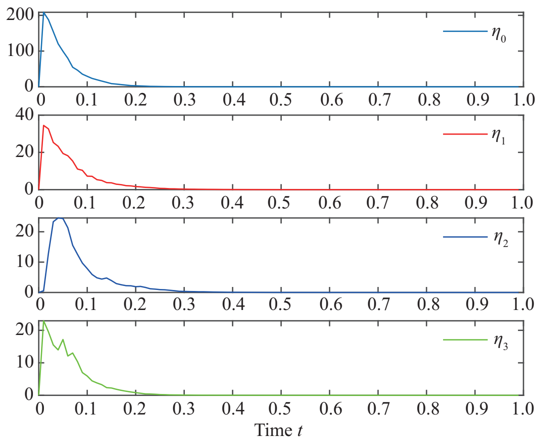

Figure 4 (Color online) Trajectories of η_i of each agent under the data-driven ETS.

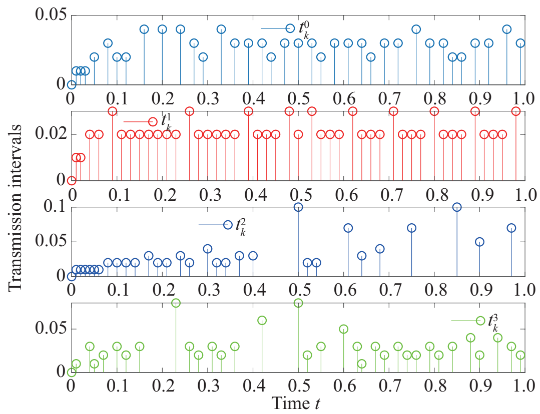

Figure 5 (Color online) Triggered instants of each agent.

disturbed system (3) by setting the discretization interval as $T_k = 0.01$, where the input is generated and sampled randomly from $u(t) \in [-1,\ 1]$. Moreover, the collected measurements are corrupted by a disturbance satisfying $\|w(T)\| \leqslant 0.001$, which fulfills Assumption 1 as in Remark 1. The matrix B_w is taken as $B_w = 0.01I$, which has full column rank. Furthermore, set the triggering-related parameters as $\sigma_0 = 0.02$, $\sigma_{10} = \sigma_{21} = \sigma_{31} = 0.05$, $\theta_i = 5$, and $\lambda_i = 0.2$, and the sampling interval is $h = 0.01$. Solving the data-based LMIs of Theorem 3 as in Remark 4, the distributed controller gains and the event-triggering matrices are given as

$$K_0 = [-683.75\ \ -71.79], K_{10} = [-719.00\ \ -93.34], K_{21} = [-233.74\ \ -35.53], K_{31} = [-203.98\ \ -23.03],$$

$$\Omega_0 = 10^5 \begin{bmatrix} 8.6465 & 0.8485 \\ 0.8485 & 0.0854 \end{bmatrix}, \ \Omega_{10} = \begin{bmatrix} 851.31 & 100.07 \\ 100.07 & 13.15 \end{bmatrix}, \ \Omega_{21} = \begin{bmatrix} 485.44 & 61.70 \\ 61.70 & 9.21 \end{bmatrix}, \ \Omega_{31} = \begin{bmatrix} 716.87 & 78.29 \\ 78.29 & 8.95 \end{bmatrix}.$$

The proposed dynamic triggering scheme (9) is numerically tested with the initial conditions $x_0(0) = [0.1\ 0.1]^{\mathrm{T}}$, $x_1(0) = [1\ 0.1]^{\mathrm{T}}$, $x_2(0) = [1\ \ -1]^{\mathrm{T}}$, and $x_3(0) = [0.2\ \ -0.1]^{\mathrm{T}}$ over a time interval of $[0, 1]$. The trajectories of all agents and the dynamic variables η_i are shown in Figures 3 and 4, respectively. Obviously, all followers approach the trajectory of the leader asymptotically, and the dynamic variables converge to zero, demonstrating the correctness of the proposed distributed data-driven triggering and control schemes. Notably, in Figure 5, only 37 out of 100 measurements for leader 0, and 46, 31, and 34 for followers 1–3, respectively, are broadcast to distributed controllers and their neighbors, while 100 samples of data are collected for each subsystem. This result proves that the proposed data-driven ETS helps reduce transmissions when achieving the consensus of MASs.

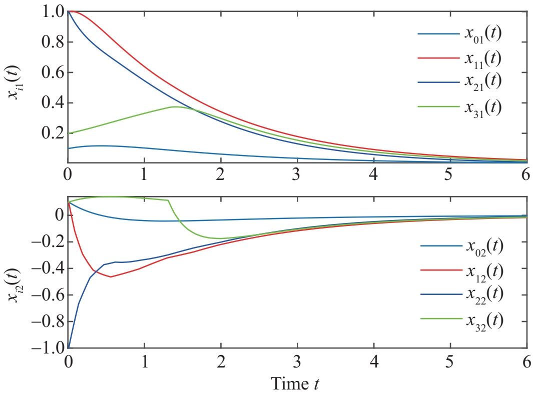

Figure 6 (Color online) Trajectories of agents under the model-based ETS.

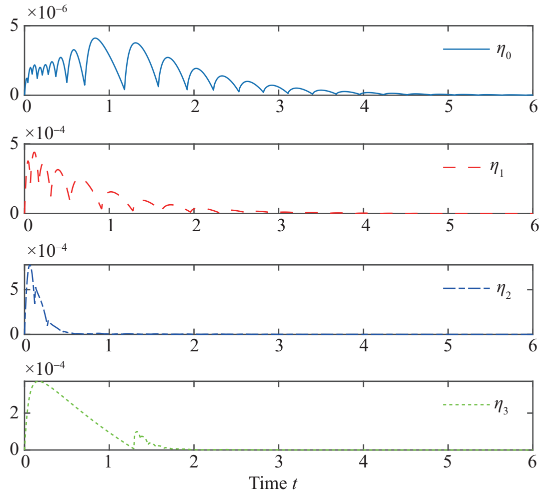

Figure 7 (Color online) Trajectories of η_i of each agent under the model-based ETS.

Table 1 Cost of system performance $\mathcal{P}_i(x)$ and ratio of transmissions $\mathcal{R}_i$ under the data-driven ETS for different θ_i over $t \in [0, 1]$

Agent		θ_i										
		2	3	4	5	10	20	50	500	1000	10^4	10^5
Leader 0	$\mathcal{P}_0(x) \cdot 10^{-3}$	0.36	0.36	0.36	0.37	0.38	0.38	0.45	0.52	0.52	0.52	0.52
	$\mathcal{R}_0$	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
Follower 1	$\mathcal{P}_1(x) \cdot 10^{-3}$	0.46	0.46	0.46	0.46	0.45	0.45	0.46	0.45	0.45	0.45	0.45
	$\mathcal{R}_1$	0.41	0.41	0.41	0.41	0.41	0.41	0.41	0.41	0.41	0.41	0.41
Follower 2	$\mathcal{P}_2(x) \cdot 10^{-3}$	0.29	0.32	0.31	0.31	0.32	0.39	0.38	0.32	0.39	0.39	0.39
	$\mathcal{R}_2$	0.21	0.20	0.21	0.21	0.20	0.20	0.20	0.20	0.20	0.20	0.20
Follower 3	$\mathcal{P}_3(x) \cdot 10^{-3}$	0.32	0.30	0.28	0.34	0.29	0.30	0.27	0.31	0.34	0.34	0.34
	$\mathcal{R}_3$	0.80	0.78	0.78	0.77	0.82	0.78	0.80	0.72	0.71	0.71	0.71
Total	$\sum_{i=0}^{3} \mathcal{P}_i(x) \cdot 10^{-3}$	1.43	1.44	1.41	1.48	1.44	1.52	1.56	1.60	1.70	1.70	1.70
	$\sum_{i=0}^{3} \mathcal{R}_i$	1.67	1.64	1.65	1.64	1.70	1.66	1.67	1.59	1.59	1.59	1.59

Comparison with the model-based method. Assume that the system matrices are known. We compute the controller gains and triggering matrices as follows, by Theorem 2 and using the same parameters as in the data-driven case,

$$K_0 = [-0.14 \ \ -0.50], \ K_{10} = [-2.28 \ \ -3.69], \ K_{21} = [-1.14 \ \ -2.15], \ K_{31} = [-0.69 \ \ -1.31],$$

$$\Omega_0 = \begin{bmatrix} 0.0028 & -0.0012 \\ -0.0012 & 0.0036 \end{bmatrix}, \Omega_{10} = \begin{bmatrix} 0.0041 & 0.0031 \\ 0.0031 & 0.0052 \end{bmatrix}, \Omega_{21} = \begin{bmatrix} 0.0025 & 0.0022 \\ 0.0022 & 0.0042 \end{bmatrix}, \Omega_{31} = \begin{bmatrix} 0.0025 & 0.0022 \\ 0.0022 & 0.0041 \end{bmatrix}.$$

The state trajectories of the MAS are depicted in Figure 6 under the same initial condition as for plotting Figure 3, and the evolution of each η_i is shown in Figure 7. The leader/follower consensus problem is also solved using the model-based method (cf. Theorem 2). Moreover, compared to Figure 3, where the consensus settling time is before $t = 1$, the steady-state instant of Figure 6 is near $t = 4$. The main reason is that Theorem 2 has less conservatism than Theorem 3 (with an introduced disturbance) at the expense of losing system performance.

Simulations for different parameter θ_i values. This part centers on searching for the value of θ_i that minimizes the number of transmitted data while maintaining the desired system performance. According to the parameter optimization method in Remark 5, the simulation results, including the cost of system performance $\mathcal{P}_i(x)$ and the ratio of transmissions $\mathcal{R}_i$ under data-driven and model-based ETSs with different θ_i (and other abovementioned parameters) values, are listed in Tables 1 and 2, respectively. From Table 1, the minimum sum of agents' transmission ratios is $\sum_{i=0}^{3} \mathcal{P}_i(x) = 1.41$ when $\theta_i = 4$ for the data-driven case. Meanwhile, under the model-based ETS (cf. Table 2), if $\theta_i = 2$, the minimum $\sum_{i=0}^{3} \mathcal{P}_i(x)$ is 1.17. Obviously, the system performance index $\mathcal{R}_i$ of each agent remains at the same level

Table 2 Cost of system performance $\mathcal{P}_i(x)$ and ratio of transmissions $\mathcal{R}_i$ under the model-based ETS for different θ_i over $t \in [0, 6]$

Agent		θ_i										
		2	3	4	5	10	20	50	500	1000	10^4	10^5
Leader 0	$\mathcal{P}_0(x) \cdot 10^{-3}$	0.25	0.25	0.25	0.26	0.26	0.27	0.27	0.28	0.28	0.28	0.28
	$\mathcal{R}_0$	0.13	0.13	0.13	0.13	0.13	0.13	0.13	0.13	0.13	0.13	0.13
Follower 1	$\mathcal{P}_1(x) \cdot 10^{-3}$	0.21	0.21	0.21	0.21	0.22	0.22	0.23	0.23	0.23	0.23	0.23
	$\mathcal{R}_1$	0.13	0.13	0.13	0.13	0.13	0.13	0.13	0.13	0.13	0.13	0.13
Follower 2	$\mathcal{P}_2(x) \cdot 10^{-3}$	0.34	0.33	0.35	0.36	0.34	0.36	0.38	0.37	0.37	0.37	0.37
	$\mathcal{R}_2$	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
Follower 3	$\mathcal{P}_3(x) \cdot 10^{-3}$	0.37	0.39	0.40	0.40	0.42	0.40	0.41	0.42	0.42	0.42	0.42
	$\mathcal{R}_3$	0.64	0.64	0.64	0.64	0.63	0.64	0.63	0.63	0.63	0.63	0.63
Total	$\sum_{i=0}^{3} \mathcal{P}_i(x) \cdot 10^{-3}$	1.17	1.18	1.21	1.23	1.24	1.25	1.29	1.30	1.30	1.30	1.30
	$\sum_{i=0}^{3} \mathcal{R}_i$	0.97	0.96	0.96	0.96	0.96	0.96	0.95	0.96	0.96	0.96	0.96

even for different θ_i values. Another finding from these tables is that as θ_i increases from 2 to 10^5, the ratio $\mathcal{P}_i(x)$ as well as the index $\mathcal{R}_i$ tends to a constant value because our triggering scheme (9) degenerates to a static ETS if $\theta_i \to \infty$.

5 Concluding remarks

This paper considered the distributed event-triggered consensus control of leader/follower MASs from a data-driven perspective. The asymptotic consensus of the MASs under the proposed dynamic periodic distributed ETS was analyzed leveraging a novel looped-functional, allowing a model-based method for obtaining the distributed controller and ETS matrices. Combining the data-based leader/follower MAS representation and the model-based condition, a data-driven codesign approach was provided and extended to the case of achieving an $\mathcal{H}_\infty$ performance. Finally, a practical example was provided to corroborate the efficacy of the proposed ETS in reducing transmissions as well as the validity of our model- and data-driven codesigning methods. Our future work will address understanding the relationship between the performance and the noise-corrupted data-driven controller design.

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